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ORIGINAL RESEARCH

Trade-in strategy for durable products in the presence of a peer-to-peer second-hand marketplace

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Abstract
An infinite-period model is established to examine a firm's long-term trade-in strategy in the presence of a P2P (peer-to-peer) second-hand marketplace. The firm can choose whether to adopt a trade-in strategy and determines the prices of new products with and without trade-ins. In addition to purchasing new products with or without trade-ins, consumers can trade used products on the P2P marketplace. We study a benchmark scenario in which the transaction fee rate of the marketplace is exogenously given and extend it to a scenario where the transaction fee rate is endogenously determined by the marketplace (called the "former scenario" and "latter scenario" for short). The main results are as follows: (1) There is a threshold for the transaction fee rate under which the marketplace exists. In the benchmark scenario, the firm will not adopt the trade-in strategy when product durability and production costs are both high. In the scenario with an endogenous transaction fee rate, the firm can always benefit from adopting the trade-in strategy since the marketplace adjusts the transaction fee rate to make it acceptable to the firm. In addition, the transaction fee rate decreases in product durability and production costs. (2) We compare the former scenario with the latter scenario and find if product durability and production costs are both high, the firm can obtain a higher profit and set a higher discount for trade-in consumers in the latter scenario; otherwise, the firm can obtain a higher profit and set a higher discount in the former scenario.

Keywords Durable product · Trade-in · Second-hand market · Transaction fee rate · Stationary equilibrium

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1 Introduction

Durable products are defined as products that can yield consumption or productive services over multiple periods (Rust, 1985). The market for durable products is very large. For example, total vehicle sales in the United States increased to 18.50 million in April of 2021 (tradingeconomics.com). However, the pricing of durable products remains a subtle task (Alev et al., 2020; Huang et al., 2001). One of the reasons for this is that, unlike perishable products, used durable products can act as substitutes for new products.

As an effective means of achieving environmental compliance (Li et al., 2019), adopting a trade-in strategy has become common practice in the sales of some durable products. For example, more than 271,400 tons of hardware were recycled through HP's trade-in program, and this number increased quickly in recent years (Guo et al., 2022). In general, a trade-in strategy allows consumers to return used products of the same brand to firms within a certain period and obtain a discount for purchasing new products. Previous studies have found that firms that adopt a trade-in strategy can benefit from new product introduction cycles that are shorter than the useful life of durable goods and price discrimination between different consumers (Cao et al., 2018a; Chen & Hsu, 2017; De, 2017; Ray et al., 2011; Zhu et al., 2016). Many durable product manufacturers employ a trade-in strategy, such as Apple, Dell, and Fujitsu (Dong et al., 2022), while many brands of computers, TVs, and vehicles do not feature a trade-in strategy.

With the rapid technological development and ubiquitous availability of internet-connected devices, many P2P (peer-to-peer) second-hand marketplaces have arisen, such as PaiPai and eBay. In 2020, eBay's gross merchandise volume was greater than $100 billion (www.marketplacepulse.com). P2P trading marketplaces allow enterprises or individuals to sell second-hand goods on the marketplace and charge transaction fees in proportion to transaction prices. Thus, P2P marketplaces provide an additional way to dispose of used products for both firms and consumers. In addition, using eBay as an example, the transaction fee rates of different kinds of goods sold on its marketplace are different (www.ebay.com).

The problem of how P2P marketplaces set transaction fee rates has been studied in several papers (Chi et al., 2022; Jiang et al., 2017; Mantin et al., 2014). However, few studies have examined the impact of P2P second-hand marketplaces on firms' trade-in strategy, especially when P2P second-hand marketplaces endogenously set transaction fee rates. In this paper, an infinite-period model involving a monopolistic durable product manufacturing firm, a P2P second-hand marketplace, and a fixed number of potential consumers is established. The firm can choose whether to adopt the trade-in strategy and sets the prices of new products with and without trade-ins. Consumers can purchase new products with or without trade-ins and trade used products on the P2P marketplace. The transaction fee rates of the P2P marketplace are exogenously given or endogenously determined by the P2P marketplace. We hope to answer the following questions:

(1) When the transaction fee rate is endogenously determined by the P2P marketplace, how does the P2P marketplace choose the transaction fee rates for different kinds of products?
(2) How do the firm's trade-in strategy and the P2P marketplace's choice of transaction fee rate interact with each other?
(3) Is it always beneficial for the firm to adopt the trade-in strategy? Is there any qualitative difference between the scenarios where the transaction fee rate is exogenously given and endogenously determined?

To the best of our knowledge, this paper is the first to explore the interaction between trade-in strategy choice for a manufacturing firm and the P2P second-hand marketplace's choice of transaction fee rate. We obtain novel insights from studying this interaction: (1) When the transaction fee rate is endogenously determined by the P2P marketplace, the firm always adopts trade-in strategy because the P2P marketplace adjusts the transaction fee rate to guarantee the firm's adoption of the trade-in strategy. The transaction fee rate decreases in product durability and production costs. (2) Compared to the scenario with an exogenous transaction fee rate, when the transaction fee rate is endogenously determined, the firm's profit becomes less sensitive to a change in production costs, while the trade-in incentive becomes more sensitive to the change of the durability and production costs. (3) For the comparison of the former scenario and the latter scenario, if the product durability and production costs are both high, the firm could obtain a higher profit and set a higher discount for trade-in consumers in the latter scenario; otherwise, the firm could obtain a higher profit and set a higher discount in the former scenario.

The remainder of this paper is organized as follows: Sect. 2 reviews the most relevant literature. Section 3 presents the basic assumptions and describes the model framework. Section 4 analyzes and compares the two scenarios where the transaction fee rate is exogenously given or endogenously decided by the P2P marketplace. Section 5 concludes this paper and indicates future research directions. The proofs of the lemmas and propositions are collected in the Appendices.

2 Literature review

The studies relevant to our work mainly concern the transaction fee rate decisions of P2P marketplaces, the dynamic pricing of durable products in infinite periods, and trade-in strategy in infinite and finite periods. In the following, we review these three streams of literature.

This paper relates to the research on P2P marketplaces' decision-making regarding the transaction fee rate. Ryan et al. (2012) studied the optimal decisions for a marketplace and a retailer when the retailer wants to sell in the marketplace but its products compete with products sold by the marketplace. They showed that the marketplace's optimal transaction fee would decrease in the retailer's marginal sales cost and the percentage of consumers who are not aware of the retailer's direct selling channel. Mantin et al. (2014) studied the strategic rationale for a retailer to introduce a third-party marketplace in a supply chain. They showed that the manufacturer should prevent the third-party marketplace when its bargaining power is large enough. The transaction fee rate of the third-party marketplace increases in the retailer's bargaining power. Jiang et al. (2017) studied the P2P marketplace's influence on traditional supply chains when consumers have valuation uncertainty. They found that the transaction fee of the marketplace would equal the market clearing price when the product cost is low while the transaction fee would be less than the market clearing price when the product cost is high. Feng et al. (2019) explored the firm's strategic decision when anticipating the trade of used products among consumers through the marketplace. They demonstrated that the marketplace's transaction fee rate can increase total production quantity. Choi and He (2019) studied the influence of the P2P marketplace on fashion products. They showed that the revenue-sharing scheme is better than the fixed service charging scheme for the marketplace. Li and Xiao (2019) compared two supply cases for sellers on a P2P marketplace: the supply shortage case and the supply surplus case. They found that when the marketplace sets a proper transaction fee rate, both the marketplace and the sellers can benefit from the supply surplus case. Chi et al. (2022) studied P2P marketplaces' choice between unilateral transaction fee rates and bilateral transaction fee rates. They showed that bilateral transaction fee rates are always better for marketplaces. Different from the above studies, we consider the impact of a P2P second-hand marketplace on a firm's trade-in strategy, especially when the P2P second-hand marketplace endogenously sets the transaction fee rate.

Our research is based on studies in the field of dynamic pricing of durable products in infinite periods. Rust (1985) first considered the problem of durable products with a second-hand market, and obtained the condition for stationary equilibrium in an infinite-period game. Konishi and Sandfort (2002) further considered the same problem when the second-hand market has a fixed transaction cost and there are different kinds of durable products. Their research laid the foundation for subsequent related work. We also refer to their research and apply the concept of stationary equilibrium. Furthermore, Huang et al. (2001) considered the leasing strategy for durable products in infinite periods. They assumed that each durable product lasts for two periods and found that the firm's strategy is a mixture of leasing and selling if the transaction fee rate is lower than a critical value; otherwise, it tends to be pure selling. Our work adopts a similar modeling method to describe product durability and consumers' valuation of the product.

Several papers have studied firms' long-term trade-in strategy by establishing an infinite-period model. However, most of them do not consider the influence of P2P marketplaces on trade-in strategy. For example, Agrawal et al. (2008) studied a firm's trade-in strategy when facing a third-party manufacturer recycling and remanufacturing used products and showed that the firm should always use a trade-in strategy to compete with the third-party manufacturer. Li and Xu (2015) compared trade-in and leasing strategies and showed that the trade-in strategy can help to protect the firm's profit from residual value risk caused by the stochastic innovation process. Chen and Hsu (2017) studied a model where a firm builds its own second-hand market and only sells high-quality used products. They found that this strategy expands the potential market. The main differences between our work and these studies are as follows: First, we consider the transaction fee rate in the P2P marketplace to be exogenous and discuss its long-term influence on both consumers' decisions and the firm's pricing strategy; second, inspired by some real-life examples, we consider the case where the transaction fee rate is endogenously decided by the P2P marketplace and characterize how the P2P marketplace determines this rate.

Regarding trade-in strategy in finite periods, although there is considerable well-established research in this field, most of these works to not consider P2P marketplaces. For example, Ray et al. (2011) studied trade-in strategy under the assumption that consumers are divided into new customers and used product owners in fixed proportion and determined the best pricing strategy for the firm under a particular market condition. Miao et al. (2018) discussed the impact of carbon emission policies on trade-in strategy and showed that carbon regulations reduce the demand for new products and improve the sales of used products. Zhu et al. (2016) studied the impact of one party's trade-in strategy in duopoly competition and demonstrated that trade-in strategy can generate a competitive advantage from market share and profits. Miao et al. (2017) discussed who should provide the trade-in service and the corresponding pricing problem in a supply chain and found that it is better to let the retailer provide the service when considering environmental performance. Ma et al. (2017) studied the pricing strategy for the coexistence of the two modes of trading old for new and trading old for remanufactured and found that a mixed strategy is not necessary. Cao et al. (2018a) studied trade-in strategy in a dual online and offline channel setting and showed that the optimal strategy is either offline channel only, online channel only, or dual-channel according to consumers' shipping cost. Cao et al. (2018b) studied the problem of whether a firm should authorize a third party to collect its used products and demonstrated that authorization can be beneficial when product durability is high. Cao et al. (2018c) studied trade-in strategy for marketplaces that sell both proprietary and third-party products. They found that the firm cannot benefit from using gift cards as a trade-in rebate when the product redemption rate is high. Bian et al. (2019) studied the strategy of providing traditional limited-period guarantee services for products or limited-period replacement services with additional charges and found that the latter is more profitable when the handling cost of used products is low. Li et al. (2019) studied the problem of whether recycled products should all be remanufactured and then sold. They showed that the manufacturer prefers to remanufacture all rather than fraction of used products in the dynamic pricing case. Xiao and Zhou (2020) explored the firm's hybrid tade-in strategy which allows the consumers to trade-in-for-cash or trade-in-for-upgrade. They showed that it is optimal to provide the trade-in-for-upgrade progrma only in the early selling periods. Feng et al. (2020) explored how the coexistence of a secondary market and trade-in program affects the firm's decisions on quality choice and pricing. They demonstrated that the firm increases product quality if the demand in the secondary market is high. Fan et al. (2022) considered the interactions between trade-in delegation and channel structure in a supply chain. They showed that trade-in delegation always benefits the manufacturer, while it may harm the retailer. Guo et al. (2022) considered a model where the firm adopts the trade-old-for-new strategy while the third-party collector adopts the trade-old-for-cash strategy. They identified the optimal pricing decisions of the firm. One limitation of the finite-period model is that after the final period, the firm stops selling or recycling, which often changes the consumers' choices in the final period. In some cases, the changes can be considerable. Different from their studies, to focus on long-term trade-in strategy, this paper chooses an infinite-period model to avoid such changes.

Some studies consider both trade-in strategy and P2P marketplaces, most of which ignore the transaction fee in the P2P marketplace. Among these works, Rao et al. (2009) first considered trade-in strategy in infinite periods when a P2P second-hand marketplace exists. They assumed that the product deteriorates stochastically after one period while ignoring the transaction fee in the second-hand market. They found that the firm can always benefit from adopting a trade-in strategy and concluded that a trade-in strategy can also improve the quality of products traded in the second-hand market. Chen and Hsu (2015) further studied the impact of recovery costs on trade-in decisions and demonstrated that the rebate of trade-in-to-high (only accepting high-quality used products for trade-in) options increases in the product deterioration rate but decreases in the recovery costs. Dong et al. (2022) compared the buyback and trade-in strategies in a finite-period model. They showed that the existence of a P2P marketplace decreases the advantages of trade-in strategy. Different from their work, our contribution addresses the transaction fee rate of the P2P marketplace and examines its impact on the trade-in strategy of the firm. Additionally, Vedantam et al. (2021) studied the firm's choice between adopting trade-in strategy or establishing its own P2P marketplace from the perspectives of profits and environmental protection. When the firm establishes its own P2P marketplace, the choice of transaction fee rate is also considered. They found that both strategies may be optimal due to product characteristics and durability. However, they did not consider a combination of the two strategies. Our research focuses on the interaction between the firm's trade-in strategy and the third-party P2P marketplace.

Different from previous research, this paper strives to understand the P2P marketplace's choice of transaction fee rate and its influence on the firm's long-term trade-in strategy. Some new findings that are novel to the literature are obtained: (1) In previous research on firms' trade-in strategy in infinite periods (Rao et al., 2009; Chen & Hsu, 2015) , the trade-in strategy is always adopted. In contrast, we find there is a threshold for the transaction fee rate under which the trade-in strategy is adopted. The trade-in strategy is not used for sale of products whose durability and production cost are both high if the P2P marketplace has a common fixed transaction fee rate. (2) In previous research on firms' trade-in strategy in finite periods (Cao et al., 2018a; Ray et al., 2011) , the proportion of trade-in consumers can be close to 0 in a certain parameter range (for example, when product durability is high). However, we find that this proportion is always higher than 0.3 when the transaction fee rate is endogenously determined by the P2P marketplace. Moreover, new findings are also obtained from the comparison of a benchmark scenario with an exogenous transaction fee rate and a scenario with an endogenous transaction fee rate, which have not been discussed in the literature. (1) If product durability and production costs are both high, the firm can obtain a higher profit and set a higher discount for trade-in consumers in the former scenario; otherwise, the firm can obtain a higher profit and set a higher discount in the latter scenario. (2) In the latter scenario, the firm's profit becomes less sensitive to changes in production costs, while the trade-in incentive becomes more sensitive to a change in durability and production costs.

3 Model framework

3.1 Basic assumptions

Our basic model framework involves a monopolistic manufacturer ("firm" for short) producing and selling a durable product to end customers. The firm produces and sells the product in each period. Denote by c the unit production cost. Each period, the firm chooses the sales price of a new product with or without a trade-in (i.e., the price for consumers who return the used product and that for those who do not return the used product. Note that the situation where the firm does not adopt a trade-in strategy is considered because when the sales price of a new product with a trade-in is high enough, no consumer will choose trade-in.). Denote by p_n^t and p_r^t the sales prices of a new product without trade-in and with trade-in in period t, respectively. The value of the product deteriorates with use time and lasts for two periods. For simplicity, we term the product in its first period a "new product" and the product in its second period a "used product".

A fixed number of potential consumers are utility-maximizing and infinitely lived. They are heterogeneous in their valuation of the product. Denote by θ consumers' valuation of a new product. For tractability, suppose that θ follows a uniform distribution on the interval $[0, 1]$. Each consumer needs at most one product per period. If a consumer uses a new product, his or her utility is $v_n(\theta) = \theta$; if he or she uses a used product, his or her utility is $v_o(\theta) = a\theta$, where $a \in (0, 1)$ indicates the durability of the product as a reverse measure of the deterioration rate.

A monopolistic P2P second-hand marketplace exists. Each period, the marketplace chooses a transaction fee rate that is charged only to the seller (it can be a consumer or the firm), and the transaction price of the product in the marketplace is endogenously set to sell all the goods offered for sale (i.e., the P2P market clearing price). Denote by $\varphi^t \in (0, 1)$ the transaction fee rate of the product in period t and by p_s^t the sales price of the used product in period t. Similar to Chen and Hsu (2017) and Rao et al. (2009), all used products are sold in the P2P marketplace after being recycled by the firm under the trade-in strategy. In addition, there is a common time discount factor $\rho \in (0, 1]$ for consumers, the firm, and the marketplace.

The timing of the game is as follows. At the beginning of each period, the marketplace chooses the transaction fee rate φ^t. Then, the firm determines the sales price of a new product

Table 1 Notation

Notation	Meaning
p_n^t	Sales price of a new product without a trade-in in period t
p_s^t	Sales price of a used product on the P2P marketplace in period t
p_r^t	Sales price of a new product with a trade-in in period t
a	Durability of the product
θ	Consumer's valuation of a new product
φ^t	Transaction fee rate of the product in period t
c	Unit production cost of the product
ρ	Discount factor

without or with a trade-in p_n^t, p_r^t. Finally, each consumer chooses his or her action (described later). All players receive their utilities in that period.

Table 1 presents our notation.

3.2 Model description

To mathematically describe the assumptions, this subsection characterizes consumers' actions, defines the state variable, obtains the state transition function, and presents all players' utility functions. The analysis of this subsection is mainly based on Huang et al. (2001) and Rao et al. (2009).

3.2.1 Consumers' actions

The action that a consumer with valuation θ can take in period t can be expressed as the following binary vector: $b^t(\theta) = (b_1^t(\theta), b_2^t(\theta), b_3^t(\theta), b_4^t(\theta), b_5^t(\theta))$, where $b_i^t(\theta) = 0$ or 1 $(i = 1, \ldots, 5)$. The meanings of b_i^t $(i = 1, \ldots, 5)$ are as follows:
$b_1^t(\theta) = 1$ represents purchasing a new product without a trade-in;
$b_2^t(\theta) = 1$ represents holding a used product;
$b_3^t(\theta) = 1$ represents purchasing a new product with a trade-in;
$b_4^t(\theta) = 1$ represents purchasing a used product in the marketplace;
$b_5^t(\theta) = 1$ represents not using any products.
Because each consumer needs at most one product during each period, the constraint $\sum_{i=1}^{5} b_i^t(\theta) = 1$ holds for all θ and t.

3.2.2 State

Denote $k_1^t(x, y)$ as the mass of consumers with valuation $\theta \in [x, y]$ who purchase a new product without a trade-in in period t (i.e., $b_1^t = 1$), $k_2^t(x, y)$ as the mass of consumers with valuation $\theta \in [x, y]$ who hold a used product in period t (i.e., $b_2^t = 1$), $k_3^t(x, y)$ as the mass of consumers with valuation $\theta \in [x, y]$ who purchase a new product with a trade-in in period t (i.e., $b_3^t = 1$), $k_4^t(x, y)$ as the mass of consumers with valuation $\theta \in [x, y]$ who purchase a used product in period t (i.e., $b_4^t = 1$), and $k_5^t(x, y)$ as the mass of consumers with valuation $\theta \in [x, y]$ who do not use any product in period t (i.e., $b_5^t = 1$). Define $g_i^t(\theta) = \lim_{d \to 0} \frac{k_i^t(\theta, \theta+d)}{d}$ for $i \in \{1, 2, 3, 4, 5\}$. $g_i^t(\theta)$ can be interpreted as the proportion

Table 2 Utility function π_θ^t of consumers with valuation θ in period t

$\pi^t(\theta)$	$b_1^{t-1}(\theta)=1$	$b_2^{t-1}(\theta)=1$	$b_3^{t-1}(\theta)=1$	$b_4^{t-1}(\theta)=1$	$b_5^{t-1}(\theta)=1$
$b_1^t(\theta)=1$	$\theta-p_n^t+(1-\varphi^t)p_s^t$	$\theta-p_n^t$	$\theta-p_n^t+(1-\varphi^t)p_s^t$	$\theta-p_n^t$	$\theta-p_n^t$
$b_2^t(\theta)=1$	$a\theta$	–	$a\theta$	–	–
$b_3^t(\theta)=1$	$\theta-p_r^t$	–	$\theta-p_r^t$	–	–
$b_4^t(\theta)=1$	$a\theta-\varphi^t p_s^t$	$a\theta-p_s^t$	$a\theta-\varphi^t p_s^t$	$a\theta-p_s^t$	$a\theta-p_s^t$
$b_5^t(\theta)=1$	$(1-\varphi^t)p_s^t$	0	$(1-\varphi^t)p_s^t$	0	0

of consumers with valuation θ who choose action $b_i^t=1$ in period t. The state of the system in period $t+1$ is defined as $g^t(\theta)=(g_1^t(\theta),g_2^t(\theta),g_3^t(\theta),g_4^t(\theta),g_5^t(\theta))$, where $g_i^t(\theta)\geq 0$ and $\sum_{i=1}^5 g_i^t(\theta)=1$.

Denote by π_θ^t the utility function of consumers with valuation θ in period t. Note that π_θ^t only depends on their actions in the previous period $b^{t-1}(\theta)$, their actions in the current period $b^t(\theta)$, the price vector $p^t=(p_n^t,p_s^t,p_r^t)$ in the current period, and the transaction fee rate φ^t in the current period. Table 2 shows the utility obtained by consumers for all combinations of previous and current actions in the current period. Note that it is assumed that only consumers who have purchased new products in the previous period can hold or trade-in. Thus, some combinations are impossible and marked as "–".

Denote by v_θ^t the maximum utility of consumers with valuation θ from period t forward; then, the Bellman equation for consumers' utility can be written as follows:

$$v_\theta^t[b^{t-1}(\theta),p^t,\varphi^t]=\max_{b^t(\theta)}\pi_\theta^t[b^{t-1}(\theta),b^t(\theta),p^t,\varphi^t]+\rho v_\theta^{t+1}[b^t(\theta),p^{t+1},\varphi^{t+1}], \quad (1)$$

where p^{t+1} and φ^{t+1} are the optimal decisions of the firm and the P2P marketplace in period $t+1$, which will be specified later.

Denote by $\{e_i\}$ $(i=1,\ldots,5)$ the basic vector group. [1] Note that the value range for $b^t(\theta)$ is $\{e_i\}$ $(i=1,\ldots,5)$. According to Table 2, when $b^{t-1}(\theta)$, p^t and φ^t are given, the optimal solution of optimization problem (1) is unique for $\theta\in(0,1)$ except for a zero measurement set. Denote it as $b^t(\theta)^*$. $b^t(\theta)^*$ can be regarded as an optimal reaction function of $b^{t-1}(\theta)$, p^t, φ^t. Denote the optimal reaction function as R_θ^t,

$$R_\theta^t[b^{t-1}(\theta),p^t,\varphi^t]=b^t(\theta)^*. \quad (2)$$

Then, the transition function of the state is as follows:

$$g^t(\theta)=\sum_{k=1}^5 R_\theta^t[e_k,p^t,\varphi^t]g_k^{t-1}(\theta), \quad (3)$$

Equation 3 also indicates that the transfer of states from $g^{t-1}(\theta)$ to $g^t(\theta)$ is entirely determined by p^t and φ^t. Thus, $g^t(\theta)$ can be expressed by $g^{t-1}(\theta)$, p^t and φ^t.

3.2.3 Utility functions

Denote the demand for new products purchased without a trade-in in period t by $B_1^t=\int_0^1 g_1^t(\theta)d\theta$ and the demand for new products purchased with a trade-in in period t by

[1] $e_1=[1,0,0,0,0]$, $e_2=[0,1,0,0,0]$, $e_3=[0,0,1,0,0]$, $e_4=[0,0,0,1,0]$, $e_5=[0,0,0,0,1]$.

$B_3^t = \int_0^1 g_3^t(\theta)d\theta$. Then, the firm's profit in period t is the following:

$$\pi_f^t[g^{t-1}(\theta), p^t, \varphi^t] = (p_n^t - c)B_1^t + [p_r^t + (1-\varphi^t)p_s^t - c]B_3^t. \tag{4}$$

Denote the demand of used products purchased in the P2P marketplace in period t as $B_4^t = \int_0^1 g_4^t(\theta)d\theta$. The profit of the P2P marketplace in period t is the following:

$$\pi_p^t[g^{t-1}(\theta), p^t, \varphi^t] = \varphi^t p_s^t B_4^t. \tag{5}$$

Denote the total quantity of used products held by consumers in period t as $B_2^t = \int_0^1 g_2^t(\theta)d\theta$. Then, the total quantity of new products in period $t-1$ (i.e., $B_1^{t-1} + B_3^{t-1}$) is equal to the total quantity of used products in period t (i.e., $B_2^t + B_4^t$).

$$B_1^{t-1} + B_3^{t-1} = B_2^t + B_4^t. \tag{6}$$

3.3 Characterization of the stationary equilibrium

Referring to previous studies (Huang et al., 2001; Rao et al., 2009) , although there is a nonstationary equilibrium (Rust, 1985) , we only focus on the stationary equilibrium, not only for the solvability of the model but also for analyzing the long-term behavior of the firm, marketplace, and consumers. In stationary equilibrium, the decisions of the firm and the marketplace and the state of the system are independent of the period number t. For a given state, the decision of the firm (resp. marketplace) is the best response in a stationary decision space to the marketplace's (resp. firm's) decision. Since the decision spaces of the firm and the marketplace are limited to be stationary, the game can be solved like a single-period game.

Since explicit time dependence has been eliminated, the superscript t from the previous equations can be removed. In this subsection, we will solve for the optimal reactions of the consumers and the optimization problems of the firm for a given transaction fee rate.

3.3.1 Stationary states

Use N, H, T, U, and I to represent the five potential actions of consumers, which are purchasing a new product without a trade-in, holding a used product, purchasing a new product with a trade-in, purchasing a used product and not using any product, respectively. Recall that the vector $b^t(\theta)$ is used to denote consumers' actions in Subsection 3.2. Therefore, N, H, T, U, I correspond to e_1, e_2, e_3, e_4, e_5, respectively.

Lemma 1 *If p_n, p_s, p_r are all larger than 0 and $\varphi \in (0, 1)$, all consumers can be divided into the following five types according to their behavior patterns: TT, NN, NU, UU, and II, where TT-type consumers purchase a new product with a trade-in in each period; NN-type consumers purchase a new product without a trade-in in each period; NU-type consumers purchase a new product if they do not have any product but otherwise hold the used product in each period; UU-type consumers purchase a used product in each period; and II-type consumers never use any product.*

Table 3 summarizes the optimal actions of these five types of consumers with different previous actions.

Lemma 2 *Transactions occur in the P2P marketplace if and only if the selling price p_s in the P2P marketplace satisfies $\frac{2ap_n - a + a^2}{(1+2\rho)a - a(1+\rho)\varphi + 1} \le p_s \le \frac{ap_n}{1+a\rho}$.*

Table 3 Optimal actions of consumers

Optimal current action \ Type / Previous action	TT	NN	NU	UU	II
N	T	N	U	H	I
H	N	N	N	U	I
T	T	N	U	H	I
U	N	N	N	U	I
I	N	N	N	U	I

Corollary 1 *Trade-in consumers exist if and only if* $p_n - p_r > \max\left\{\frac{(1-\varphi)(2ap_n-a+a^2)}{(1+2\rho)a-a(1+\rho)\varphi+1}, 0\right\}$.

The difference between the price of a new product with and without a trade-in can be regarded as a subsidy for trade-in consumers. In previous studies (Rao et al., 2009; Ray et al., 2011) , this difference is called a trade-in incentive or a trade-in rebate. By Corollary 1, there is a lower bound of the trade-in incentive for the existence of trade-in consumers. Note that this lower bound is related to the decision variables p_n and φ. It decreases in φ and increases in p_n. An increase in φ leads to a reduction in the revenue from selling used products. Thus, the trade-in incentive is correspondingly reduced. An increase in p_n leads to a decrease in the supply of used products. Thus, the revenue from selling used products increases and leads to an increase in the trade-in incentive.

Next, use S_i $(i = I, \ldots, IV)$ to denote the following four regions of (p_n, p_r). The expressions for S_i are as follows:

$$S_I = \left\{(p_n, p_r)\middle| p_n - p_r < \frac{(1-\varphi)(2ap_n - a + a^2)}{(1+2\rho)a - a(1+\rho)\varphi + 1}, \frac{1-a}{2} \le p_n \le \frac{(1-a)(1+a\rho)}{(1+a\rho\varphi + a\varphi - a)}\right\},$$

$$S_{II} = \left\{(p_n, p_r)\middle| p_n - p_r < \frac{(1-\varphi)(2ap_n - a + a^2)}{(1+2\rho)a - a(1+\rho)\varphi + 1}, \frac{(1-a)(1+a\rho)}{(1+a\rho\varphi + a\varphi - a)} \le p_n \le 1 + a\rho\right\},$$

$$S_{III} = \left\{(p_n, p_r)\middle| p_n - p_r \ge \frac{(1-\varphi)(2ap_n - a + a^2)}{(1+2\rho)a - a(1+\rho)\varphi + 1}, p_n - p_r < \frac{ap_r - a + a^2 + ap_r\rho}{1+a\rho},\right.$$
$$\left.(1+\rho)p_r - p_n \le 1 - a\right\},$$

$$S_{IV} = \left\{(p_n, p_r)\middle| p_n - p_r \ge \frac{(1-\varphi)(2ap_n - a + a^2)}{(1+2\rho)a - a(1+\rho)\varphi + 1}, p_n - p_r \ge \frac{ap_r - a + a^2 + ap_r\rho}{1+a\rho},\right.$$
$$\left.(1+\rho)p_r - p_n \le 1 - a\right\}.$$

Define Case i as the case where $(p_n, p_r) \in S_i$. The stationary states and valuation interval of the consumer types that exist in the four cases are expressed in Proposition 1.

Proposition 1 *The consumer types that exist in the four cases are as follows:*
Case I: Trade-in consumers do not exist. New products are purchased by both NU-type and NN-type consumers.
Case II: Trade-in consumers do not exist. New products are purchased by NU-type consumers only.

Table 4 Valuation interval of consumer types and stationary states

Case	Valuation interval	Consumer types	$g^*(\theta)$
Case I	$\theta \in \left[\frac{p_n-(1+\rho)(1-\varphi)p_s}{1-a}, 1\right]$	NN	$(1, 0, 0, 0, 0)$
	$\theta \in \left[\frac{p_n-(1+\rho)p_s}{1-a}, \frac{p_n-(1+\rho)(1-\varphi)p_s}{1-a}\right]$	NU	$\left(\frac{1}{2}, \frac{1}{2}, 0, 0, 0\right)$
	$\theta \in \left[\frac{p_s}{a}, \frac{p_n-(1+\rho)p_s}{1-a}\right]$	UU	$(0, 0, 0, 1, 0)$
	$\theta \in \left[0, \frac{p_s}{a}\right]$	II	$(0, 0, 0, 0, 1)$
Case II	$\theta \in \left[\frac{p_n}{1+a\rho}, 1\right]$	NU	$\left(\frac{1}{2}, \frac{1}{2}, 0, 0, 0\right)$
	$\theta \in \left[0, \frac{p_n}{1+a\rho}\right]$	II	$(0, 0, 0, 0, 1)$
Case III	$\theta \in \left[\frac{(1+\rho)p_r-p_n}{1-a}, 1\right]$	TT	$(0, 0, 1, 0, 0)$
	$\theta \in \left[\frac{p_n-(1+\rho)p_s}{1-a}, \frac{(1+\rho)p_r-p_n}{1-a}\right]$	NU	$\left(\frac{1}{2}, \frac{1}{2}, 0, 0, 0\right)$
	$\theta \in \left[\frac{p_s}{a}, \frac{p_n-(1+\rho)p_s}{1-a}\right]$	UU	$(0, 0, 0, 1, 0)$
	$\theta \in \left[0, \frac{p_s}{a}\right]$	II	$(0, 0, 0, 0, 1)$
Case IV	$\theta \in \left[\frac{(1-\rho)p_n+\rho p_r-p_s}{1-a}, 1\right]$	TT	$(0, 0, 1, 0, 0)$
	$\theta \in \left[\frac{p_s}{a}, \frac{(1-\rho)p_n+\rho p_r-p_s}{1-a}\right]$	UU	$(0, 0, 0, 1, 0)$
	$\theta \in \left[0, \frac{p_s}{a}\right]$	II	$(0, 0, 0, 0, 1)$

Case III: Trade-in consumers exist. New products are purchased by both TT-type and NU-type consumers.
Case IV: Trade-in consumers exist. New products are purchased by TT-type consumers only.
Table 4 lists the valuation interval of consumer types and stationary states in these four cases.

Denote p_{si} as the second-hand price (i.e., the sales price in the P2P marketplace) in Case i. According to Eq. (6), the quantity of new products purchased is equal to the quantity of used products in each period. Thus, the sum of lengths of the valuation intervals of TT-type and NN-type consumers is equal to that of UU-type consumers. Note that there is no transaction in P2P marketplace in Case II; the second-hand prices in the other three cases are as follows:

$$\begin{cases} p_s^I = \frac{2ap_n-a+a^2}{(1+2\rho)a-(1+\rho)a\varphi+1}, \\ p_s^{III} = \frac{(1+\rho)ap_r-a+a^2}{a\rho+1}, \\ p_s^{IV} = \frac{2a(1-\rho)p_n+2a\rho p_r-a+a^2}{a+1}. \end{cases} \tag{7}$$

3.3.2 The optimal decision of the firm

Define $\theta_{31} = \frac{(1+\rho)p_r-p_n}{1-a}$, $\theta_{32} = \frac{p_n-(1+\rho)p_s}{1-a}$ and $\theta_{41} = \frac{(1-\rho)p_n+\rho p_r-p_s}{1-a}$. Denote π_{fi} as the firm's profit in Case i ($(p_n, p_r) \in S_i$). π_{fi} can be expressed as follows:

$$\begin{cases} \pi_{fI} = \frac{(p_n-c)(2+2a\rho-2p_n-a\varphi-a\rho\varphi)}{2a+4a\rho-2a\varphi-2a\rho\varphi+2}, \\ \pi_{fII} = \frac{(1+a\rho-p_n)(p_n-c)}{2+2a\rho}, \\ \pi_{fIII} = (1-\theta_{31})(p_r + (1-\varphi)\frac{ap_r-a+a^2+ap_r\rho}{1+a\rho} - c) + 0.5(\theta_{31}-\theta_{32})(p_n-c), \\ \pi_{fIV} = (1-\theta_{41})(p_r + (1-\varphi)\frac{2a(1-\rho)p_n+2a\rho p_r-a+a^2}{1+a} - c). \end{cases} \tag{8}$$

The firm's optimal decision problem can be expressed as follows:

$$\max_{p_n, p_r} \pi_f \qquad \pi_f = \pi_{fi}, \quad if \quad (p_n, p_r) \in S_i. \tag{9}$$

For the ease of exposition, the subsequent work is based on $\rho = 1$. This simplification assumes that consumers are maximizers of the average payoff per period, which has been adopted by most of the previous research on trade-in strategy in infinite periods (Agrawal et al., 2008; Li & Xu, 2015; Rao et al., 2009) . In the Appendix, we analyze the case when ρ is not too small and find that the main qualitative results still hold. In fact, most firms' update cycles for durable products are currently approximately one year, as is done by Apple and Samsung. The common discount factor ρ can be approximated as $\frac{1}{1+r}$, where r denotes the reciprocal of the bank's annual interest rate. Since r is less than 5% in general, $\rho \geq 0.95$ holds. Therefore, our analysis can cover most of the situations in reality.

When $\rho = 1$, define $(p_n^*, p_r^*) = \arg\max_{p_n, p_r} \pi_f$, which is the firm's optimal pricing decision. Define $(p_{n2}, p_{r2}) = (\frac{1+a+c}{2}, +\infty)$, which is the optimal point in region S_{II}. Define $(p_{n3}, p_{r3}) = (\frac{(1-a)(4a+c+3ac-3a\varphi-3a^2\varphi+3a^2-ac\varphi+1)}{2(-a^2\varphi^2+2a^2\varphi-3a^2-2a\varphi+2a+1)}, \frac{(1-a)(4a+c+ac-3a\varphi+a^2\varphi-a^2-a^2\varphi^2+1)}{2(-a^2\varphi^2+2a^2\varphi-3a^2-2a\varphi+2a+1)})$, which is the optimal point in region S_{III}.

Lemma 3 *For a given φ, (p_n^*, p_r^*) can be expressed as follows:*

$$(p_n^*, p_r^*) = \begin{cases} (p_{n2}, p_{r2}), & \text{if } a \leq 0.5, c > 1-a-2a^2, \varphi > \frac{(1-a)(1+a-c)}{a+ac+a^2} \text{ or} \\ & a > 0.5, \varphi > \frac{(1-a)(1+a-c)}{a+ac+a^2} \\ (p_{n3}, p_{r3}), & \text{otherwise.} \end{cases} \tag{10}$$

4 Model analysis

This section conducts the model analysis. It starts with the benchmark scenario when the transaction fee rate is exogenously given (Subsection 4.1), then proceeds to the scenario when the transaction fee rate is endogenously set by the marketplace (Subsection 4.2), and finally, the two scenarios are compared (Subsection 4.3).

4.1 Benchmark scenario: exogenous transaction fee rate

In this subsection, it is assumed that the transaction fee rate of the marketplace is exogenously given. Denote it as φ_0. We will focus on the market structure[2] and the trade-in incentive in stationary equilibrium.

Define $\varphi_h = \frac{(1-a)(1+a-c)}{a+ac+a^2}$. Denote (p_{nb}^*, p_{rb}^*) as the firm's optimal decision in the benchmark Scenario. According to Lemma 3, (p_{nb}^*, p_{rb}^*) is as follows:

$$(p_{nb}^*, p_{rb}^*) = \begin{cases} (p_{n3}, p_{r3}), & \text{if } \varphi_0 \leq \varphi_h. \\ (p_{n2}, p_{r2}), & \text{if } \varphi_0 > \varphi_h. \end{cases} \tag{11}$$

Proposition 2 is directly derived from Lemma 3.

[2] In this paper, when we discuss the market structure, we refer to the problems of which consumer types exist, whether the firm chooses to adopt the trade-in strategy, and whether there are transactions in the marketplace. These results will facilitate understanding the long-term selling of different durable products.

Proposition 2 *When $\varphi_0 \leq \varphi_h$, the condition of Case III is satisfied under which trade-in consumers exist and new products are purchased by both TT-type and NU-type consumers; otherwise, the condition of Case II is satisfied under which trade-in consumers do not exist and new products are purchased by NU-type consumers only.*

According to Proposition 2, there is a threshold φ_h for the transaction fee rate, and the firm adopts the trade-in strategy if and only if the transaction fee rate of the P2P marketplace does not exceed the threshold. Transactions occur in the P2P marketplace if and only if there are trade-in consumers. In Case II, no trade-in consumer exists. Thus, no transaction occurs in the P2P marketplace. Huang et al. (2001) also obtained a similar result about the condition for the existence of transactions in the P2P marketplace. In particular, if $\varphi_0 = 0$, trade-in consumers always exist, which is consistent with some existing studies (Chen & Hsu, 2015; Rao et al., 2009) .

In addition, the firm adopts the trade-in strategy and sells recycled products on the P2P marketplace as long as the transaction fee rate does not exceed φ_h. Thus, φ_h is the firm's maximum acceptable transaction fee rate for the trade-in strategy. The threshold φ_h is analyzed in Proposition 3.

Proposition 3 *When $c + a + 2a^2 \leq 1$, $\varphi_h \geq 1$ holds. Thus, any $\varphi_0 \in [0, 1]$ is acceptable to the firm. Otherwise, the firm's maximum acceptable transaction fee rate is φ_h. Furthermore, $\frac{\partial \varphi_h}{\partial a} < 0$, $\frac{\partial \varphi_h}{\partial c} < 0$.*

It is shown that when durability and production costs are both low, the firm will always adopt the trade-in strategy, even if it cannot obtain profits from selling used products; otherwise, there is a maximum acceptable transaction fee rate (which is less than 1) for the firm, which decreases in durability and production costs. Note that the decline in durability has no effect on TT-type consumers, but it significantly reduces the utility of NU-type consumers. Therefore, the lower the durability is, the greater the advantage of adopting the trade-in strategy. Note that the profit margin of new products purchased with trade-ins is smaller than that without trade-ins. Thus, the decrease in production costs will increase the profit margin of new products purchased with trade-ins more significantly, which also encourages the firm to adopt the trade-in strategy. Therefore, when the durability and production cost are both low, the advantage from adopting the trade-in strategy is so significant that the firm always chooses to adopt it. When the durability and production cost are not too low, the maximum acceptable transaction fee rate is less than 1 and decreases in both durability and production costs.

Finally, the impact of the transaction fee rate, durability, and production costs effect on the trade-in incentive is shown in Proposition 4.

Proposition 4 *When trade-in consumers exist, $\frac{\partial (p_{nb}^* - p_{rb}^*)}{\partial a} > 0$, $\frac{\partial (p_{nb}^* - p_{rb}^*)}{\partial c} > 0$. In addition, when $c + a + 2a^2 \geq 1$, $\frac{\partial (p_{nb}^* - p_{rb}^*)}{\partial \varphi_0} < 0$.*

When durability increases, the utility of TT-type consumers is unchanged, while the utility of all other consumers increases. Therefore, to implement the trade-in strategy, the firm needs to provide a higher trade-in incentive. This result is similar to those of Rao et al. (2009) and Li et al. (2019). Moreover, an increase in the price of a new product with a trade-in (p_r) will lead to a decrease in the demand for trade-ins, a decline in the supply of used products, and an increase in the transaction price of used products. According to Eq. (7), $p_s = \frac{2ap_r - a + a^2}{a+1}$. Consider the firm's profit from a new product with a trade-in (denoted as $u_r = p_r + (1-\varphi_0)p_s - c$) and without a trade-in (denoted as $u_n = p_n - c$). Note that u_r can be expressed as $k_1(p_r - \frac{c}{k_1}) - k_2$, where $k_1 = 1 + \frac{2(1-\varphi_0)a}{1+a}$ and $k_2 = \frac{-a^2+a}{a+1}$. The influence of production costs c on p_r is lower than that on p_n since $k_1 \geq 1$. Therefore, when the increase of production costs leads to an increase in the sales prices of new products with and without trade-ins (p_r and p_n), the increasing magnitude of p_r is lower. Although the increase of production costs has a greater adverse impact on the profit from trade-in products, the trade-in incentive still increases in production costs. Finally, the lower transaction fee rate increases the profits of trade-in products, so the trade-in incentive increases.

4.2 Scenario with an endogenous transaction fee rate

In this scenario, it is assumed that the transaction fee rate φ is a decision variable of the P2P marketplace. The optimal transaction fee rate of the P2P marketplace and the market structure are discussed. We also analyze the marketplace's profit through a numerical study.

In Subsection 4.1, it is found that there is a maximum acceptable transaction fee rate for the firm. Thus, the transaction fee rate will always be less than this threshold when the P2P marketplace decides this rate. As φ_h is always greater than 0, this ensures that the P2P marketplace obtains positive revenue.

Proposition 5 *The condition of Case III is always satisfied in stationary equilibrium under which trade-in consumers exist and new products are purchased by both TT-type and NU-type consumers.*

In stationary equilibrium, consumers who purchase new products with and without trade-ins coexist, which is similar to the findings of some previous studies (Chen & Hsu, 2015; Rao et al., 2009) that assume no transaction fee. This result implies that the firm can always benefit from adopting the trade-in strategy even when the transaction fee rate is a decision variable of the P2P marketplace.

In addition, compared to the market structure in the basic scenario, interestingly, in stationary equilibrium, there is no NN-type consumer. Consumers who purchase new products without trade-ins must be NU-type consumers. Consumers purchase new products with trade-ins unless they do not have any product to trade-in, and then they will buy a new product without a trade-in. A similar result was obtained in Huang's leasing model (Huang et al., 2001) under the condition that the transaction fee rate of the marketplace charged to consumers is higher than that charged to the firm. In our trade-in model, the results still hold even if these two rates are equal. This result shows that when the trade-in strategy is adopted, consumers would always return used products for trade-in instead of selling them on the P2P marketplace.

Next, according to Proposition 5 and Eq. (5), the optimization problem of the P2P marketplace can be expressed as follows:

$$\begin{aligned}
&\max_{\varphi} \pi_p \\
&\quad = \frac{-a\varphi(2a+c-a\varphi)(a+c-2ac+a\varphi+a^2c-a^3\varphi+a^2-a^3+ac\varphi-a^2c\varphi-1)}{2(-a^2\varphi^2+2a^2\varphi-3a^2-2a\varphi+2a+1)^2} \\
&\quad s.t.\ 0 \leq \varphi \leq \min\{1, \varphi_h\}.
\end{aligned} \tag{12}$$

Denote $\varphi^*(a, c)$ as the optimal solution of the optimization problem in (12). Proposition 6 presents some properties of the optimal transaction fee rate.

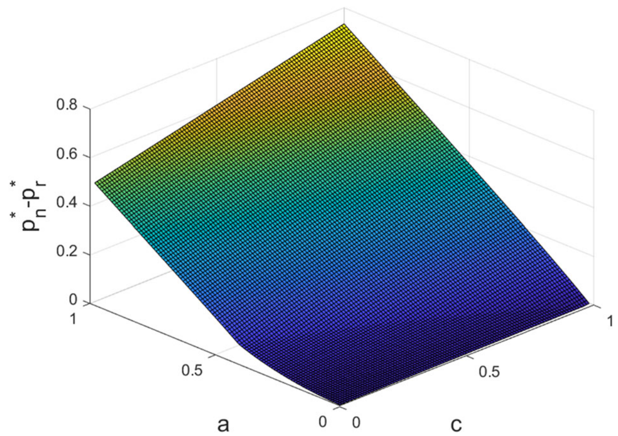

Fig. 1 Trade-in incentive with respect to a and c

Proposition 6 *There exists a unique* $\varphi^*(a, c)$. *When* $3a^2 + 2c^2 + 2ac + 2a + 2c \leq 1$, $\varphi^*(a, c) = 1$; *otherwise,* $\varphi^*(a, c) < 1$ *and* $\frac{\partial \varphi^*(a,c)}{\partial a} < 0$, $\frac{\partial \varphi^*(a,c)}{\partial c} < 0$.

Proposition 6 indicates that when both durability and production costs are not too low, the optimal transaction fee rate $\varphi^*(a, c)$ decreases in durability a and production costs c [(the result on production costs's influence on the optimal transaction fee rate is similar to that in Ryan et al. (2012)], and the optimal transaction fee rate can reach 1 when both durability and production costs are low. These results imply that the choice of the transaction fee rate by the P2P marketplace mainly depends on the firm's advantage of adopting the trade-in strategy. The interpretation is as follows. When the durability a or production costs c increases, the firm will allow fewer consumers to trade in, and the transaction volume in the P2P marketplace decreases. Thus, the P2P marketplace charges a lower transaction fee rate to guarantee the transaction volume. However, when both durability a and production costs c are low enough, the firm induces as many consumers as possible to trade-in even if it cannot obtain profits from selling used products.

There is some evidence from eBay's setting of transaction fee rates for products offered with trade-in services. The transaction fee rate for cars on eBay is lower than that for cell phones. Note that, in our model, the durability of cars would be higher than that of cell phones because cell phones deteriorate faster in general. According to Proposition 6, the transaction fee rate of cars should be lower than that of cell phones, which conforms to reality.

Unfortunately, the optimal transaction fee rate $\varphi^*(a, c)$ cannot be expressed as an explicit analytical formula. However, due to the existence and uniqueness of $\varphi^*(a, c)$, we can use numerical study to analyze more properties in this scenario including the trade-in incentive and the P2P marketplace's profit. A sensitivity analysis of the trade-in incentive when $\varphi^*(a, c) < 1$ is conducted first. Note that in the benchmark scenario, the trade-in incentive increases in both durability and production costs when the trade-in strategy is adopted. We find that the result in the current scenario is similar to that in the benchmark scenario (Fig. 1).

In the following, the profit of the P2P marketplace is investigated since it is now a decision-maker.

Observation 1 *(1)* π_p^* *first increases and then decreases in* a. *(2)* π_p^* *decreases in* c.

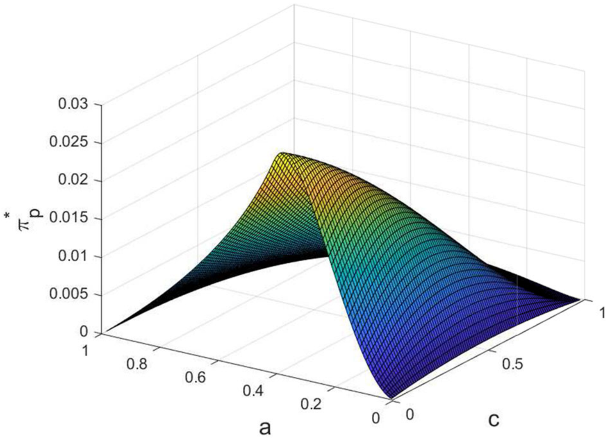

Fig. 2 Marketplace's profit with respect to a and c

From Fig. 2, it can be observed that in stationary equilibrium, the profit of the P2P marketplace first increases and then decreases in durability, while it always decreases in production costs. An increase in durability will lead to an increase in the demand for used products. The transaction volume and transaction price in the P2P marketplace will increase. Furthermore, according to Proposition 3, when durability is low, the decision space of the transaction fee rate of the P2P marketplace is independent of durability; otherwise, an increase in durability will lead to a decrease in the decision space of the transaction fee rate. Thus, when durability is low, the decision space is relatively large, and the profit of the P2P marketplace increases in durability. When durability is high, the decision space is so small that it becomes the main driver of the profit of the P2P marketplace. Therefore, the profit of the P2P marketplace decreases in durability. In addition, an increase in production costs will lead the firm to increase the prices of new products both with and without a trade-in, which reduces the demand for both new and used products. Thus, the marketplace's profit decreases since the whole market shrinks.

4.3 Comparison between the two scenarios

Denote the benchmark scenario as Scenario 1 and the scenario with an endogenous transaction fee rate as Scenario 2. Previous analysis has shown that the trade-in strategy is always adopted in Scenario 2, while the trade-in strategy is not adopted in Scenario 1 when product durability and production costs are high. This subsection further compares these two scenarios from the perspectives of the proportion of trade-in consumers (denoted as β), the marketplace and the firm's profits, and the trade-in incentive.

Observation 2 *In Scenario 2, $\beta > 0.3$ always holds. β approaches 1 when both a and c approach* 0.

According to the discussion of Proposition 3, the higher the durability and production costs are, the smaller the advantage of the trade-in strategy is. According to Fig. 3, when durability and production costs are both low, most consumers would purchase new products with trade-ins, while when durability and production costs are both high, there is still a proportion of trade-in consumers. In comparison, in Scenario 1 this result does not hold

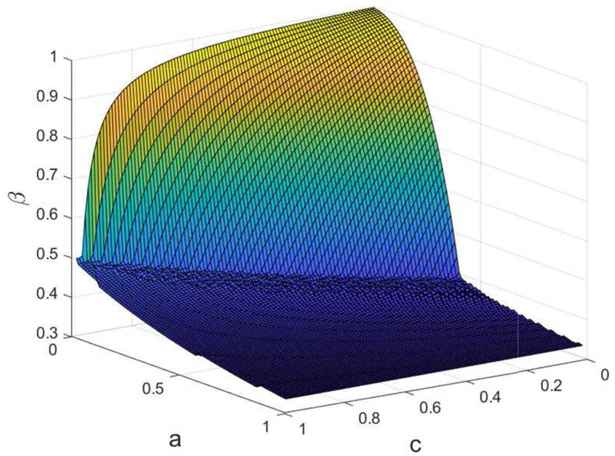

Fig. 3 Proportion of trade-in consumers with respect to a and c

because the firm will not adopt the trade-in strategy when durability and production costs are high, which indicates that the P2P marketplace's choice of transaction fee rate increases the market share of purchasing with trade-ins. This result differs from the result in previous studies that the proportion of trade-in consumers can be close to 0, which is due to the endogenous choice of the transaction fee rate. When durability and production costs increase, the firm's maximum acceptable transaction fee rate decreases. Thus, the P2P marketplace actively reduces the transaction fee rate to guarantee that the firm's profit margin from trade-in products can balance with the profit margin from non-trade-in products, which finally results in the proportion of trade-in consumers not being too low.

Denote π^*_{p1}, π^*_{f1} as the marketplace and the firm's profits in Scenario 1 and π^*_{p2}, π^*_{f2} as the marketplace and the firm's profits in Scenario 2, respectively. Denote i_1, i_2 as the trade-in incentive in Scenario 1 and Scenario 2, respectively. The next two propositions compare the profits and the trade-in incentives in the two scenarios.

Proposition 7 (1) $\pi^*_{p1} \leq \pi^*_{p2}$.
(2) *When* $\varphi_0 < \varphi^*$, $\pi^*_{f1} > \pi^*_{f2}$. *When* $\varphi^* \leq \varphi_0 \leq \varphi_h$, $\pi^*_{f1} \leq \pi^*_{f2}$.

It is not difficult to understand that the profit of the marketplace is higher in Scenario 2 since the marketplace can decide the transaction fee rate. Proposition 7 also shows that when the trade-in strategy is adopted ($\varphi_0 \leq \varphi_h$), which scenario the firm can obtain a higher profit in depends on which scenario features a lower transaction fee rate, since an increase in the transaction fee rate decreases the value of recycled products from trade-ins. Proposition 6 demonstrates that the transaction fee rate in Scenario 2 decreases in both durability and production costs. Thus, when durability and production costs are both so low that $\varphi_0 \leq \varphi^*$,[3] the firm can obtain a higher profit in Scenario 1 where the marketplace has a fixed transaction fee rate. When durability and production costs are both so high that $\varphi^* \leq \varphi_0$, the firm can obtain a higher profit in Scenario 2 where the marketplace decides the transaction fee rate. Moreover, note that the firm's profit decreases in production costs in both scenarios, and thus the firm's profit is less sensitive to a change in production costs when the transaction fee rate is endogenously determined.

[3] According to Proposition 6, the range of φ^* is [0, 1]. Therefore, for any $\varphi_0 \in [0, 1]$, φ^* can be higher or lower than φ_0.

Proposition 8 *When* $2a + 2c + 2ac + 3a^2 + 2c^2 > 1$, $i_1 > i_2$ *if* $\varphi_0 < \varphi^*$; $i_1 \leq i_2$ *if* $\varphi^* \leq \varphi_0 \leq \varphi_h$.

Proposition 8 indicates that when the trade-in strategy is adopted and the optimal transaction fee rate in Scenario 2 is less than 1 ($2a + 2c + 2ac + 3a^2 + 2c^2 > 1$, $\varphi_0 \leq \varphi_h$), the scenario in which the trade-in incentive is higher depends on which scenario features a lower transaction fee rate, since an increase in the transaction fee rate decreases the profit from trade-in consumers. According to Proposition 6, similar to Proposition 7, when durability and production costs are both so low that $\varphi_0 \leq \varphi^*$, the firm sets a higher trade-in incentive when the marketplace has a fixed transaction fee rate. When durability and production costs are both so high that$\varphi^* \leq \varphi_0$, the firm sets a higher trade-in incentive when the marketplace determines its transaction fee rate. Note that the trade-in incentive increases in durability and production costs in both scenarios. Thus, the trade-in incentive is more sensitive to changes in durability and production costs when the transaction fee rate is endogenously determined.

5 Conclusion

In this paper, we establish an infinite-period model to study a firm's long-term trade-in strategy in the presence of a P2P second-hand marketplace. The firm can choose whether to adopt the trade-in strategy and determines the prices of new products with and without trade-ins. Consumers can purchase new products with or without trade-ins and trade used products on the P2P marketplace. The transaction fee rates of the P2P marketplace are exogenously given or endogenously determined by the P2P marketplace.

First, regarding the market structure, our study describes the potential consumer action patterns under different conditions. If the transaction fee rate is endogenously determined by the P2P marketplace, the firm will always benefit from adopting the trade-in strategy and consumers who purchase new products with and without trade-ins coexist. However, if the transaction fee rate is exogenously given, when product durability and production costs are both high, the firm does not benefit from trade-in strategy, and the optimal pricing strategy is only selling to consumers who purchase new products when they have no products to use and continue to hold used products they currently own. Moreover, there are no transactions in the P2P marketplace.

Second, for firms selling durable goods, our study provides suggestions on pricing strategies. We first discuss the conditions for the firm to adopt a trade-in strategy. There is a threshold for the transaction fee rate under which the trade-in strategy is beneficial to the firm. The threshold decreases in product durability and production costs and is also the upper bound of the P2P marketplace's decision on the transaction fee rate. It is also shown that the trade-in incentive increases in product durability and production costs, which holds in both scenarios with exogenous or endogenous transaction fee rates.

Third, for P2P second-hand marketplaces, our work provides a method for determining the transaction fee rate. If the marketplace does not determine the transaction fee rate, products with high durability and high production cost will not be traded in the marketplace. If the marketplace determines it, transactions will always occur in the P2P marketplace. In addition, in stationary equilibrium, the transaction fee rate decreases in product durability and production costs. The profit of the P2P marketplace first increases and then decreases in product durability, while it always decreases in production costs.

Finally, our work examines the difference between the two scenarios with exogenous and endogenous transaction fee rates. Having the P2P marketplace determine the transaction fee rate increases the market share of purchases with trade-ins. When product durability and production costs are both high, the firm can obtain a higher profit and set a higher trade-in incentive in the scenario with an endogenous transaction fee rate; otherwise, the firm can obtain a higher profit and set a higher trade-in incentive in the scenario with an exogenous transaction fee rate. In addition, when the transaction fee rate is endogenously determined, the firm's profit is less sensitive to a change in production costs, while the trade-in incentive is more sensitive to the change in durability and production costs.

Our study may be helpful to the decision-making of manufacturers who provide trade-in services and P2P second-hand marketplaces who set the transaction fee rate. However, there are some limitations of our model that may be addressed in the future. First, market competition is ignored, which is fairly common in reality. A model with multiple firms selling competitive durable products may yield some different findings. Second, recycling raw materials as another alternative to address used products is not considered, but some firms, such as Apple and Xerox, save several hundred million dollars in this way each year (Li & Xu, 2015) . Thus, adding this alternative to our model could be considered in future studies. Third, in our model, the consumers do not sell used products in the P2P marketplace in stationary equilibrium, but in reality, they do. The main reason is that consumers' valuations of products are assumed to be fixed to obtain a clear characterization of their behavior. However, consumers' valuations may change over time. Thus, models with time-dependent valuation are closer to reality.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Appedix A: Proof of Lemma 1

First of all, it is noted that for any θ, $R_\theta[N, p, \varphi] = R_\theta[T, p, \varphi], v_\theta[N, p, \varphi] = v_\theta[T, p, \varphi]$,$R_\theta[U, p, \varphi] = R_\theta[H, p, \varphi] = R_\theta[I, p, \varphi], v_\theta[U, p, \varphi] = v_\theta[H, p, \varphi] = v_\theta[I, p, \varphi]$.

All θ are classified as follows according to the optimal reaction function $R_\theta[N, p, \varphi]$.

If $R_\theta[N, p, \varphi] = I$, considering $R_\theta[I, p, \varphi]$, if $R_\theta[I, p, \varphi] = I$, the optimal choice in any Case is I, so it is a II-type consumer; if $R_\theta[I, p, \varphi] = U$, then $R_\theta[U, p, \varphi] = U$, it is a UU-type consumer; if $R_\theta[I, p, \varphi] = N$, it indicates that the optimal action of this type of the consumers from a certain N action to the next N decision is I, and its utility is $(1 - \varphi)p_s + \rho(\theta - p_n)$ in this process. However, if the intermediate action is replaced by N, the utility becomes $(1 + \rho)\{\theta - p_n + (1 - \varphi)p_s\}$ which is strictly increased compared with I for any $\theta \geq p_n$, so $\theta < p_n$ can be obtained. On the other hand, $v_\theta[I, p, \varphi] = (\theta - p_n) + \rho(1 - \varphi)p_s + \rho^2(\theta - p_n) + \rho^3(1 - \varphi)p_s + \cdots = ((\theta - p_n) + \rho(1 - \varphi)p_s)/(1 - \rho^2)$, but this consumer's total utility is $(\theta - p_n + (1 - \varphi)p_s)/(1 - \rho)$ if choosing action N each period after an action I, under the condition $\theta < p_n$ and $(\theta - p_n) + \rho(1 - \varphi)p_s > 0$ it is strictly greater than the former, which leads to contradiction, so there is no consumer with $R_\theta[N, p, \varphi] = I$ and $R_\theta[I, p, \varphi] = N$.

If $R_\theta[N, p, \varphi] = H$, considering $R_\theta[H, p, \varphi]$, if $R_\theta[H, p, \varphi] = I$, then $R_\theta[I, p, \varphi] = I$, which indicates its type is II; if $R_\theta[H, p, \varphi] = O$, then it is a UU-type consumer; if $R_\theta[H, p, \varphi] = N$, then it is a NH-type consumer.

If $R_\theta[N, p, \varphi] = T$, then $R_\theta[T, p, \varphi] = R_\theta[N, p, \varphi] = T$, it is a TT-type consumer.

If $R_\theta[N, p, \varphi] = N$, then it is a NN-type consumer.

$R_\theta[N, p, \varphi]$ can never be U, because at this time U is dominated by H.

All possibilities are considered above, so the proof is completed.

Appendix B: Proof of Lemma 2

Suppose $p_s > \frac{ap_n}{1+a\rho}$. From lemma 1, we know that the buyer of the second-hand market must be a UU-type consumer, if its type is θ, then $v_\theta[U, p, \varphi] = \frac{a\theta - p_s}{1-\rho}$, then the condition $\theta > \frac{ap_n}{1+a\rho}$ must be satisfied. If this consumer adopts the NH action pattern, the total utility after a certain U action is $\frac{(\theta-p_n)+\rho a\theta}{1-\rho^2}$, then $\frac{a\theta-p_s}{1-\rho} \geq \frac{(\theta-p_n)+\rho a\theta}{1-\rho^2}$ implies $\theta < \frac{p_n}{1+a\rho}$, which is contradictory, so $p_s \leq a/(1+a\rho)p_n$.

Suppose $p_s < \frac{2ap_n-a+a^2}{(1+2\rho)a-a(1+\rho)\varphi+1}$. Similarly, the seller must be a NN-type consumer, supposing its type as θ, then $v_\theta[N, p, \varphi] = \frac{\theta-p_n+(1-\varphi)p_s}{1-\rho}$, if this consumer adopts NH strategy, then the total utility after a certain N action is $\frac{(\theta-p_n)+\rho a\theta}{1-\rho^2}$, then $\frac{\theta-p_n+(1-\varphi)p_s}{1-\rho} \geq \frac{(\theta-p_n)+\rho a\theta}{1-\rho^2}$. If the consumer adopts the TT strategy, then the total utility after a certain N action is $\frac{\theta-p_r}{1-\rho}$, then $\frac{\theta-p_n+(1-\varphi)p_s}{1-\rho} \geq \frac{\theta-p_r}{1-\rho}$. Considering the supply of used products in the second-hand market, it is known from lemma 1 that it can only come from NN-type consumers and TT-type consumers. Because there are NN-type consumers at the same time, the total utility of TT-type consumers will not be higher than NN-type consumers. Therefore, the two types of consumers must meet the requirements of $\theta \geq \frac{p_n-(1+\rho)(1-\varphi)p_s}{1-a}$, so the second-hand supply is less than $1 - \frac{p_n-(1+\rho)(1-\varphi)p_s}{1-a}$. Now considering the demand, for NH-type consumers, $v_\theta[H, p, \varphi] = \frac{(\theta-p_n)+\rho a\theta}{1-\rho^2}$; for UU-type consumers, $v_\theta[O, p, \varphi] = \frac{a\theta-p_s}{1-\rho}$. So the demand $\frac{p_n-(1+\rho)p_s}{1-a} - \frac{p_s}{a}$ can be obtained by comparing. However, when $p_s < \frac{2ap_n-a+a^2}{(1+2\rho)a-a(1+\rho)\varphi+1}$, $\frac{p_n-(1+\rho)p_s}{1-a} - \frac{p_s}{a} > 1 - \frac{p_n-(1+\rho)(1-\varphi)p_s}{1-a}$, it is inconsistent with the liquidation condition. To sum up, $p_s \geq \frac{2ap_n-a+a^2}{(1+2\rho)a-a(1+\rho)\varphi+1}$.

Appendix C: Proof of Proposition 1

We only consider Case I here, and other cases can be obtained similarly.

In Case I, when $p_n - p_r < (1-\varphi)\frac{2ap_n-a+a^2}{(1+2\rho)a-a(1+\rho)\varphi+1}$, if there is a TT-type consumer whose type is θ, then $v_\theta[T, p, w] = \frac{\theta-p_r}{1-\rho}$. If the consumer adopts NN strategy after a certain T state, its total utility is $\frac{\theta-p_n+(1-\varphi)p_s}{1-\rho}$. Thus $\frac{\theta-p_r}{1-\rho} \geq \frac{\theta-p_n+(1-\varphi)p_s}{1-\rho}$, $p_n - p_r \geq (1-\varphi)p_s$ is obtained. From lemma 2, we know that $p_s \geq \frac{2ap_n-a+a^2}{(1+2\rho)a-a(1+\rho)\varphi+1}$, which leads to contradiction. Therefore, only NN-type,NU-type,UU-type and II-type consumers may exist in stationary equilibrium.

For all NN-type consumers, $v_\theta[N, p, \varphi] = (\theta - p_n + (1-\varphi)p_s)/(1-\rho)$; for all NH-type consumers, $v_\theta[N, p, \varphi] = (\rho(\theta - p_n) + a\theta)/(1-\rho^2)$, $v_\theta[H, p, \varphi] = ((\theta - p_n) +$

$\rho a\theta)/(1-\rho^2)$; for all UU-type consumers, $v_\theta[U, p, \varphi] = (a\theta - p_s)/(1-\rho)$; for all II-type consumers, $v_\theta[I, p, \varphi] = 0$. The critical values are obtained by comparing the above formulas.

The $g^*(\theta)$ of each type consumers is obtained by (3).

Appendix D: Proof of Lemma 3

First we consider the utility function of the firm in Case IV, the determinant of the Hessian matrix is $\frac{-(\rho-1)^2}{(a+1)^2}$, thus this matrix is always not negative definite as soon as $\rho \neq 1$. And it indicates that there is not a interior point of local optimum in Case IV, so we can ignore Case IV because its boundary would be considered in other cases. Since we assume $\rho = 1$ for tractability and conciseness, Case IV is still abandoned because we want to make the result close to the condition when $\rho \to 1$.

The four utility functions of the firm in each case are as follows:

$$\begin{cases} \pi_{f1} = \frac{(p_n-c)(2+2a-2p_n-2a\varphi)}{6a-4a\varphi+2}, \\ \pi_{f2} = \frac{(1+a-p_n)(p_n-c)}{2+2a}, \\ \pi_{f3} = (1-\frac{2p_r-p_n}{1-a})\left(p_r+\frac{(1-\varphi)(2ap_r-a+a^2)}{1+a}-c\right)+0.5\left(\frac{2p_r-p_n}{1-a}-\frac{p_n-2\frac{2ap_r-a+a^2}{1+a}}{1-a}\right) \\ (p_n-c), \\ \pi_{f4} = \left(1-\frac{(1-\rho)p_n+\rho p_r-\frac{2a(1-\rho)p_n+2a\rho p_r-a+a^2}{a+1}}{1-a}\right)\left(p_r+\frac{(1-\varphi)(2a(1-\rho)p_n+2a\rho p_r-a+a^2)}{1+a}-c\right). \end{cases}$$

Denote the Hessian matrix of π_{f3} as H_3.

$$H_3 = \begin{pmatrix} -\frac{2}{1-a} & -\frac{12a-4a\varphi+4}{(2a+2)(a-1)} \\ -\frac{12a-4a\varphi+4}{(2a+2)(a-1)} & -\frac{4(2a\varphi-3a-1)}{(a+1)(a-1)} \end{pmatrix}.$$

Next, by calculating the the determinant of H_3 we can acknowledge H_3 is always strict negative definite if and only if $\varphi < \frac{a+\sqrt{2-2a^2}-1}{a}$. And when $\varphi < \frac{a+\sqrt{2-2a^2}-1}{a}$, the optimal point is as follows:

$$(p_{n3}^*, p_{r3}^*) = (\frac{(1-a)(4a+c+3ac-3a\varphi-3a^2\varphi+3a^2-ac\varphi+1)}{2(-a^2\varphi^2+2a^2\varphi-3a^2-2a\varphi+2a+1)}, \frac{(1-a)(4a+c+ac-3a\varphi+a^2\varphi-a^2-a^2\varphi^2+1)}{2(-a^2\varphi^2+2a^2\varphi-3a^2-2a\varphi+2a+1)}).$$

By plugging the above result to $p_n - p_r = \frac{(1-\varphi)(2ap_n-a+a^2)}{(1+2\rho)a-a(1+\rho)\varphi+1}$, it is not hard to prove that optimal point always satisfy the limit condition of $p_n - p_r > \frac{(1-\varphi)(2ap_n-a+a^2)}{(1+2\rho)a-a(1+\rho)\varphi+1}$ because there is not a root of (a, c, φ) in the defined area. Besides, Case I can be regard as a special case of Case III on the hyperplane of $p_n - p_r = \frac{(1-\varphi)(2ap_n-a+a^2)}{(1+2\rho)a-a(1+\rho)\varphi+1}$. So, when $\varphi < \frac{a+\sqrt{2-2a^2}-1}{a}$, Case I is dominated by Case III. Now we define $\varphi_1 = \frac{a+\sqrt{2-2a^2}-1}{a}$.

Then, by solving $2p_r - p_n = 1-a$ we can acknowledge the optimal point satisfy $2p_r - p_n \leq 1-a$ if and only if $\varphi \leq \frac{(1-a)(1+a-c)}{a+ac+a^2}$. Define $\varphi_2 = \frac{(1-a)(1+a-c)}{a+ac+a^2}$. Remember in Case III if $2p_r - p_n \geq 1-a$, the HH consumers and UU consumers do not exist and it turn into Case II, so Case II can be regard as a special case of Case III on the hyperplane of

$2p_r - p_n = 1 - a$. Therefore, when $\varphi < min(\varphi_1, \varphi_2)$, the optimal point in Case III is the firm's optimal decision.

Define $\varphi_3 = \frac{(3a+c+3)(1-a)}{2a(a+c+1)}$, when $\varphi \geq \varphi_3$, the optimal point in Case I $p_n = \frac{(1-\varphi)a+c+1}{2}$ beyond the limit range $\frac{1-a^2}{1+2a\varphi-a}$. It indicates that Case I is dominated by Case II.

Finally, following inequalities hold since there is not a root of $c \in (0, 1)$ for the equation that makes the two sides of the unequal sign equal, which is not difficult to prove; 1) For all $\varphi \in (0, 1)$, if $\varphi \leq \varphi_2$, then $\varphi \leq \varphi_1$; 2) For all $\varphi \in (0, 1)$, if $\varphi \geq \varphi_2$, then$\varphi \geq \varphi_3$. So, when $\varphi \leq \varphi_2, \varphi \leq \varphi_1$, and the optimal point in Case III is the firm's optimal decision. When $\varphi \geq \varphi_2$, if $\varphi \leq \varphi_1$, then Case I is dominated by Case III and Case III is dominated by Case II; if $\varphi \geq \varphi_1$, then Case III can be abandoned, also because $\varphi \leq \varphi_1$, Case I is dominated by Case II. Therefore the optimal point in Case II is the firm's optimal decision as long as $\varphi \geq \varphi_2$.

Appendix E: Proof of Proposition 3

By derivation and inequality reduction, we obtain:

$$\frac{\partial \varphi_h}{\partial a} = \frac{-2a^2c - a^2 + 2ac^2a + c^2 - 1}{(a + ac + a^2)^2} < \frac{-2a^2c - a^2}{(a + ac + a^2)^2} < 0,$$
$$\frac{\partial \varphi_h}{\partial c} = \frac{2a^2 - 2}{a(1 + a + c)^2} < 0.$$

Appendix F: Proof of Proposition 4

By taking derivative of $p_{nb}^* - p_{rb}^*$ with respect to φ_0,
$\frac{\partial (p_{nb}^* - p_{rb}^*)}{\partial \varphi_0} = \frac{a(a-1)g(\varphi_0)}{2(-a^2\varphi_0^2+2a^2\varphi_0-3a^2-2a\varphi_0+2a+1)^2}$, where $g(\varphi_0) = (2a^2 + 2a^3 + a^2c)\varphi_0^2 - (2a + 4a^2 + 2a^3 + 4a^2c)\varphi_0 + (4a + c - 2ac + a^2c - 4a^3)$. The extreme point is $\frac{2a+4a^2+2a^3+4a^2c}{4a^2+4a^3+2a^2c} > 1$, so $g(\varphi_0) > g(\varphi_h) = \frac{2c(1-a^2)(3a^2+2ac+2a+c^2+2c-1)}{(1+a+c)^2} > 0$. So $\frac{\partial (p_{nb}^* - p_{rb}^*)}{\partial \varphi_0} < 0$;

By taking derivative of $p_{nb}^* - p_{rb}^*$ with respect to a,
$\frac{\partial (p_{nb}^* - p_{rb}^*)}{\partial a} = \frac{(\varphi_0 - 2)B}{2(-a^2\varphi_0^2+2a^2\varphi_0-3a^2-2a\varphi_0+2a+1)^2}$.

By taking derivative of B with respect to c, $\frac{\partial B}{\partial c} = -a^2\varphi_0^2 - a^2 + 2a - 1 < 0$. So we only need to prove $B < 0$ when $c = 0$. Assume $c = 0$, then $B = a(\varphi_0 - 2)h(\varphi_0)$, $h(\varphi_0) = a^3\varphi_0^2 - (2a^3 - 4a^2 + 2a)\varphi_0 + (1 - a)(2 + a - 3a^2)$. If $a > 1/2$, then $\Delta = -4a^2(1 - a)^2(2a^2 + 4a - 1) < 0$, so $h(\varphi_0) > 0$, else if $a \leq 1/2$, then the extreme point $\frac{2a^3-4a^2+2a}{2a^3} \geq 1$, so $h(\varphi_0) > h(1) = 2a^3 - 3a + 2 > 0$. Thus $h(\varphi_0) > 0$, $B < 0$, $\frac{\partial p_{nb} - p_{rb}}{\partial a} > 0$.

By taking derivative of $p_{nb}^* - p_{rb}^*$ with respect to c,
$\frac{\partial (p_{nb}^* - p_{rb}^*)}{\partial c} = \frac{(1-a)(2a-a\varphi_0)}{2(-a^2\varphi_0^2+2a^2\varphi_0-3a^2-2a\varphi_0+2a+1)} \geq 0$.

Appendix G: Proof of Proposition 6

First, π_p has three zero points: 0, $\frac{(1-a)(1+a-c)}{a+ac+a^2}$ and $\frac{2a+c}{a}$. Denote $F(\varphi, a, c)$ as the denominator of $\frac{\partial \pi_p}{\partial \varphi}$ and φ_2 as $\frac{(1-a)(1+a-c)}{a+ac+a^2}$. The two zero points of the denominator of π_p are $\frac{a-1\pm\sqrt{2-2a^2}}{a}$. We denote φ_1 as the bigger one and it is easy to find the smaller root is negative. When $\varphi_1 \leq \varphi_2$, $3a^2 + 2c^2 + 2ac + 2a + 2c \leq 1$. Besides $\varphi_2 \geq 1$ is equivalent to $a + 2a^2 + c \leq 1$, $\varphi_1 \geq 1$ is equivalent to $a \leq \frac{\sqrt{2}}{2}$. So when $\varphi_1 \leq \varphi_2$, $\varphi_2 \geq \varphi_1 > 1$. Consider the size relationship of φ_1, φ_2 and 1, there are three potential cases: 1) $\varphi_1 > \varphi_2$. 2) $\varphi_2 \geq \varphi_1 > 1$. 2) $\varphi_1 \geq \varphi_2 > 1$. Define them as Case 1, Case 2 and Case 3.
Case 1 If $\varphi_1 > \varphi_2, \pi_p$ is smooth in $(0, \varphi_2)$, then F at least has one zero points for φ in $(0, \varphi_2)$. Noticing that F is a quartic function for φ, if F has more than one zero point in $(0, \varphi_2)$, then F must have three zero points. Then $\frac{\partial^2 F}{\partial^2 \varphi}$ has a zero point in $(0, \varphi_2)$. We solved $\frac{\partial^2 F}{\partial^2 \varphi} = 0$ and the smaller root is:

$$\varphi_r = \frac{2a + c + 3ac + c^2 + 2 - \sqrt{\Delta}}{2a(1 + a + c)},$$

where

$$\Delta = 10a^4 + 14a^3c + 12a^3 + 11a^2c^2 + 18a^2c - 4a^2 + 4ac + 10ac^2 + 2ac - 4a + c^4 + 4c^3 + 3c^2 - 2c + 2.$$

φ_r must be smaller than φ_2. By simplification,

$$2a^5 + 2a^4 + 6a^3c - 3a^3 + 2a^2c^2 + a^2c - 5a^2 + ac^2 - 8ac - 3a + c^3 - c^2 - 3c - 1 > 0.$$

However,

$$\begin{aligned} &2a^5 + 2a^4 + 6a^3c - 3a^3 + 2a^2c^2 + a^2c - 5a^2 + ac^2 - 8ac - 3a + c^3 - c^2 - 3c - 1 \\ &\leq 5a^2 + 8ac - 3a^3 - 5a^2 + a - 8ac - 3a + c^2 - c^2 - 3c - 1 \\ &= -3a^3 - 2a - 3c - 1 < 0, \end{aligned}$$

which leads to contradiction.

So F has only one zero points in $(0, \varphi_2)$. Also it is obvious that $\frac{\partial \pi_p}{\partial \varphi}$ is positive on 0 and negative on φ_2. So, π_p is convex in $(0, \varphi_2)$ and φ^* is existing and unique.

Note that all the roots of $\frac{\partial^2 F}{\partial^2 \varphi}$ is larger than φ_2 and its quadratic coefficient is positive, F is concave in $(0, \varphi_2)$, so $\varphi^* \leq \varphi_2 + \varphi^* \frac{F(\varphi_2)}{F(0)}$, $\varphi^* \geq -F(0)/\frac{\partial F(0)}{\partial \varphi}$.

Now the implicit function $\varphi^*(a, c)$ exists. First, for any a, $\varphi^*(a, 0) \geq \varphi^*(a, 1)$ by the above estimates. Then assume there exists c_1 and $\frac{\partial \varphi^*(c_1)}{\partial c} = 0$. Because $\frac{\partial \varphi^*(c_1)}{\partial c} = -\frac{\partial F(a,c_1,\varphi^*(c_1))}{\partial c} / \frac{\partial F(a,c_1,\varphi^*(c_1))}{\partial \varphi}$, $\frac{\partial F(a,c_1,\varphi^*(c_1))}{\partial c} = 0$. Also we know $F(a, c_1, \varphi^*(c_1)) = 0$, so c_1 is a fold root for quadratic function $F(a, c, \varphi^*(c_1))$. By $\Delta = 0$, $(a^2\varphi^2 + 2a^2\varphi + 3a^2 - 2a\varphi - 2a - 1)(a^2\varphi^2 - 2a^2\varphi + 3a^2 + 2a\varphi - 2a - 1)^3 = 0$. The four roots of φ are $\frac{a-1\pm\sqrt{2-2a^2}}{a}$, $\frac{1-a\pm\sqrt{2-2a^2}}{a}$. However, in this case $\varphi_2 < \frac{a-1+\sqrt{2-2a^2}}{a} < \frac{1-a+\sqrt{2-2a^2}}{a}$ and $0 > \frac{1-a-\sqrt{2-2a^2}}{a} > \frac{a-1-\sqrt{2-2a^2}}{a}$, so $\varphi^*(c_1)$ can not be any one of them which leads to contradiction. So $\frac{\partial \varphi^*}{\partial c} < 0$ in $(0, \varphi_2)$.

Also the implicit function $\varphi^*(a, c)$ (Here $\varphi^*(a, c)$ is used as the the implicit function induced by $F(a, c, \varphi) = 0$) for a exists, for any c, $\lim_{a\to 0} \varphi^*(a, c) \to +\infty$ by the above estimates and $\varphi^*(1, c) = 0$ because $\varphi_h = 0$. So $\frac{\partial \varphi^*}{\partial a} < 0$ in $0+$ and $1-$. Assume there exists

a_1 which satisfies $\frac{\partial\varphi^*(a_1)}{\partial a}=0$, then a_1 is a multiple root for 5-order function $F(a,c,\varphi^*(a_1))$. Also, there exists a_2 which satisfies $\varphi^*(a_1,c)=\varphi^*(a_2,c)$, so a_2 is a root for 5-order function $F(a,c,\varphi^*(a_1))$.

Now consider $F(a,c,\varphi^*(a_1))$. $F(0,c,\varphi^*(a_1))=c-c^2>0$, but $F\to-\infty$ when $a\to-\infty$, so F has at least one negative real root. We find F have 5 real roots and denote them as a_1,a_1,a_2,a_3,a_4, also by coefficients of F, $a_3a_1a_1a_2a_4=\frac{c^2-c}{\varphi^4-15\varphi^2+10\varphi+6}<0$, so we can assume $a_3<0$ and $a_4>0$.

Consider the root of F as a function of φ between the three roots of π_p. When $a\to 0$, these roots are all negative for all roots are not positive. But when $a\to-1-c$, $\varphi_2\to+\infty$, so there exist root which is bigger than $2-\frac{c}{1+c}$, it is bigger than 1 and thus bigger than $\varphi^*(a_1)$. According to continuity there exists $a_0\in(-1-c,0)$ which satisfy $\varphi^*(a_0)=\varphi^*(a_1)$, so a_0 is a root for 5-order function $F(a,c,\varphi^*(a_1))$, so $a_0=a_3$.

Then $a_4>\frac{a_3a_1a_1a_2a_4}{-1-c}$, $a_4<(a_3+a_2+a_1+a_1+a_4)+1+c$, so $\frac{a_3a_1a_1a_2a_4}{-1-c}<(a_3+a_2+a_1+a_1+a_4)+1+c$. By substituting the coefficients of F we can simplify it to $c^3+6c\varphi^3-9c\varphi^2-12c\varphi+8c+4\varphi^3-12\varphi^2+6\varphi+10<0$. However $c^3+6c\varphi^3-9c\varphi^2-12c\varphi+8c+4\varphi^3-12\varphi^2+6\varphi+10>c^3+6c\varphi^3-9c\varphi^2-12c\varphi+8c+2>c^3-12c\varphi+5c+8>0$ which leads to contradiction. So $\frac{\partial\varphi^*}{\partial a}<0$ in $(0,\varphi_2)$.

Case 2: If $\varphi_2\geq\varphi_1>1$ and $\varphi_2>\frac{2a+c}{a}$. It is obvious that $\frac{\partial\pi_p}{\partial\varphi}$ is positive on 0 and 1 and we will show that F retain positive in $(0,1)$. At this time when $\varphi_1>\frac{2a+c}{a}$, then $\frac{\partial\pi_p}{\partial\varphi}$ is negative on $\frac{2a+c}{a}$, so there is a root for F in $(1,\frac{2a+c}{a})$. By Case I F can not have three or more roots in $(0,\varphi_2)$, so F retain positive in $(0,1)$. When $\varphi_1<\frac{2a+c}{a}$, then there is a root for F in $(\frac{2a+c}{a},\varphi_2)$, F retain positive in $(0,1)$ for the same reason. so in this case $\varphi^*=1$.

Case 3: If $\varphi_2\geq\varphi_1>1$ and $\varphi_2<\frac{2a+c}{a}$. It is obvious that $\frac{\partial\pi_p}{\partial\varphi}$ is positive on 0 and 1 and we will also show that F retain positive in $(0,1)$. Assume F has two roots in $(0,1)$, then F has four real roots: two of them are in $(0,1)$, one of them is in $(\varphi_2,\frac{2a+c}{a})$ and the last one is in $(\frac{2a+c}{a},+\infty)$. Denote z_1,z_2,z_3,z_4 as the four roots. Then $z_4\geq(z_1+z_2+z_3+z_4)-3-\frac{2a+c}{a}$, by using coefficient of F, $z_4\geq\frac{ac+c^2+c+4-4a^2}{a+c+1}$. Besides, $\varphi_2\frac{ac+c^2+c+4-4a^2}{a+c+1}<z_3z_4<z_1z_2+z_1z_3+z_1z_4+z_2z_3+z_2z_4+z_3z_4$ which can be simplified as

$$15a^4+21a^3c+37a^3+3a^2c^2+25a^2c+25a^2-3ac^3+3ac^2+7ac-a+c^3+3c-4>0.$$

However, as $3a^2+2c^2+2ac+2a+2c\leq 1$,

$$\begin{aligned}
&15a^4+21a^3c+37a^3+3a^2c^2+25a^2c+25a^2-3ac^3+3ac^2+7ac-a+c^3+3c-4\\
&\leq 15a^4+21a^3c+37a^2+3a^2c^2+25a^2c+13a^2-3ac^3+3ac^2-ac-9a+c^3-5c-8c^2\\
&\leq 10.5a^4+18a^3c+34a^3+22a^2c+14.5a^2-3ac^3+3ac^2-ac-9a+c^3-5c-8c^2\\
&\leq 10.5a^4+14.5a^3c+34a^3+19a^2c+14.5a^2-6ac^3+0.5ac-9a+c^3-5c-8c^2\\
&\leq \frac{10.5}{27}a+0.5c+\frac{34}{9}a+\frac{19}{9}c+\frac{14.5}{3}a+0.2c-9a+c^2-5c-8c^2\\
&< -4.3c-7c^2<0,
\end{aligned}$$

which leads to contradiction. So, F retains positive in $(0,1)$ and $\varphi^*=1$.

Appendix H: Proof of Proposition 7

It is obvious that $\pi_{p1}^* \leq \pi_{p2}^*$. When $\varphi < \varphi_h$, $\frac{\partial \pi_f}{\partial \varphi} = \frac{a(a-1)(a\varphi-c-2a)(c-ac+a\varphi+a^2\varphi+a^2+ac\varphi-1)}{2(-a^2\varphi^2+2a^2\varphi-3a^2-2*a\varphi+2a+1)^2} <$ 0. So when $\varphi_0 < \varphi^*$, $\pi_{f1}^* > \pi_{f2}^*$. Otherwise, $\pi_{f1}^* \leq \pi_{f2}^*$.

Appendix I: Proof of Proposition 8

According to Proposition 4, When $c + a + 2a^2 \geq 1$, $\frac{\partial(p_n^* - p_r^*)}{\partial \varphi} < 0$. So $i_1 > i_2$ if $\varphi_0 < \varphi^*$; $i_1 \leq i_2$ if $\varphi^* \leq \varphi_0 \leq \varphi_h$.

Appendix J: Results When ρ is not too small

This section shows the results when ρ is not too small. All the results in the text part are examined when ρ is not too small and the main qualitative results still hold. Except for Proposition 6 and a part of Proposition 4 which are hard to prove and examined by numerical study, all the propositions and lemmas of $\rho = 1$ in the text part are rediscussed here and they have the same numbers after “J." (for example, Lemma 3 and Proposition 3 correspond to Lemma J.3 and Proposition J.3). All the observations of $\rho = 1$ in the text part are also reobserved here and they have the same numbers after “J." (for example, Observation 1 corresponds to Observation J.1).

We list the results of the scenario when the transaction fee rate is exogenously given (Subsection J.1), the results of the scenario when the transaction fee rate is endogenously determined by the marketplace (Subsection J.2) and the comparison between the two scenarios (Subsection J.3).

The definitions of a “not too small" ρ is shown later in the first two subsections.

J.1 Scenario with an exogenously given transaction fee rate

In this subsection we assume that the transaction fee rate of the marketplace is exogenously given and denote it as φ_0. We will focus on the market structure and the trade-in incentive. For a given φ_0, define $\rho_0(a, \varphi_0) = \max(\rho_1(\varphi_0), \rho_2(a), \rho_3(a, \varphi_0))$, where

$$\rho_1(\varphi_0) = 2(1 - \varphi_0)/(2 - \varphi_0),$$

$$\rho_2(a) = \begin{cases} 0, \; if \;\; 5092a^4 + 2960a^3 + 1884a^2 - 1736a + 81 < 0 \\ (68a^2 + (5092a^4 + 2960a^3 + 1884a^2 - 1736a + 81)^{1/2} - 9)/(2(13a^2 + 62a)), \\ if \;\; 5092a^4 + 2960a^3 + 1884a^2 - 1736a + 81 \geq 0 \end{cases}$$

$$\rho_3(a, \varphi_0)) = (a - a\varphi_0 + [(a\varphi_0 - a + 1)^2 + (2a - 2a\varphi_0)(8a - 4a\varphi_0)]^{1/2} - 1)/(4a - 2a\varphi_0).$$

Figure 4 shows ρ_0 with respect to a and φ_0.

The results of this subsection is proved when $\rho \geq \rho_0$. As explained in the text part, $\rho \geq 0.95$ holds in general. Therefore as shown in Table 5, this sufficient condition can be satisfied in reality at most times.

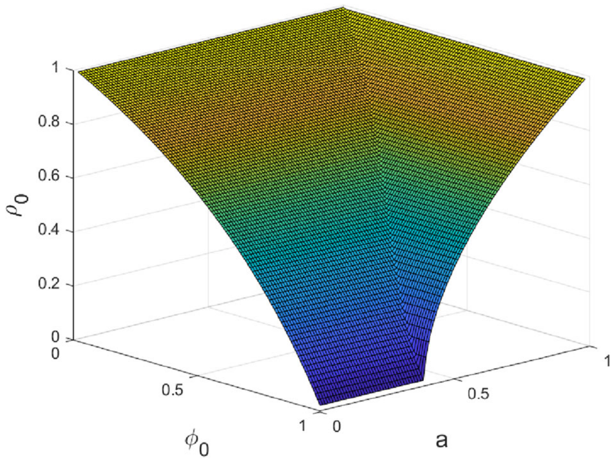


Fig. 4 ρ_0 with respect to a and φ_0

Table 5 Parameter conditions for $\rho \geq \rho_0$

ρ	Parameter condition for $\rho \geq \rho_0$
0.9	$\varphi_0 > 0.19, a < 0.88$
0.95	$\varphi_0 > 0.096, a < 0.95$
0.97	$\varphi_0 > 0.059, a < 0.96$

Define $(p_n^*, p_r^*) = \arg\max_{p_n, p_r} \pi_f$ as the firm's optimal pricing decision (π_f is defined in Eq.(8)). Define

$$
\begin{aligned}
(p_{n2}, p_{r2}) &= (\frac{1 + a\rho + c}{2}, +\infty),\\
p_{n3} =&(a-1)(6a + 3c + 2\rho + 3ac - 8a\varphi_0 + 20a\rho + 4c\rho + 6a\rho^2 + 6a^2\rho + c\rho^2\\
&+ 14a^2\rho^2 + 4a^2\rho^3\\
&- 12a^2\varphi_0\rho^2 - 4a^2\varphi_0\rho^3 - 2ac\varphi_0 + 10ac\rho - 12a\varphi_0\rho + 9ac\rho^2 + 2ac\rho^3\\
&- 4a\varphi_0\rho^2 - 8a^2\varphi_0\rho\\
&- 2ac\varphi_0\rho^2 - 4ac\varphi_0\rho + 6)/(4a^2\varphi_0^2\rho^2 + 8a^2\varphi_0^2\rho + 4a^2\varphi_0^2 + 8a^2\varphi_0\rho^3\\
&- 4a^2\varphi_0\rho^2 - 24a^2\varphi_0\rho\\
&- 12a^2\varphi_0 + 4a^2\rho^4 - 4a^2\rho^3 + 13a^2\rho^2 + 26a^2\rho + 9a^2 + 12a\varphi_0\rho^2 + 16a\varphi_0\rho + 4a\varphi_0\\
&+ 4a\rho^3 - 22a\rho^2 - 16a\rho + 2a + \rho^2 - 10\rho - 7),\\
p_{r3} =&((a-1)(7a + 4c - 10a\varphi_0 + 26a\rho + 4c\rho + 12a^2\varphi_0 - a\rho^2 - 8a^2\rho\\
&- 9a^2 - 4a^2\varphi_0^2 + 11a^2\rho^2\\
&- 2a^2\rho^3 - 10a^2\varphi_0\rho^2 - 4a^2\varphi_0^2\rho + 4ac\rho - 14a\varphi_0\rho + 4ac\rho^2 + 6a^2\varphi_0\rho + 8))\\
&/(4a^2\varphi_0^2\rho^2 + 8a^2\varphi_0^2\rho + 4a^2\varphi_0^2 + 8a^2\varphi_0\rho^3 - 4a^2\varphi_0\rho^2 - 24a^2\varphi_0\rho - 12a^2\varphi_0 + 4a^2\rho^4\\
&- 4a^2\rho^3 + 13a^2\rho^2 + 26a^2\rho + 9a^2 + 12a\varphi_0\rho^2 + 16a\varphi_0\rho + 4a\varphi_0 + 4a\rho^3\\
&- 22a\rho^2 - 16a\rho + 2a + \rho^2 - 10\rho - 7),
\end{aligned}
$$

$$\varphi_h = (3ac - c - \rho - 3a + 6a\rho - 3c\rho - 3a\rho^2 - 3a^2\rho + a^2\rho^2 - 2a^2\rho^3 + 3ac\rho - 2ac\rho^2 + 5) \\ /(2a(\rho + 1)(c + a\rho + 1)).$$

(p_{n2}, p_{r2}) is the optimal point in S_{II} (S_i are defined in Subsubsection 3.3.1). (p_{n3}, p_{r3}) is the optimal point in S_{III}.

Lemma J.3 *For a given φ_0, when $\rho \geq \rho_0$, (p_n^*, p_r^*) can be expressed as follows:*

$$(p_n^*, p_r^*) = \begin{cases} (p_{n2}, p_{r2}), & \text{if } \varphi_0 > \varphi_h. \\ (p_{n3}, p_{r3}), & \text{if } \varphi_0 \leq \varphi_h. \end{cases}$$

Proof First, by the proof of Lemma 3, the Hessian matrix of π_{f4} is not negative definite. Thus there is not a optimal point inside of region S_{IV}. we can ignore this case since it must be dominated by one of the other three cases.

The utility function of the firm in other three cases are:

$$\begin{aligned} \pi_{f1} =& (p_n - c)(2 + 2a\rho - 2p_n - a\varphi_0 - a\rho\varphi_0)/(2a + 4a\rho - 2a\varphi_0 - 2a\rho\varphi_0 + 2), \\ \pi_{f2} =& (1 + a\rho - p_n)(p_n - c)/(2 + 2a\rho), \\ \pi_{f3} =& (1 - ((1 + \rho)p_r - p_n)/(1 - a))(p_r + (1 - \varphi_0)(ap_r - a + a^2 + ap_r\rho)/(1 + a\rho) - c) \\ & + 0.5(p_n - c)(((1 + \rho)p_r - p_n)/(1 - a) \\ & - (p_n - (1 + \rho)((1 + \rho)ap_r - a + a^2)/(a\rho + 1))/(1 - a)). \end{aligned}$$

In Case I and Case II, the utility functions are concave. The optimal points are:

$$\begin{aligned} p_{n1} &= c/2 - (a\varphi_0)/4 + (a\rho)/2 - (a\varphi_0\rho)/4 + 1/2, \\ p_{n2} &= c/2 + (a\rho)/2 + 1/2. \end{aligned}$$

Substituting them into the conditions of region S_I ($(1 - a)(1 + a\rho)/(1 + a\rho\varphi_0 + a\varphi_0 - a) - p_n > 0$) and S_2 ($p_n - (1 - a)(1 + a\rho)/(1 + a\rho\varphi_0 + a\varphi_0 - a) > 0$), respectively, we can obtain $(1 - a)(1 + a\rho)/(1 + a\rho\varphi_0 + a\varphi_0 - a) - p_{n1} > 0$ if and only if $\varphi_0 < \varphi_1$, $p_{n2} - (1 - a)(1 + a\rho)/(1 + a\rho\varphi_0 + a\varphi_0 - a) > 0$ if and only if $\varphi_0 > \varphi_2$. φ_1 and φ_2 are expressed as follows:

$$\begin{aligned} \varphi_1 =& (a + 2ac + a\rho + 3a^2\rho + a^2 - (a^2(\rho + 1)^2(4a^2\rho^2 + 12a^2\rho + a^2 + 8ac\rho \\ & - 4ac - 4a\rho + 10a + 4c^2 + 12c - 7))^{1/2} + 2a^2\rho^2 + 2ac\rho)/(2a^2(\rho + 1)^2), \\ \varphi_2 =& (1 - a)(a\rho - c + 1)/(a(1 + \rho)(c + a\rho + 1)). \end{aligned}$$

Define

$$\begin{aligned} x_1 &= (c + 1 + a\rho)(a^2(\rho + 1)^2(4a^2\rho^2 + 12a^2\rho + a^2 + 8ac\rho \\ &\quad - 4ac - 4a\rho + 10a + 4c^2 + 12c - 7))^{1/2}, \\ x_2 &= a(\rho + 1)(2a^2\rho^2 + 3a^2\rho + 4ac\rho - ac + a\rho + 3a + 2c^2 + 5c - 1). \end{aligned}$$

By simplification, $\varphi_1 - \varphi_2 > 0$ is equivalent to $x_1 < x_2$. As $x_1^2 - x_2^2 = -8a^2(1 + \rho)^2(1 - a)^2(1 + a\rho)(1 + a\rho - c) < 0$, so $\varphi_1 > \varphi_2$. When $\varphi_0 > \varphi_1$, Case I is dominated by Case II.

Denote the Hessian matrix of π_{f3} as H_3.

$$H_3 = \begin{pmatrix} \frac{2}{a-1} & -\frac{3a+\rho-2a\varphi_0+7a\rho+2a\rho^2-2a\varphi_0\rho+3}{2(a\rho+1)(a-1)} \\ -\frac{3a+\rho-2a\varphi_0+7a\rho+2a\rho^2-2a\varphi_0\rho+3}{2(a\rho+1)(a-1)} & \frac{-2(a\varphi_0-a-2a\rho+a\varphi_0\rho-1)(\rho+1)}{(a\rho+1)(a-1)} \end{pmatrix}.$$

Define

$$\begin{aligned} a_3 &= -4a^2(\rho+1)^2, \\ b_3 &= -4a(\rho+1)(3\rho-3a-3a\rho+2a\rho^2+1), \\ c_3 &= -4a^2\rho^4+4a^2\rho^3-13a^2\rho^2-26a^2\rho-9a^2-4a\rho^3+22a\rho^2 \\ &\quad +16a\rho-2a-\rho^2+10\rho+7. \end{aligned}$$

H_3 is negative definite if and only if its determinant $|H_3| > 0$, which is equivalent to $a_3\varphi_0^2 + b_3\varphi_0 + c_3 > 0$. It is not difficult to note that $a_3 < 0, c_3 > 0$, so when $\varphi_0 > 0$, $a_3\varphi_0^2 + b_3\varphi_0 + c_3 > 0$ if and only if φ_0 is smaller than the larger root of the quadratic equation $a_3\varphi_0^2 + b_3\varphi_0 + c_3 = 0$. Denote this root as φ_3,

$$\varphi_3 = \frac{-(a+4a\rho+3a\rho^2-6a^2\rho-3a^2-a^2\rho^2+2a^2\rho^3-2(-2a^2(a\rho+1)(a-1)(\rho+1)^4)^{1/2})}{2a^2(\rho+1)^2}.$$

Therefore, when $\varphi_0 < \varphi_3$, H_3 is negative definite. The optimal point (p_{n3}, p_{r3}) is the optimal solution in Case III.

Consider the conditions for Case III:

$$p_n - (1+\rho)p_r + 1 - a > 0. \tag{13a}$$

$$p_n - \frac{(1+2\rho)a-(1+\rho)a\varphi_0+1}{(2\rho+\varphi_0-\varphi_0\rho-1)a+1}p_r + \frac{a(1-a)(1-\varphi_0)}{(2\rho+\varphi_0-\varphi_0\rho-1)a+1} > 0. \tag{13b}$$

By solving the equations that makes the two sides of the unequal sign equal, when $\rho \geq \rho_1(\varphi_0)$, $\frac{a(1-a)(1-\varphi_0)}{(2\rho+\varphi_0-\varphi_0\rho-1)a+1} > 1-a$. When $\rho \geq \rho_3(a,\varphi_0)$, $\frac{(1+2\rho)a-(1+\rho)a\varphi_0+1}{(2\rho+\varphi_0-\varphi_0\rho-1)a+1} < 1+\rho$. Thus, if p_{n3}, p_{r3} satisfy Eq. (13a), they also satisfy Eq. (13b). Substitute p_{n3}, p_{r3} into Eq. (13a), we can obtain $p_{n3} - (1+\rho)p_{r3} + 1 - a > 0$ if and only if $\varphi_0 < \varphi_4$, where

$$\varphi_4 = \frac{3ac-c-\rho-3a+6a\rho-3c\rho-3a\rho^2-3a^2\rho+a^2\rho^2-2a^2\rho^3+3ac\rho-2ac\rho^2+5}{2a(\rho+1)(c+a\rho+1)}.$$

Now compare φ_3 and φ_4. Define

$$x_3 = a(\rho+1)(c+2\rho-3ac-4a\rho+3c\rho+4a\rho^2-2a^2\rho^2+2a^2\rho^3-3ac\rho+2ac\rho^2-2).$$
$$x_4 = (1+c+a\rho)2^{1/2}(-a^2(a\rho+1)(a-1)(\rho+1)^4)^{1/2}.$$

By simplification, $\varphi_3 - \varphi_4 > 0$ is equivalent to $x_3 < x_4$. Note that x_3, x_4 is linear in c, we only need to prove $x_3 < x_4$ when $c = 0$ and $c = 1$. When $c = 0$, $x_3 < 0, x_4 > 0$. When $c = 1$,

$$\begin{aligned} x_3^2 - x_4^2 =& a^2(\rho+1)^2(4a^4\rho^6-6a^4\rho^5+8a^4\rho^4+2a^4\rho^3+22a^3\rho^5-46a^3\rho^4+34a^3\rho^3 \\ &+22a^3\rho^2+46a^2\rho^4-112a^2\rho^3+39a^2\rho^2+58a^2\rho+9a^2+44a\rho^3-106a\rho^2 \\ &-16a\rho+14a+17\rho^2-26\rho-7) \\ <& a^2(\rho+1)^2(-2a^4\rho^5-14a^3\rho^4-10a^2\rho^3+39a^2\rho^2+58a^2\rho \\ & a^2+44a\rho^3-106a\rho^2-16a\rho+14a \end{aligned}$$

$$
\begin{aligned}
&+17\rho^2-26\rho-7) < a^2(\rho+1)^2(-10a^2\rho^3+7a^2\rho^2+58a^2\rho\\
&a^2+44a\rho^3-106a\rho^2+14a+17\rho^2-26\rho-7)\\
&<a^2(\rho+1)^2(-13a^2\rho^2+68a^2\rho+9a^2-52a\rho^2+14a+17\rho^2-26\rho-7)\\
&<a^2(\rho+1)^2(-13a^2\rho^2+68a^2\rho+9a^2-62a\rho^2+14a-9\rho-7).
\end{aligned}
$$

When $\rho \geq \rho_2(a)$, the quadratic function $-13a^2\rho^2+68a^2\rho+9a^2-62a\rho^2+14a-9\rho-7$ is negative. So, $\varphi_3 > \varphi_4$. By the similar method, we can also obtain $\varphi_4 > \varphi_1$.

Therefore, when $\varphi_0 < \varphi_4$, Case I and Case II are dominated by Case III because they can be regarded as a special case of Case III on the hyperplane of $p_n - (1+\rho)p_r + 1 - a = 0$, $p_n - \frac{(1+2\rho)a-(1+\rho)a\varphi_0+1}{(2\rho+\varphi_0-\varphi_0\rho-1)a+1} p_r + \frac{a(1-a)(1-\varphi_0)}{(2\rho+\varphi_0-\varphi_0\rho-1)a+1} = 0$. Since $\varphi_3 > \varphi_4$, the optimal point (p_{n3}, p_{r3}) in Case III is the optimal solution of Eq. (9). When $\varphi_0 > \varphi_4$, Case III must be dominated since the optimal solution in Case III is on the boundary. Since $\varphi_4 > \varphi_1$, Case I is dominated by Case II, the optimal point $(p_{n2}, +\inf)$ in Case II is the optimal solution of Equation (9). □

Proposition J.2 is directly derived from Lemma J.3.

Proposition J.2 *When $\varphi_0 \leq \varphi_h$, the condition of Case III is satisfied under which trade-in consumers exist and new products are purchased by both TT-type and NU-type consumers; otherwise, the condition of Case II is satisfied under which trade-in consumers do not exist and new products are purchased by NU-type consumers only.*

Similar to Proposition 2, Proposition J.2 indicates that the firm adopts trade-in strategy and sell the recycled products on the P2P marketplace as long as the transaction fee rate does not exceed φ_h. Thus, φ_h is the firm's maximum acceptable transaction fee rate for trade-in strategy. The threshold φ_h is analyzed in Proposition J.3.

Proposition J.3 $\frac{\partial \varphi_h}{\partial a} < 0$, $\frac{\partial \varphi_h}{\partial c} < 0$.

Proof By simplification, $\frac{\partial \varphi_h}{\partial a} < 0$ is equivalent to $2a^2c\rho^2+6a^2c\rho-a^2\rho^3+5a^2\rho^2-6ac\rho^2-2ac\rho-2a\rho^2+10a\rho-3c^2\rho-c^2-4c\rho+4c-\rho+5>0$.

$$
\begin{aligned}
&2a^2c\rho^2+6a^2c\rho-a^2\rho^3+5a^2\rho^2-6ac\rho^2-2ac\rho-2a\rho^2+10a\rho-3c^2\rho\\
&\quad -c^2-4c\rho+4c-\rho+5 > 2a^2c\rho^2+6a^2c\rho4a^2\rho^2\\
&\quad -6ac\rho^2-2ac\rho-2a\rho^2+10a\rho-3c^2\rho-c^2-4c\rho+4c-\rho+5\\
&\quad > 2a^2c\rho^2+6a^2c\rho4a^2\rho^2-3c^2\rho-c^2-4c\rho+4c-\rho+5\\
&\quad > 2a^2c\rho^2+6a^2c\rho4a^2\rho^2-4c\rho-\rho+5>0.
\end{aligned}
$$

By simplification, $\frac{\partial \varphi_h}{\partial c} < 0$ is equivalent to $(1-a)(3+\rho)(1+a\rho)/(a(1+\rho)(1+a\rho+c)^2) > 0$, which is obvious. □

Similar to Proposition 3, Proposition J.3 shows that the maximum acceptable transaction fee rate decreases in durability and production costs.

Define

$$
\begin{aligned}
g(a,c,\rho) =&2a^2\rho^4+5a^2\rho^3+8a^2\rho^2+9a^2\rho+4ac\rho^3+8ac\rho^2+4ac\rho+3a\rho^3+3a\rho^2\\
&+a\rho+9a+2c^2\rho^2+4c^2\rho+2c^2+4c\rho^2+8c\rho+4c+\rho^2-2\rho-7>0.
\end{aligned}
$$

It is not difficult to note that $\frac{g(a,c,\rho)}{\partial c} > 0$, $\frac{g(a,c,\rho)}{\partial a} > 0$.

The following proposition examines how the transaction fee rate and production costs effect on trade-in incentives.

Proposition J.4 (1) *When* $\varphi_0 \le \varphi_h$ *and* $a < -(\rho - 1)/(3\rho - 2\varphi_0 - 2\varphi_0\rho + 2\rho^2 + 3)$, $\frac{\partial(p_n^* - p_r^*)}{\partial c} < 0$*; otherwise,* $\frac{\partial(p_n^* - p_r^*)}{\partial c} \ge 0$.
(2) *When* $\varphi_0 \le \varphi_h$, $\rho + a + ap + 2a\rho^2 > 1$ *and* $g(a, c, \rho) > 0$, $\frac{\partial(p_n^* - p_r^*)}{\partial \varphi_0} < 0$.

Proof By simplification, $\frac{\partial(p_n^* - p_r^*)}{\partial c} < 0$ is equivalent to $3a + \rho - 2a\varphi_0 + 3a\rho + 2a\rho^2 - 2a\varphi_0\rho - 1 < 0$ which is linear about a. By taking derivative of $p_n^* - p_r^*$ with respect to φ_0,

$$\frac{\partial p_n^* - p_r^*}{\partial \varphi_0} = 2a(a-1)f,$$

where

$$\begin{aligned} f = & 34a + 9c - 5\rho - 14ac - 20a\varphi_0 + 103a\rho + 32c\rho + 9a^2c \\ & + 12a^2\varphi_0 + 89a\rho^2 - 31a^2\rho + 23a\rho^3 \\ & - 27a^3\rho + 17a\rho^4 - 10a\rho^5 + 30c\rho^2 - 7c\rho^4 - 27a^2 \\ & + \rho^2 + 9\rho^3 - 2\rho^4 + 4a^2\varphi_0^2 + 59a^2\rho^2 \\ & + 55a^2\rho^3 - 65a^3\rho^2 - 28a^2\rho^4 - 47a^3\rho^3 - 12a^2\rho^5 - 39a^3\rho^4 \\ & - 16a^2\rho^6 - 50a^3\rho^5 - 20a^3\rho^6 \\ & - 8a^3\rho^7 + 4a^2c\varphi_0^2 + 34a^2c\rho^2 + 24a^2c\rho^3 + a^2c\rho^4 - 20a^2c\rho^5 \\ & - 12a^2c\rho^6 - 128a^2\varphi_0\rho^2 \\ & + 28a^2\varphi_0^2\rho - 104a^2\varphi_0\rho^3 - 4a^3\varphi_0\rho^2 + 4a^3\varphi_0^2\rho \\ & - 12a^2\varphi_0\rho^4 - 68a^3\varphi_0\rho^3 - 60a^3\varphi_0\rho^4 \\ & - 8a^3\varphi_0\rho^5 + 4ac\varphi_0 - 36ac\rho - 60a\varphi_0\rho + 52a^2\varphi_0^2\rho^2 + 36a^2\varphi_0^2\rho^3 \\ & + 28a^3\varphi_0^2\rho^2 + 8a^2\varphi_0^2\rho^4 \\ & + 52a^3\varphi_0^2\rho^3 + 36a^3\varphi_0^2\rho^4 + 8a^3\varphi_0^2\rho^5 - 12a^2c\varphi_0 - 20ac\rho^2 + 28a^2c\rho \\ & - 8ac\rho^3 - 30ac\rho^4 - 20ac\rho^5 \\ & - 44a\varphi_0\rho^2 - 24a^2\varphi_0\rho - 4a\varphi_0\rho^3 + 12a^3\varphi_0\rho + 24a^2c\varphi_0^2\rho^2 + 16a^2c\varphi_0^2\rho^3 \\ & + 4a^2c\varphi_0^2\rho^4 - 48a^2c\varphi_0\rho \\ & - 8ac\varphi_0\rho^3 - 4ac\varphi_0\rho^4 - 80a^2c\varphi_0\rho^2 + 16a^2c\varphi_0^2\rho - 72a^2c\varphi_0\rho^3 \\ & - 36a^2c\varphi_0\rho^4 - 8a^2c\varphi_0\rho^5 + 8ac\varphi_0\rho - 3. \end{aligned}$$

By taking derivative of f with respect to φ_0,

$$\begin{aligned} \frac{\partial f}{\partial \varphi_0} = & 3a + c - 10\rho - 3ac + 2a\varphi_0 - 9a\rho + c\rho - 23a\rho^2 + 3a^2\rho - 3a\rho^3 \\ & - c\rho^2 - c\rho^3 - \rho^2 - 4a^2\rho^2 \\ & - 13a^2\rho^3 - 2a^2\rho^4 + 12a^2\varphi_0\rho^2 + 14a^2\varphi_0\rho^3 + 4a^2\varphi_0\rho^4 + 2ac\varphi_0 \\ & - 9ac\rho + 12a\varphi_0\rho - 11ac\rho^2 \end{aligned}$$

$$- 7ac\rho^3 - 2ac\rho^4 + 14a\varphi_0\rho^2 + 2a^2\varphi_0\rho + 4a\varphi_0\rho^3 + 6ac\varphi_0\rho^2 + 2ac\varphi_0\rho^3 + 6ac\varphi_0\rho - 5.$$

By taking derivative of $\frac{\partial f}{\partial \varphi_0}$ with respect to c,

$$\frac{\partial(\frac{\partial f}{\partial \varphi_0})}{\partial c} = (\rho+1)^2(2a\varphi_0 - \rho - 3a - 3a\rho - 2a\rho^2 + 2a\varphi_0\rho + 1) < (\rho+1)^2(-\rho - a - a\rho - 2a\rho^2 + 1) < 0.$$

When $c = 0$,

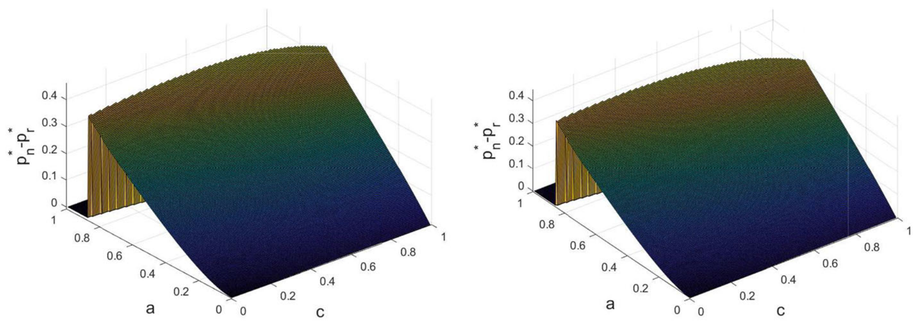
$$\frac{\partial f}{\partial \varphi_0} = (a\rho+1)(3a - 10\rho + 2a\varphi_0 - 4a\rho - 13a\rho^2 - 2a\rho^3 - \rho^2 + 12a\varphi_0\rho + 14a\varphi_0\rho^2 + 4a\varphi_0\rho^3 - 5) < -10\rho - 4a\rho - 13a\rho^2 - 2a\rho^3 - \rho^2 + 12a\varphi_0\rho + 14a\varphi_0\rho^2 + 4a\varphi_0\rho^3 < 0.$$

Therefore $\frac{\partial f}{\partial \varphi_0} < 0$. When $\varphi_0 = \varphi_h$,

$$f = 4(a-1)(a\rho+1)(\rho - c - a\rho - 2c\rho + a\rho^2 - c\rho^2 - 1)(2a^2\rho^4 + 5a^2\rho^3 + 8a^2\rho^2 + 9a^2\rho + 4ac\rho^3 + 8ac\rho^2 + 4ac\rho + 3a\rho^3 + 3a\rho^2 + a\rho + 9a + 2c^2\rho^2 + 4c^2\rho + 2c^2 + 4c\rho^2 + 8c\rho + 4c + \rho^2 - 2\rho - 7)/(c + a\rho + 1)^2 > 0.$$

So when $\varphi_0 < \varphi_h$, $f > 0$, $\frac{\partial(p_n^* - p_r^*)}{\partial \varphi_0} < 0$. □

Note that the case of $a \le \frac{1-\rho}{3\rho - 2\varphi - 2\varphi\rho + 2\rho^2 + 3}$ can be ignored since it actually requires a very low a (according to previous approximation, in this case $a < 0.01$). Thus, similar to Proposition 4, Proposition J.4 shows that when trade-in consumers exist, the trade-in incentive increases in production costs. When trade-in consumers exist and both durability and production costs are not too low ($\rho + a + ap + 2a\rho^2 > 1$ and $g(a, c, \rho) > 0$), the trade-in incentive decreases in the transaction fee rate. Besides, it is hard for us to examine



(a) ($\varphi_0 = 0.2, \rho = 0.95$)

(b) ($\varphi_0 = 0.2, \rho = 0.97$)

Fig. 5 Trade-in Incentive with Respect to a and c

how durability effect on trade-in incentives. The numerical result shows that when $\varphi_0 \leq \varphi_h$, the trade-in incentive increases in durability a (Fig. 5)[4].

Figure 5 show that when trade-in consumers exist, the trade-in incentive increases in durability and production costs. The result of Proposition 4 is now examined by Proposition J.4 and Fig. 5.

J.2 Scenario with an endogenous transaction fee rate

In this scenario, we consider the transaction fee rate φ as a decision variable of the P2P marketplace. We discuss the optimal transaction fee rate of the P2P marketplace, and analyze the trade-in incentive and the marketplace's profit through a numerical study. In order to define the P2P marketplace's transaction fee rate in stationary equilibrium φ^*, we first assume φ^* satisfies $\rho \geq \rho_0(a, \varphi^*)$. Then we obtain a numerical approximation of φ^* under this assumption and show that $\rho \geq \rho_0(a, \varphi^*)$ holds in most of the parameter area.

We first prove that the maximum acceptable transaction fee rate for the firm φ_h is always positive.

Lemma J.4 $\varphi_h \geq 0$.

Proof $\varphi_h = \frac{x}{2a(\rho+1)(c+a\rho+1)}$, where

$$x = 3ac - c - \rho - 3a + 6a\rho - 3c\rho - 3a\rho^2 - 3a^2\rho + a^2\rho^2 - 2a^2\rho^3 + 3ac\rho - 2ac\rho^2 + 5.$$

By taking derivative of x with respect to c, $\frac{\partial x}{\partial c} = 3a - 3\rho + 3a\rho - 2a\rho^2 - 1 < 1.5a - 3\rho - 0.5a\rho^2 - 1 < 0$. When $c = 1$, $x = -2a^2\rho^3 + a^2\rho^2 - 3a^2\rho - 5a\rho^2 + 9a\rho - 4\rho + 4 \geq -2a^2\rho^3 + a^2\rho^2 - 3a^2\rho - 5a\rho^2 + 9a\rho \geq 0$. Therefore $x \geq 0$. □

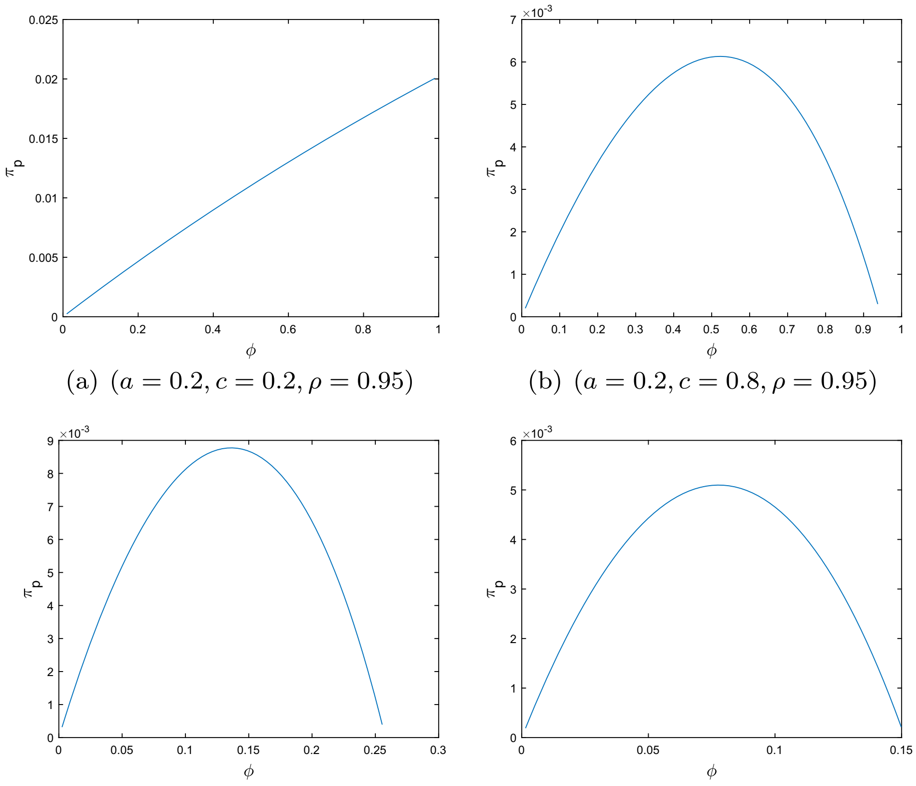
Similar to Proposition 5, when the P2P marketplace determines the transaction fee rate, the transaction fee rate will always be less than the maximum acceptable transaction fee rate for the firm since it is always greater than 0.

Proposition J.5 *When $\rho \geq \rho_0(a, \varphi^*)$, the condition of Case III is always satisfied in stationary equilibrium under which trade-in consumers exist and new products are purchased by both TT-type and NU-type consumers.*

Define

$$\begin{aligned}
\pi_p(\varphi) = &(a\varphi(a-1)(\rho+1)(9a + 4c - 2\rho - 6a\varphi + 16a\rho + 8c\rho + 5a\rho^2 + 2a\rho^3 \\
&+ 4c\rho^2 + \rho^2 - 8a\varphi\rho - 2a\varphi\rho^2 + 1)(3a + c + \rho - 3ac \\
&+ 2a\varphi - 6a\rho + 3c\rho + 3a\rho^2 + 3a^2\rho - a^2\rho^2 + 2a^2\rho^3 \\
&+ 2a^2\varphi\rho^2 + 2ac\varphi - 3ac\rho + 2a\varphi\rho + 2ac\rho^2 \\
&+ 2a^2\varphi\rho + 2ac\varphi\rho - 5))/(4a^2\varphi^2\rho^2 + 8a^2\varphi^2\rho \\
&+ 4a^2\varphi^2 + 8a^2\varphi\rho^3 - 4a^2\varphi\rho^2 - 24a^2\varphi\rho - 12a^2\varphi + 4a^2\rho^4 \\
&- 4a^2\rho^3 + 13a^2\rho^2 + 26a^2\rho \\
&+ 9a^2 + 12a\varphi\rho^2 + 16a\varphi\rho + 4a\varphi + 4a\rho^3 - 22a\rho^2 - 16a\rho \\
&+ 2a + \rho^2 - 10\rho - 7)^2.
\end{aligned}$$

[4] In Fig. 5, for ease of expression, trade-in incentive is assumed to be 0 when $\varphi_0 > \varphi_h$.



(a) $(a = 0.2, c = 0.2, \rho = 0.95)$ (b) $(a = 0.2, c = 0.8, \rho = 0.95)$

(c) $(a = 0.8, c = 0.2, \rho = 0.95)$ (d) $(a = 0.8, c = 0.8, \rho = 0.95)$

Fig. 6 π_p with respect to φ

According to Proposition J.4 and Eq. (5), the optimization problem of the P2P marketplace can be expressed as follows:

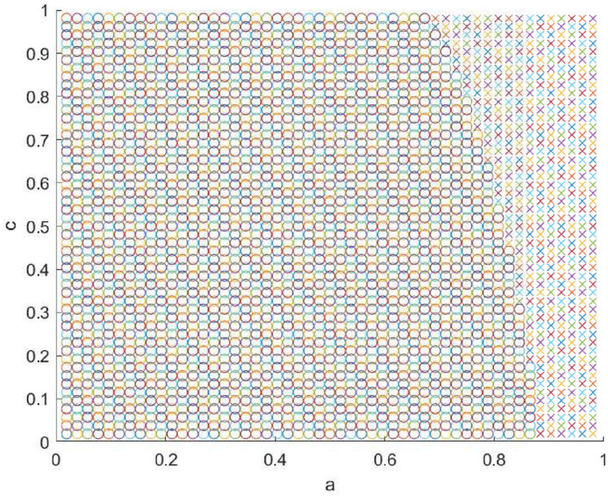
$$\begin{aligned} &\max_{\varphi} \pi_p(\varphi) \\ &s.t.\ 0 \le \varphi \le \min\{1, \varphi_h\}. \end{aligned} \tag{14}$$

Denote $\varphi^*(a, c, \rho)$ as the optimal solution of (5). The numerical result shows that there exists a unique $\varphi^*(a, c, \rho)$ (Fig. 6).

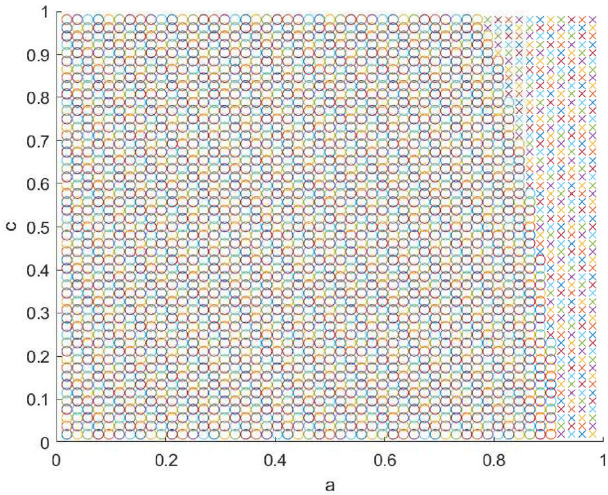
Then, denote $\rho_0^{'}(a, c, \rho) = \rho_0(a, c, \varphi^*(a, c, \rho))$. As shown in Fig. 7, the area of "o" indicates $\rho \ge \rho_0^{'}(a, c, \rho)$ and the area of "x" indicates $\rho < \rho_0^{'}(a, c, \rho)$. We find the assumption $\rho \ge \rho_0^{'}(a, c, \rho)$ can cover most of the parameter area. Thus the results in this subsection do not lose much generality.

In addition, the numerical result shows that the optimal transaction fee rate $\varphi^*(a, c, \rho)$ decreases in durability a and production costs c when $\rho \ge \rho_0^{'}(a, c, \rho)$ (Fig. 8).

In summary, when $\rho \ge \rho_0^{'}(a, c, \rho)$, according to the numerical results there exists a unique $\varphi^*(a, c, \rho)$ and $\varphi^*(a, c, \rho)$ decreases in both durability a and production costs. The major results of Proposition 6 are examined. Now, we use numerical study to analyze the trade-in incentive and the P2P marketplace's profit. We first find the trade-in incentive increases



(a) ($\rho = 0.95$)



(b) ($\rho = 0.97$)

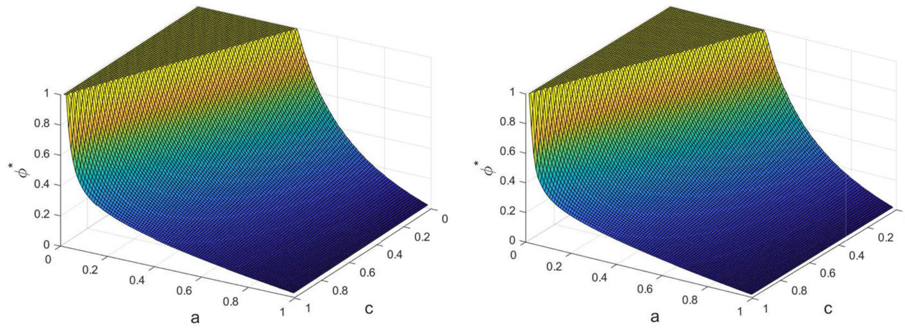
Fig. 7 Parameter area of $\rho \geq \rho_0^{'}(a, c, \rho)$ and $\rho \leq \rho_0^{'}(a, c, \rho)$

in both durability and production costs when $\rho \geq \rho_0^{'}(a, c, \rho)$, which is similar to that in Subsection 1 (Fig. 9).

In the following, we investigate the profit of the P2P marketplace since it is now a decision-maker.

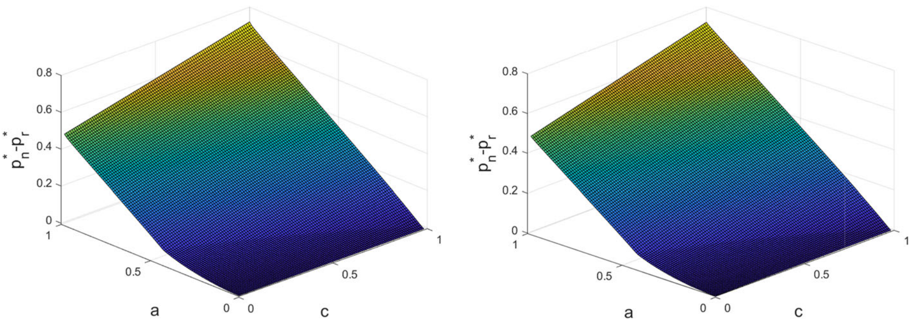
Observation J.1 (1) π_p^* *first increases and then decreases in* a. (2) π_p^* *decreases in* c.

According to Fig. 10, when $\rho \geq \rho_0^{'}(a, c, \rho)$, the profit of the P2P marketplace first increases and then decreases in durability, while it always decreases in production costs. The result is similar to Observation 1.



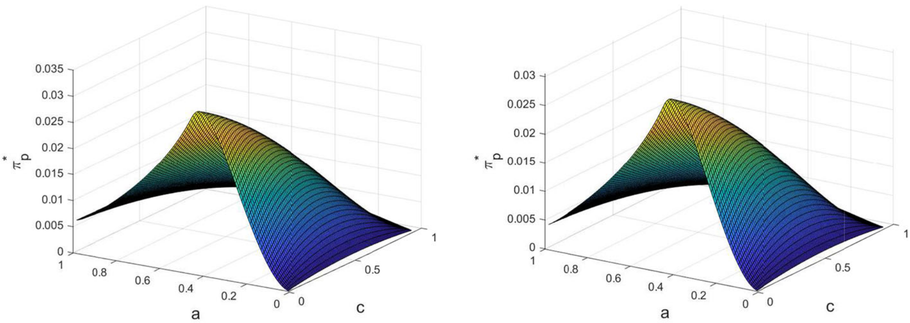
(a) ($\rho = 0.95$) (b) ($\rho = 0.97$)

Fig. 8 φ^* with respect to a and c



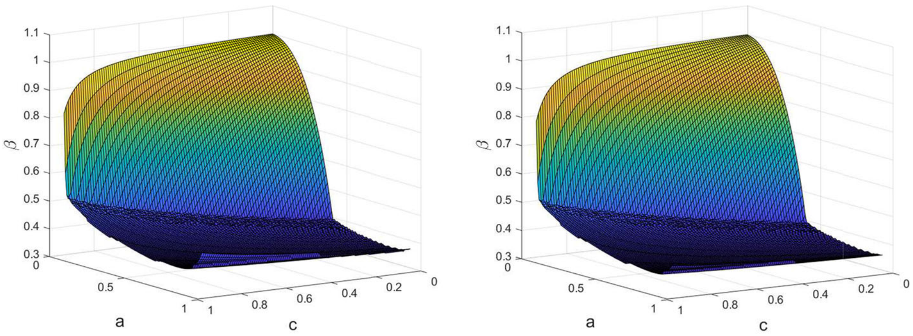
(a) ($\rho = 0.95$) (b) ($\rho = 0.97$)

Fig. 9 Trade-in incentive with respect to a and c



(a) ($\rho = 0.95$) (b) ($\rho = 0.97$)

Fig. 10 Marketplace's profit with respect to a and c



(a) ($\rho = 0.95$) (b) ($\rho = 0.97$)

Fig. 11 Proportion Of trade-in consumers with respect to a and c

J.3 Comparison between the two scenarios

Denote the scenario in Subsection J.1 as Scenario J.1, and the scenario in Subsection J.2 as Scenario J.2. We have found that trade-in strategy is always adopted in Scenario J.2, while trade-in strategy is not adopted in Scenario J.1 when product durability and production cost are high. This subsection further compares these two scenarios from the perspectives of the proportion of trade-in consumers which is denoted as β, the marketplace and the firm's profits, and the trade-in incentive.

Observation J.2 *In Scenario J.2,* $\beta > 0.3$ *always holds.* β *approaches 1 when both a and c approach* 0.

According to Fig. 11, similar to Observation 2, when durability and production costs are both low, most of the consumers will purchase new product with trade-ins, while when durability and production costs are both high, there is still a proportion of trade-in consumers.

Denote π^*_{p1}, π^*_{f1} as the marketplace and the firm's profits in Scenario J.1 and π^*_{p2}, π^*_{f2} as the marketplace and the firm's profits in Scenario J.2 respectively. Denote i_1, i_2 as the trade-in incentive in Scenario J.1 and Scenario J.2, respectively. The following two propositions compare the profits and the trade-in incentives in the two scenarios.

Proposition J.7 *1)* $\pi^*_{p1} \le \pi^*_{p2}$.
2) When $\varphi_0 < \varphi^*$, $\pi^*_{f1} > \pi^*_{f2}$. *When* $\varphi^* \le \varphi_0 \le \varphi_h$, $\pi^*_{f1} \le \pi^*_{f2}$.

Proof When $\varphi < \varphi_h$, it is obvious that $\pi^*_{p1} \le \pi^*_{p2}$.

By taking derivative of π_f with respect to φ,

$$
\begin{aligned}
\frac{\partial \pi_f}{\partial \varphi} =& a(\rho+1)(a-1)(6a\varphi - 6\rho - 9a - 16a\rho \\
& - 5a\rho^2 - 2a\rho^3 - 5\rho^2 + 8a\varphi\rho + 2a\varphi\rho^2 - 5) \\
& (4\rho + 4a\varphi - 9a\rho + 5a\rho^2 + 3a^2\rho - a^2\rho^2 + 2a^2\rho^3 \\
& + 2a^2\varphi\rho^2 + 4a\varphi\rho + 2a^2\varphi\rho - 4) \\
& /(4a^2\varphi^2\rho^2 + 8a^2\varphi^2\rho + 4a^2\varphi^2 + 8a^2\varphi\rho^3 - 4a^2\varphi\rho^2 - 24a^2\varphi\rho \\
& - 12a^2\varphi + 4a^2\rho^4 - 4a^2\rho^3
\end{aligned}
$$

$$+13a^2\rho^2+26a^2\rho+9a^2+12a\varphi\rho^2+16a\varphi\rho+4a\varphi$$
$$+4a\rho^3-22a\rho^2-16a\rho+2a+\rho^2-10\rho-7)^2.$$

When $\varphi<\varphi_h, 4\rho+4a\varphi-9a\rho+5a\rho^2+3a^2\rho-a^2\rho^2+2a^2\rho^3+2a^2\varphi\rho^2+4a\varphi\rho+2a^2\varphi\rho-4<0$. Note that

$$6a\varphi-6\rho-9a-16a\rho-5a\rho^2-2a\rho^3-5\rho^2+8a\varphi\rho+2a\varphi\rho^2-5$$
$$\leq -6\rho-3a-6a\rho-5a\rho^2-2a\rho^3-5\rho^2-5<0.$$

So $\frac{\partial \pi_f}{\partial \varphi}<0$. When $\varphi_0<\varphi^*$, $\pi_{f1}^*>\pi_{f2}^*$. Otherwise, $\pi_{f1}^*\leq\pi_{f2}^*$. □

Similar to Proposition 7, Proposition J.7 shows that: 1) The marketplace 's profit in Scenario J.2 is always higher. 2) When trade-in strategy is adopted ($\varphi_0\leq\varphi_h$), which scenario the firm can obtain a higher profit in depends on which scenario features a lower transaction fee rate.

Proposition J.8 *When $\rho+a+ap+2a\rho^2>1$ and $g(a,c,\varphi)>0$, $i_1>i_2$ if $\varphi_0<\varphi^*$; $i_1\leq i_2$ if $\varphi^*\leq\varphi_0\leq\varphi_h$.*

Proof According to Proposition J.4, When $\varphi<\varphi_h$, $\rho+a+ap+2a\rho^2>1$ and $g(a,c,\varphi)>0$, $\frac{\partial p_n^*-p_r^*}{\partial\varphi}<0$. So $i_1>i_2$ if $\varphi_0<\varphi^*$; $i_1\leq i_2$ if $\varphi^*\leq\varphi_0\leq\varphi_h$. □

Similar to Proposition 8, Proposition J.8 indicates that when trade-in strategy is adopted and both durability and production costs are not too low, the scenario in which the trade-in incentive is higher depends on which scenario features a lower transaction fee rate.

References

Agrawal, V., Ferguson, M., & Souza, G. (2008). Trade-in rebates for price discrimination and product recovery. *IEEE Transactions on Engineering Management, 63*(3), 326–339.

Alev, L., Agrawal, V., & Atasu, A. (2020). Extended producer responsibility for durable products. *Manufacturing and Service Operations Management, 22*(2), 364–382.

Bian, Y., Xie, J., Archibald, T. W., & Sun, Y. (2019). Optimal extended warranty strategy: Offering trade-in service or not. *European Journal of Operational Research, 278*(1), 240–254.

Cao, K., Wang, J., Dou, G., & Zhang, Q. (2018). Optimal trade-in strategy of retailers with online and offline sales channels. *Computers and Industrial Engineering, 123*, 148–156.

Cao, K., Bo, Q., & He, Y. (2018). Optimal trade-in and third-party collection authorization strategies under trade-in subsidy policy. *Kybernetes, 47*(5), 854–872.

Cao, K., Xu, X., Bian, Y., & Sun, Y. (2018). Optimal trade-in strategy of business-to-consumer platform with dual-format retailing model. *Omega, 82*, 181–192.

Chen, J., & Hsu, Y. (2015). Trade-in strategy for a durable goods firm with recovery cost. *Journal of Industrial and Production Engineering, 32*(6), 396–407.

Chen, J., & Hsu, Y. (2017). Revenue management for durable goods using trade-ins with certified pre-owned options. *International Journal of Production Economics, 186*, 55–70.

Chi, X., Fan, Z. P., & Wang, X. (2022). The entry quality threshold setting and commission rate contract selection of a peer-to-peer service sharing platform. *Kybernetes*. https://doi.org/10.1108/K-02-2022-0248

Choi, T. M., & He, Y. (2019). Peer-to-peer collaborative consumption for fashion products in the sharing economy: Platform operations. *Transportation Research Part E: Logistics and Transportation Review, 126*, 49–65.

De, P. (2017). Closed-loop supply chain coordination through incentives with asymmetric information. *Annals of Operations Research, 253*(1), 133–167.

Dong, C., Lei, Y., & Liu, Q. (2022). (Un)conditional collection policies on used products with strategic customers. *Production and Operations Management*. https://doi.org/10.1111/poms.13826

Fan, X., Guo, X., & Wang, S. (2022). Optimal collection delegation strategies in a retail-/dual-channel supply chain with trade-in programs. *European Journal of Operational Research, 303*(2), 633–649.
Feng, L., Li, Y., & Fan, C. (2020). Optimization of pricing and quality choice with the coexistence of secondary market and trade-in program. *Annals of Operations Research*. https://doi.org/10.1007/s10479-020-03588-7
Feng, L., Zheng, X., Govindan, K., & Xue, K. (2019). Does the presence of secondary market platform really hurt the firm? *International Journal of Production Economics, 213*, 55–68.
Guo, X., Fan, X., & Wang, S. (2022). Trade-in for cash or for new? Optimal pricing decisions under the government subsidy policy. *Annals of Operations Research*. https://doi.org/10.1007/s10479-022-04664-w
Huang, S., Yang, Y., & Anderson, K. (2001). A theory of finitely durable goods monopoly with used-goods market and transaction costs. *Management Science, 47*(11), 1515–1532.
Jiang, L. F., Dimitrov, S., & Mantin, B. (2017). P2P marketplaces and retailing in the presence of consumers' valuation uncertainty. *Production and Operations Management, 26*(3), 509–524.
Konishi, H., & Sandfort, M. T. (2002). Existence of stationary equilibrium in the markets for new and used durable goods. *Journal of Economic Dynamics and Control, 26*(6), 1029–1052.
Li, K., & Xiao, L. (2019). Peer-to-peer short-term sharing on the bilateral commission platform. In *2019 16th international conference on service systems and service management (ICSSSM)*. https://doi.org/10.1109/ICSSSM.2019.8887628
Li, K. J., & Xu, S. H. (2015). The comparison between trade-in and leasing of a product with technology innovations. *Omega, 54*, 134–146.
Li, Y., Feng, L., Gocindan, K., & Xu, F. (2019). Effects of a secondary market on original equipment manufactures' pricing, trade-in remanufacturing, and entry decisions. *European Journal of Operational Research, 279*(3), 751–766.
Ma, Z. J., Zhou, Q., Dai, Y., & Sheu, J. B. (2017). Optimal pricing decisions under the coexistence of "trade old for new" and "trade old for remanufactured" programs. *Transportation Research Part E: Logistics and Transportation Review, 106*, 337–352.
Mantin, B., Krishnan, H., & Dhar, T. (2014). The strategic role of third-party marketplaces in retailing. *Production and Operations Management Society, 23*(11), 1937–1949.
Miao, Z., Fu, K., Xia, Z., & Wang, Y. (2017). Models for closed-loop supply chain with trade-ins. *Omega, 66*, 308–326.
Miao, Z., Mao, H., Fu, K., & Wang, Y. (2018). Remanufacturing with trade-ins under carbon regulations. *Computers and Operations Research, 89*, 253–268.
Rao, R. S., Narasimhan, O., & John, G. (2009). Understanding the role of trade-ins in durable goods markets: Theory and evidence. *Marketing Science, 28*(5), 950–967.
Ray, S., Boyaci, T., & Aras, N. (2011). Optimal prices and trade-in rebates for durable, remanufacturable products. *Manufacturing and Service Operations Management, 7*(3), 208–228.
Rust, J. (1985). Stationary equilibrium in a market for durable assets. *Econometrica, 53*(4), 783–805.
Ryan, J. K., Sun, D., & Zhao, X. (2012). Competition and coordination in online marketplaces. *Production and Operations Management, 21*(6), 997–1014.
Vedantam, A., Demirezen, E. M., & Kumar, S. (2021). Trade-in or sell in my P2P marketplace: A game theoretic analysis of profit and environmental impact. *Production and Operations Management, 30*(11), 3923–3942.
Xiao, Y., & Zhou, S. X. (2020). Trade-in for cash or for upgrade? Dynamic pricing with customer choice. *Product and Operations Management, 29*(4), 856–881.
Zhu, X., Wang, M., Chen, G., & Chen, X. (2016). The effect of implementing trade-in strategy on duopoly competition. *European Journal of Operational Research, 248*(3), 856–868.