

CUMCM-2002 Problems

(Note: University-level teams can choose any one from problems A and B; College-level teams can choose any one from problems C and D)

Problem A: Optimal Design for Light Source of Headlights

The shape of headlights is a rotational paraboloid, the symmetric axis of headlights horizontally face to the straight forward, its open radius is 36mm and deep 21.6mm. Through its focal point, symmetrically put a linear light source with uniform distribution. Please determine the length of the light source under some rule.

After simplifying the rule, describe it as follows. Put a test-screen on a point A which at straight front 25 meters of the focal point F, which is vertical to FA and is used to test the reflect light of headlights. On the screen, draw a straight line through point A, which parallels to the earth. Take points B and C at the same side of A in the line such that $AC+2AB=2.6\text{m}$. Demand that the light strongness of point C is not less than a fixed value (take 1 unit), that of point B is not less than double of the value (just consider a reflect).

Please solve the following problems:

- (1) Under the condition of satisfying the rule, compute the length of light source with minimal power.
- (2) For the getable length of light source, draw the brightness of reflect light at coordinate system with staff.
- (3) Discuss the reasonableness of the design rule.

(Problem A is proposed by Prof. Tan Yong-ji from Fudan University and Prof. Yu Wen-ci from East China University of Science and Technology)

Problem B: Mathematics in Lottery Ticket

Recently “Lottery Hurricane” has swept across China, huge amount of money tempt more and more people to become “lottery men”. At present the popular lottery has two classes, “tradition” and “over-joy”.

The “tradition” adopts a scheme named “take 6+1 from 10” : first, wave out 6 basic numbers from 6 classes of ball with number 0~9, pick out one from each class. Then, wave and pick out a special number from balls with number 0~4 to make up of the prize number. Throwing man arbitrarily pick out 6 basic numbers (allow repeat) from number 0~9, and pick out a special number from 0~4 to consist a stakes. Determine the prize grade according to numbers and order of single stakes number tally with prize number. Use the prize number “abcdef+g” as an example to explain prize grade, see Table 1 (X denotes the number which is not pitched on).

Table 1

Prize Grade	Pick out 6+1 from 10 (6+1/10)	Explanation
	Basic Number	Special Number
1 st Prize	abcdef	g
2 nd Prize	abcdef	
3 rd Prize	abcdeX Xbcdef	
4 th prize	abcdXX XbcdeX XXcdef	
5 th prize	abcXXX XbcdXX XXcdeX XXXdef	
6 th prize	abXXXX XbcXXX XXcdXX XXXdeX XXXXef	

The “over-joy” has many different forms such as the scheme called “pick out 7 from 33”: first, wave out 7 basic numbers from balls with number 01~33 one by one. Then, pick out a special number from the rest 26 numbers. Throwing man arbitrarily pick out 7 numbers from numbers 01~33 (do not allow repetitions) to consist a stake. Determine the prize grade according to numbers of single stakes number tally with prize number, and not consider the given order. Another example is the scheme “pick out 6+1 from 36”: first, wave out 7 basic numbers from balls with number 01~33 one by one. Then, pick out a special number from the rest 26 numbers. Throwing man arbitrarily pick out 7 numbers from numbers 01~33 (do not allow repetitions) to consist a stakes. Determine the prize grade according to numbers of single stakes number tally with prize number, and not consider the given order. Please refer to Table 2 for the prize grades of these two schemes.

Table 2

Prize Grade	Pick out 7 from 33 (7/33)			Pick out 6+1 from 36 (6+1/36)		
	Basic Number	Special Number	Note	Basic Number	Special Number	Note
1 st Prize	●●●●●●●		Win 7 of 7	●●●●●●● ★		Win 6+1 of 7
2 nd Prize	●●●●●●○ ★		Win 6+1 of 7	●●●●●●●		Win 6 of 7
3 rd Prize	●●●●●●○		Win 6 of 7	●●●●●●○ ★		Win 5+1 of 7
4 th prize	●●●●●○○ ★		Win 5+1 of 7	●●●●●○○		Win 5 of 7
5 th prize	●●●●●○○		Win 5 of 7	●●●●○○○ ★		Win 4+1 of 7
6 th prize	●●●●○○○ ★		Win 4+1 of 7	●●●●○○○		Win 4 of 7
7 th prize	●●●●○○○		Win 4 of 7	●●●○○○○ ★		Win 3+1 of 7

Notes:

- Basic numbers you win; ★ Special numbers you win; ○ The numbers you do not win.

The proportion of total prize of the above two types is usually 50% of total sales. A single stake is 2 yuan. If a single stake wins high grade prize, low grade prize is wined no longer. Now the familiar sales rule and corresponding prize setting are referred to Table 3, where the high prize grades include first, second, 3rd prize and the behind are low grades. The amount of low grades are fixed and the high prizes are assigned in proportion. But the lowest amount of first prize for a single stake is 60,000 yuan and the highest is 500,000 yuan, the amount of high prize is given by:

[(The present total sales multiplied by the proportion of total prize) minus the total of low prize] multiplied by the proportion of single prize

- (1) According to these plans, synthetically analyze the factors including the appearance possibility of diversified prize grade, the setting of prize grade and prize amount and charm for lottery man, value the rationality of each plan.
- (2) Develop a better plan and corresponding algorithm and then give advice to the lottery administration.
- (3) Write a memo to newspaper to give lottery man a reference.

(Problem B is proposed by Prof. Han Zhong-geng from PLA University of Information Engineering)

Problem C

(Same as Problem A)

Problem D: Match Schedule Arrangement

Your grade has five classes, each class has a ball team to hold on a single cycle match at a same court and hold 10 matches in all. How to arrange the schedule such that it is fair for each class? The following is a schedule arranged casually: Denote 5 teams A, B, C, D, E, respectively. Casually write 1, 2, ... 10 in 10 blank spaces at right-triu of left-half part of the below table, then obtain a schedule. That is, A vs. B in the first match, B vs. C in the second match, ... C vs. E in the 10th match. For convenience, write these numbers in the left-tril symmetrically to the diagonal.

How about the fairness of the schedule? It might as well to see weather the rest time for each class between two matches is equal. The right-half part of table is interval-match numbers in between matches. Obviously, the schedule is benefit for A and E and unfair for D.

According to the above example, discuss the following problems:

- 1) For a match among 5 teams, arrange a schedule that each team has at least a interval-number match in between matches.
- 2) When the nth team is holding match, compute the maximum interval numbers in between matches for each class.
- 3) Under the condition of reaching the maximum given in 2), give arrangements for n=8 and n=9, and explain them.
- 4) Except the index that the interval match in between matches, what you can give to scale a schedule is good or bad. Estimate the schedule given in 3) according to the index.

(Problem D is proposed by Prof. Jiang Qi-yuan from Tsinghua University)