Short Communication

A note on ''Two-warehouse inventory model with deterioration under FIFO dispatch policy''

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Abstract

A recently published paper by Lee [C.C. Lee, Two-warehouse inventory model with deterioration under FIFO dispatching policy, European Journal of Operational Research 174 (2006) 861–873] considers different dispatching models for the two-warehouse inventory system with deteriorating items, in which Pakkala and Achary's LIFO (last-in–first-out) model [T.P.M. Pakkala, K.K. Achary, A deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate, European Journal of Operational Research 57 (1992) 71–76] is first modified, and then the author concludes that the modified LIFO model always has a lower cost than Pakkala and Achary's LIFO model under a particular condition specified by him. The present note points out that this conclusion is incorrect and misleading. Alternatively, we provide a new sufficient condition such that the modified LIFO model always has a lower cost than Pakkala and Achary's model. Besides, we also compare Pakkala and Achary's original LIFO model with Lee's FIFO (first-in–first-out) model for the special case where the two warehouses have the same deteriorating rates. Finally, numerical examples are provided to investigate and examine the impact of corresponding parameters on policy choice. The results in this note give a much clearer picture than those at Lee's paper about the relationships between the different dispatching policies for the two-warehouse inventory system with deterioration items.
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1. Introduction

In a recent paper, Lee [1] considers the two-warehouse inventory problem for deteriorating items and has modified Pakkala and Achary's [2] LIFO (last-in–first-out) model where inventory in RW (rented warehouse), which is stored last, will be consumed before those in OW (own warehouse). In Theorem 1 of Lee's paper [1], he concludes that the modified LIFO model always has a lower cost than Pakkala and Achary's LIFO model under a particular condition specified by him. However, there is a contradiction in the comparison of the modified LIFO model and Pakkala and Achary's model (we will show this in the next section). As a result, his conclusion is incorrect. In Section 2 of the present note, a new sufficient condition is

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proposed such that the modified LIFO model has a lower cost than Pakkala and Achary's model.

Lee's paper [1] also proposes a FIFO (first-in–first-out) dispatching model in which inventory in OW, which is stored first, will be consumed before those in RW. Then he concludes that the FIFO model is less expensive to operate than the modified LIFO, if the mixed effects of deterioration and holding cost in RW are less than that of OW (specifically, this is the condition of Theorem 2 in Lee [1]). However, he did not compare the FIFO model with Pakkala and Achary's original LIFO model. In Section 3 of the present note, we propose an observation concerning the FIFO model with Pakkala and Achary's original LIFO model under the same conditions. In Section 4, we conduct numerical experiments to investigate and examine the impact of the major parameters on policy choice.

To save space, we omit the discussion about the existing literature on the importance of different dispatching policies for the two-warehouse inventory system with deterioration items, and the discussion about the characteristics, merits and demerits of RW and OW. For such information, the readers are referred to Lee's paper [1] and the references cited therein.

2. Comparison between two LIFO models

The notation and assumptions in this paper are the same as those of Lee [1]. Four kinds of cost parameters are included: cost of a unit deteriorating item (C_1), shortage cost per unit item per unit time (C_2), unit setup cost (C_3), and holding cost per unit item per unit time (H and F for OW and RW respectively). In order to guarantee the models are reasonable and to avoid any degenerate situations, we further assume the constant production rate (P) satisfies the condition $P > D + \alpha W$, where D is the constant demand rate, W is the constant capacity of OW, and α is the deteriorating rate in OW (the deteriorating rate in RW is denoted by β with $0 < \alpha, \beta < 1$).

For convenience, denote Pakkala and Achary's original LIFO model [2] and the modified LIFO model in Lee [1] as LIFO_P and LIFO_L, respectively.

The basic assumption for both models is that inventory items are stored in RW only after OW is fully utilized. In LIFO_P model, it is also assumed that the items deteriorated in inventory stored in OW are not replaced by good ones, thus the inven-

tory situation for this model can be represented as shown in Fig. 1 (same as Fig. 1 in [1]).

For the LIFO_P model, denoting $T_{PB} = T_{P1} + T_{P6}$, it is easy to obtain the expressions of the time period T_{Pi} of stage i ($i = 1, 2, 4, 5, 6$) in a production cycle as below [1,2]:

$$T_{P1} = DT_{PB}/P, \tag{1}$$

$$T_{P2} = \frac{1}{\alpha} \ln \left[\frac{P - D}{P - D - \alpha W} \right], \tag{2}$$

$$T_{P4} = \frac{1}{\beta} \ln \left[\frac{P - (P - D)e^{-\beta T_{P3}}}{D} \right], \tag{3}$$

$$T_{P5} = \frac{1}{\alpha} \ln \left[1 + \frac{\alpha W e^{\alpha(T_{P3} + T_{P4})}}{D} \right], \tag{4}$$

$$T_{P6} = (P - D)T_{PB}/P. \tag{5}$$

Therefore, the total cost per unit time for the model LIFO_P, TC_P, can be expressed as below (that is, Eq. (20) in [1] with some typo errors being corrected):

$$\begin{aligned} TC_P = (1/T_P)\{ &F[PT_{P3} - D(T_{P3} + T_{P4})]/\beta \\ &+ H[PT_{P2} - D(T_{P2} + T_{P5})]/\alpha \\ &+ C_1[P(T_{P2} + T_{P3}) - D(T_{P2} + T_{P3} + T_{P4} + T_{P5})] \\ &+ C_2 D(P - D)T_{PB}^2(2P) + C_3\}, \end{aligned} \tag{6}$$

where $T_P = \sum_{i=1}^{6} T_{Pi}$. Please note that TC_P can be regarded as a nonlinear function explicitly only in terms of two decision variables T_{PB} and T_{P3}.

Alternatively, in LIFO_L model, it is assumed the items deteriorated in inventory stored in OW are replaced by good ones, thus the inventory situation for this model can be represented as shown in Fig. 2 (same as Fig. 2 in [1]).
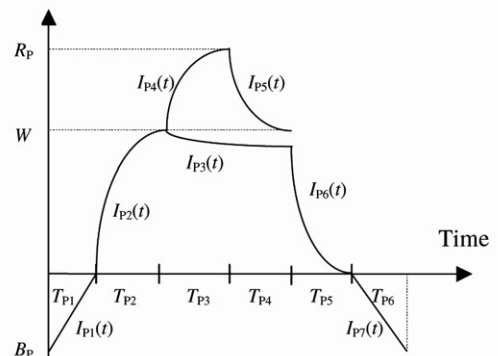

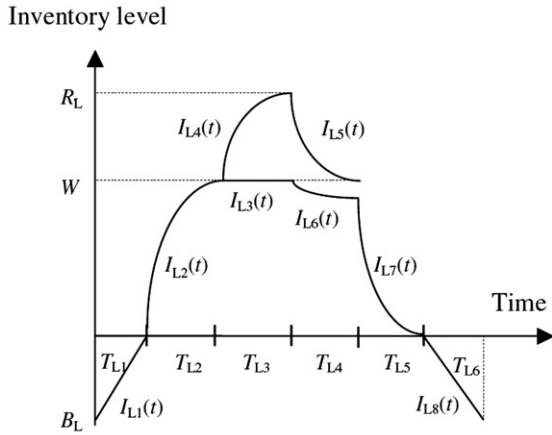Fig. 1. Inventory level of LIFO_P model.

Inventory level

Fig. 2. Inventory level of LIFO_L model.

Denoting $T_{LB} = T_{L1} + T_{L6}$, the variables T_{Li} ($i = 1, 2, 4, 5, 6$) can be expressed as functions of T_{LB} and T_{L3} as below [1]:

$$T_{L1} = DT_{LB}/P, \tag{7}$$

$$T_{L2} = \frac{1}{\alpha} \ln\left[\frac{P-D}{P-D-\alpha W}\right], \tag{8}$$

$$T_{L4} = \frac{1}{\beta} \ln\left[\frac{(P-\alpha W) - (P-D-\alpha W)e^{-\beta T_{L3}}}{D}\right], \tag{9}$$

$$T_{L5} = \frac{1}{\alpha} \ln\left[1 + \frac{\alpha W e^{-\alpha T_{L4}}}{D}\right], \tag{10}$$

$$T_{L6} = (P-D)T_{LB}/P. \tag{11}$$

The total cost per unit time for the model LIFO_L, TC_L, can be expressed as a nonlinear function explicitly only in terms of T_{LB} and T_{L3} as below (that is, Eq. (14) in [1] with a minor typo error being corrected):

$$\begin{aligned}
TC_L = (1/T_L)\{ & F[PT_{L3} - D(T_{L3}+T_{L4}) - \alpha W T_{L3}]/\beta \\
& + H[PT_{L2} - D(T_{L2}+T_{L5}) + \alpha W T_{L3}]/\alpha \\
& + C_1[P(T_{L2}+T_{L3}) - D(T_{L2}+T_{L3}+T_{L4}+T_{L5})] \\
& + C_2 D(P-D)T_{LB}^2(2P) + C_3 \},
\end{aligned} \tag{12}$$

where $T_L = \sum_{i=1}^{6} T_{Li}$.

In order to compare the difference between these two models, Lee [1] presented the following Theorem (please note that the original statement of Theorem 1 in Lee [1] has a typo error, i.e., the condition $H - \alpha F/\beta > 0$ should be $H - \alpha F/\beta < 0$).

Theorem 1. *Modified LIFO two-warehouse model (LIFO_L) always has a lower cost than Pakkala and Achary's LIFO model (LIFO_P) if $H - \alpha F/\beta < 0$.*

In the proof of Theorem 1 in Lee [1], it is assumed $T_{Pi} = T_{Li}$, for $i = 1, \ldots, 6$. Obviously, since T_{Pi} and T_{Li}, $i = 1, \ldots, 6$, are not independent deci-

sion variables, this assumption has lost generality. For example, when $T_{P3} = T_{L3}$ and $T_{P4} = T_{L4}$, then $\alpha W(1 - e^{-\beta T_{L3}}) = 0$ from Eqs. (3) and (9). Since $\alpha, \beta > 0$, we have $W = 0$ or $T_{L3} = 0$. This contradicts $W T_{L3} > 0$ which is an essential assumption in the proof. Therefore, the proof of Theorem 1 in Lee [1] is incorrect.

This contradiction can also be seen from another viewpoint. If $T_{PB} = T_{LB}$ and $T_{P3} = T_{L3}$, then it is easy to show $T_{Pi} = T_{Li}$ for $i = 1, 2, 3, 6$. However, if $T_{P3} = T_{L3} > 0$, then $T_{P4} > T_{L4}$ and $T_{P5} < T_{L5}$. Thus the situation $T_{Pi} = T_{Li}$ for $i = 1, \ldots, 6$ is impossible unless $T_{P3} = T_{L3} = 0$, which is only a degenerative case.

In fact, one can easily find counterexamples to show that this theorem is incorrect. For example, take $P = 32,000$, $D = 8000$, $C_1 = 8$, $C_2 = 20$, $C_3 = 2000$, $W = 1200$, $H = 4$, $F = 2$, $\alpha = 0.24$, $\beta = 0.06$, then the condition $H - \alpha F/\beta < 0$ holds. However, one can numerically calculate the optimal costs for these two models as $TC_P^* = 11,493$ and $TC_L^* = 11,544$ (hereafter the superscript star (*) means the minimum cost for the corresponding model), respectively, and thus the LIFO_L model has a higher cost than the LIFO_P model.

In order to obtain a fair comparison between these two models, a new observation is shown between the two policies as follows, which can be regarded as a revised version of Theorem 1 in Lee [1].

Theorem 2. *Suppose the two warehouses have the same deterioration rate, i.e., $\alpha = \beta$. Then the modified LIFO two-warehouse model (LIFO_L) has a lower optimal cost than Pakkala and Achary's LIFO model (LIFO_P) if and only if $H < F$. That's to say, $TC_L^* < TC_P^*$ if $H < F$; otherwise $TC_L^* > TC_P^*$ if $H > F$.*

In order to prove this theorem, we present the following lemma first.

Lemma 1. *Suppose $\alpha = \beta$, $T_{PB} = T_{LB}$ and $T_{P3} = T_{L3} > 0$, then the following will hold:*

(i) $T_{P4} + T_{P5} = T_{L4} + T_{L5}$ and $T_P = T_L$.
(ii) $\frac{\alpha W}{D} T_{L3} + T_{L4} > T_{P4}$.

Proof of Lemma (i). From Eqs. (3) and (4),

$$\begin{aligned}
T_{P4} + T_{P5} &= T_{P4} + \frac{1}{\alpha} \ln\left[1 + \frac{\alpha W e^{-\alpha(T_{P3}+T_{P4})}}{D}\right] \\
&= \frac{1}{\alpha} \ln\left(e^{\alpha T_{P4}} + \frac{\alpha W e^{-\alpha T_{P3}}}{D}\right).
\end{aligned} \tag{13}$$

Substitute Eq. (3) and $\alpha = \beta$ into the right hand side of Eq. (13) and after simplification, we have

$$T_{P4} + T_{P5} = \frac{1}{\alpha} \ln \left(\frac{P - (P - D - \alpha W)e^{-\alpha T_{P3}}}{D} \right). \quad (14)$$

From Eqs. (9) and (10),

$$T_{L4} + T_{L5} = T_{L4} + \frac{1}{\alpha} \ln \left[1 + \frac{\alpha W e^{-\alpha T_{L4}}}{D} \right]$$
$$= \frac{1}{\alpha} \ln \left(e^{\alpha T_{L4}} + \frac{\alpha W}{D} \right). \quad (15)$$

Substitute Eq. (9) and $\alpha = \beta$ into the right hand side of Eq. (15) and after simplification, we have

$$T_{L4} + T_{L5} = \frac{1}{\alpha} \ln \left(\frac{P - (P - D - \alpha W)e^{-\alpha T_{L3}}}{D} \right)$$
$$= T_{P4} + T_{P5}. \quad (16)$$

Noticing that $T_{P3} = T_{L3}$ and $T_{L2} = T_{P2}$ $= \frac{1}{\alpha} \ln \left[\frac{P-D}{P-D-\alpha W} \right]$ from Eqs. (2) and (8), we have

$$T_{\mathrm{PB}} + T_{P2} + T_{P3} + T_{P4} + T_{P5}$$
$$= T_{\mathrm{LB}} + T_{L2} + T_{L3} + T_{L4} + T_{L5}, \quad \text{i.e.,}$$
$$T_P = T_L. \quad \square$$

Proof of Lemma (ii). Substitute $T_{P3} = T_{L3}$ and $\alpha = \beta$ into the expressions of T_{L4} and T_{P4} in Eqs. (3) and (9), we have

$$T_{L4} = \frac{1}{\alpha} \ln \left[\frac{(P - \alpha W) - (P - D - \alpha W)e^{-\alpha T_{L3}}}{D} \right], \quad (17)$$

$$T_{P4} = \frac{1}{\alpha} \ln \left[\frac{P - (P - D)e^{-\alpha T_{L3}}}{D} \right]. \quad (18)$$

Define $f(T_{L3}) = \frac{\alpha W}{D} T_{L3} + T_{L4} - T_{P4}$, it is easy to see that $f(0) = 0$. When $T_{L3} > 0$, we have

$$f(T_{L3}) = \frac{\alpha W}{D} T_{L3} + \frac{1}{\alpha} \ln \left[\frac{(P - \alpha W) - (P - D - \alpha W)e^{-\alpha T_{L3}}}{D} \right]$$
$$- \frac{1}{\alpha} \ln \left[\frac{P - (P - D)e^{-\alpha T_{L3}}}{D} \right], \quad (19)$$

$$f'(T_{L3}) = \frac{\alpha W}{D} + \frac{P - D - \alpha W}{(P - \alpha W)e^{\alpha T_{L3}} - (P - D - \alpha W)} - \frac{P - D}{P e^{\alpha T_{L3}} - (P - D)}$$
$$= \frac{\alpha W}{D} + \frac{P - D - \alpha W}{P e^{\alpha T_{L3}} - (P - D) - (e^{\alpha T_{L3}} - 1)\alpha W} - \frac{P - D}{P e^{\alpha T_{L3}} - (P - D)}$$
$$\text{by } (e^{\alpha T_{L3}} - 1)\alpha W > 0$$
$$> \frac{\alpha W}{D} + \frac{P - D - \alpha W}{P e^{\alpha T_{L3}} - (P - D)} - \frac{P - D}{P e^{\alpha T_{L3}} - (P - D)}$$
$$= \frac{\alpha W}{D} - \frac{\alpha W}{P e^{\alpha T_{L3}} - (P - D)}$$
$$= \frac{\alpha W P(e^{\alpha T_{L3}} - 1)}{D[P e^{\alpha T_{L3}} - (P - D)]}$$
$$> 0.$$

This implies that $f(T_{L3}) > 0$ for $T_{L3} > 0$, hence we have $\frac{\alpha W}{D} T_{L3} + T_{L4} > T_{P4}$. $\square$

Proof of Theorem 2. From the expressions of TC_P (Eq. (6)) and TC_L (Eq. (12)), and using Lemma 1(i) just proved, the cost difference between TC_P and TC_L is given by

$$\mathrm{TC}_P - \mathrm{TC}_L = \left(\frac{1}{T_L} \right) \left[\frac{F}{\beta} (D T_{L4} - D T_{P4} + \alpha W T_{L3}) \right.$$
$$\left. + \frac{H}{\alpha} (D T_{L5} - D T_{P5} - \alpha W T_{L3}) \right]$$
$$= \frac{D(F - H)}{\alpha T_L} \left(\frac{\alpha W}{D} T_{L3} + T_{L4} - T_{P4} \right). \quad (20)$$

Therefore, Lemma (ii) implies that $\mathrm{TC}_L^* < \mathrm{TC}_P^*$ if $H < F$, and $\mathrm{TC}_L^* > \mathrm{TC}_P^*$ if $H > F$. This completes the Proof of Theorem 2. $\square$

3. Comparison between LIFO models and FIFO model

Lee's paper [1] also proposes a FIFO (first-in–first-out) dispatching model in which inventory in OW, which is stored first, will be consumed before those in RW. The inventory situation for this model can be represented as shown in Fig. 3 (same as Fig. 3 in [1]).

Similarly as in the last section, denoting $T_{\mathrm{FB}} = T_{F1} + T_{F6}$, the variables T_{Fi} ($i = 1, 2, 4, 5, 6$) can be expressed as functions of T_{FB} and T_{F3} as below [1]:

$$T_{F1} = D T_{\mathrm{FB}} / P, \quad (21)$$

$$T_{F2} = \frac{1}{\alpha} \ln \left[\frac{P - D}{P - D - \alpha W} \right], \quad (22)$$

$$T_{F4} = \frac{1}{\alpha} \ln \left[1 + \frac{\alpha W e^{-\alpha T_{F3}}}{D} \right], \quad (23)$$

$$T_{F5} = \frac{1}{\beta} \ln \left[1 + \frac{(P - D)(1 - e^{-\beta T_{F3}})e^{-\beta T_{F4}}}{D} \right], \quad (24)$$

$$T_{F6} = (P - D) T_{\mathrm{FB}} / P. \quad (25)$$

The total cost per unit time for the model FIFO, TC_F, can be expressed as a nonlinear function explicitly only in terms of T_{FB} and T_{F3} as below (that is, Eq. (23) in [1]):
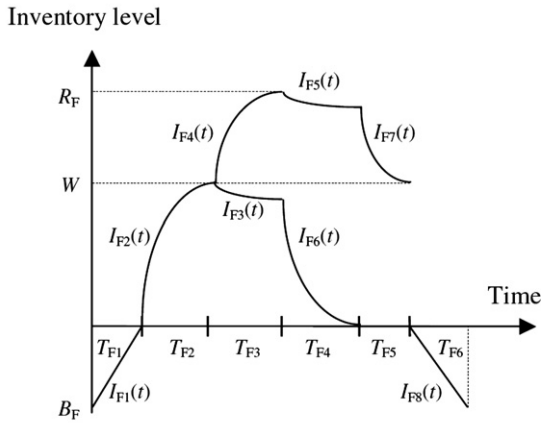
Inventory level

Fig. 3. Inventory level of FIFO model.

$$\mathrm{TC}_F = (1/T_F)\{F[PT_{F3} - D(T_{F3} + T_{F4})]/\beta$$
$$+ H[PT_{F2} - D(T_{F2} + T_{F4})]/\alpha$$
$$+ C_1[P(T_{F2} + T_{F3}) - D(T_{F2} + T_{F3} + T_{F4} + T_{F5})]$$
$$+ C_2 D(P - D)T_{\mathrm{FB}}^2(2P) + C_3\}, \tag{26}$$

where $T_F = \sum_{i=1}^{6} T_{Fi}$.

In order to compare the difference between TC_F and TC_L, Lee [1] presented the following Theorem (that is Theorem 2 in [1]. Please notice that we add a superscript star to the total cost TC_F and TC_L since the theorem should be understood to hold for the optimal costs only).

Theorem 3. *If the two warehouses have the same deterioration rate, i.e., $\alpha = \beta$, then $TC_F^* > TC_L^*$ if $H < F$; otherwise $TC_F^* < TC_L^*$ if $H > F$.*

However, Lee [1] did not compare the difference between TC_F and TC_P. Here we present a similar observation concerning the TC_F and TC_P as the following theorem.

Theorem 4. *If the two warehouses have the same deterioration rate, i.e., $\alpha = \beta$, then $\mathrm{TC}_F^* > \mathrm{TC}_P^*$ if $H < F$; otherwise $\mathrm{TC}_F^* < \mathrm{TC}_P^*$ if $H > F$.*

The Proof of Theorem 4 needs the following lemma, which can be proved similarly as the proof of Lemma 1(i) in the last section and we omit the details here.

Lemma 2. *Suppose $\alpha = \beta$, $T_{PB} = T_{FB}$ and $T_{P3} = T_{F3}$, then $T_{P4} + T_{P5} = T_{F4} + T_{F5}$ and $T_P = T_F$.*

Proof of Theorem 4. From the expressions of TC_P (Eq. (6)) and TC_F (Eq. (26)), and using Lemma 2, the cost difference between TC_P and TC_F is given by

$$\mathrm{TC}_P - \mathrm{TC}_F = \frac{1}{T_F}\left(\frac{HD}{\alpha}(T_{F4} - T_{P5}) - \frac{FD}{\alpha}(T_{F5} - T_{P4})\right)$$
$$= \frac{D(H - F)}{\alpha T_F}(T_{F4} - T_{P5}). \tag{27}$$

Comparing Eqs. (4) and (23), it is easy to see $T_{F4} > T_{P5}$. Therefore, $\mathrm{TC}_F^* > \mathrm{TC}_L^*$ if $H < F$; otherwise $\mathrm{TC}_F^* < \mathrm{TC}_L^*$ if $H > F$. This completes the Proof of Theorem 4.

Finally, combining the Theorems 2–4, we have the following conclusion, which gives the whole picture about the relationships between these three different dispatching policies for the two-warehouse inventory system with deterioration items.

Corollary. *If the two warehouses have the same deterioration rate, i.e., $\alpha = \beta$, then $\mathrm{TC}_L^* > \mathrm{TC}_P^* > \mathrm{TC}_F^*$ when $H > F$, and $\mathrm{TC}_L^* < \mathrm{TC}_P^* < \mathrm{TC}_F^*$ when $H < F$.*

4. Numerical examples and summary

In previous sections, we have analytically compared the costs for the models LIFO_P, LIFO_L, FIFO only for the case where the two warehouses have the same deterioration rate, i.e., $\alpha = \beta$. The important aspect to be considered in two-warehouse inventory models is that the warehouses have different deterioration rates, due to the effect of storage environment and hence different holding costs. However, if $\alpha \neq \beta$, since the costs for the three models are complicated nonlinear functions with respect to the decision variables, it is not easy to compare them analytically. In this section, numerical experiments are conducted to investigate the impact of $\{H, F, \alpha, \beta\}$ on the policy choice when $\alpha \neq \beta$.

When $\alpha > \beta$, it is only interesting to consider the case $H \leqslant F$. Otherwise, the mixed effects of deterioration and holding cost in OW are obviously more than that of RW, thus FIFO model is always less expensive to operate (In fact, it is doubtable to use a two-warehouse system under this condition since it is essentially economical to use RW than OW). Similarly, when $\alpha < \beta$, it is only interesting to consider the case $H \geqslant F$. Otherwise, the mixed effects of deterioration and holding cost in OW are obviously less than that of RW, thus LIFO model is always less expensive to operate. Therefore, the following numerical experiments only focus on the

cases of either $\alpha > \beta$ and $H \leqslant F$, or $\alpha < \beta$ and $H \geqslant F$.

Before introducing the results of the numerical experiments, we recall that the basic assumption in this paper is that inventory items are stored in RW only after OW is fully utilized. If the capacity W of the OW is large enough, it might be profitable not to use the OW to its full capacity and not to use the RW at all, thus the L_1 system will be economically less than the L_2 system [1]. Since the focus of this paper is on the two-warehouse inventory system, a relatively small value for W is used in the following numerical experiments.

In the numerical examples below, most of the values of parameters are taken from Lee [1]: $P = 32{,}000$, $D = 8000$, $C_1 = 8$, $C_2 = 20$, $C_3 = 2000$, $W = 400$. Compared with the values used in the original paper of Lee [1], only the value for W is changed from 1200 to 400 for the reason mentioned above.

Case 1. $\alpha > \beta$ and $H \leqslant F$

Three sets of deterioration rates, i.e., $(\alpha, \beta) = (0.06, 0.05)$, $(\alpha, \beta) = (0.06, 0.03)$ and $(\alpha, \beta) = (0.06, 0.01)$, are tested. The holding cost in OW is fixed at $H = 1$ and the holding cost in RW is set at $F = 1$, 1.5, 2, 4, 6 and 8 respectively. The numerical results are summarized in Tables 1–3.

Examinations of the three tables in this case reveal the following observations:

(i) If F is not significantly greater than H, then the best choice is to use FIFO model. The reason is that under these conditions the effect of deterioration cost dominates the mixed effects of deterioration and holding cost, i.e., the mixed effects of deterioration and holding cost in RW are less than that of OW.

(ii) On the contrary, if F is significantly greater than H, then the best choice is to use LIFO_L model. The reason is that under these conditions the effect of holding cost dominates the mixed effects of deterioration and holding cost, i.e., the mixed effects of deterioration and holding cost in RW are more than that of OW.

Case 2. $\alpha < \beta$ and $H \geqslant F$

Three sets of deterioration rates, i.e, $(\alpha, \beta) = (0.05, 0.06)$, $(\alpha, \beta) = (0.03, 0.06)$ and $(\alpha, \beta) = (0.01, 0.06)$, are tested. The holding cost in RW is

Table 1
Comparison of policies by varying holding costs when $(\alpha, \beta) = (0.06, 0.05)$

F	TC_P^*	TC_L^*	TC_F^*	Policy choice
1	6267.019	6267.348	6225.387	FIFO
1.5	6687.745	6687.354	6744.462	LIFO_L
2	7033.272	7032.398	7176.936	LIFO_L
4	7978.900	7977.161	8385.965	LIFO_L
6	8555.586	8553.632	9135.434	LIFO_L
8	8948.905	8946.938	9645.251	LIFO_L

Table 2
Comparison of policies by varying holding costs when $(\alpha, \beta) = (0.06, 0.03)$

F	TC_P^*	TC_L^*	TC_F^*	Policy choice
1	5850.872	5852.072	5719.127	FIFO
1.5	6355.836	6356.004	6333.250	FIFO
2	6759.709	6759.214	6833.180	LIFO_L
4	7827.424	7825.785	8189.528	LIFO_L
6	8457.463	8455.532	9007.270	LIFO_L
8	8879.671	8877.700	9555.584	LIFO_L

Table 3
Comparison of policies by varying holding costs when $(\alpha, \beta) = (0.06, 0.01)$

F	TC_P^*	TC_L^*	TC_F^*	Policy choice
1	5333.475	5336.002	5101.285	FIFO
1.5	5959.071	5960.032	5848.547	FIFO
2	6440.823	6440.844	6436.816	FIFO
4	7659.566	7658.055	7972.656	LIFO_L
6	8351.238	8349.340	8868.499	LIFO_L
8	8805.767	8803.797	9459.585	LIFO_L

fixed at $F = 1$, and the holding cost in OW is set at $H = 1$, 1.5, 2, 4, 6 and 8 respectively. The numerical results are summarized in Tables 4–6.

Examinations of the three tables in this case reveal the following observations:

(i) If H is not significantly greater than F, then the best choice is to use LIFO_L model. This can be explained by the arguments that under these conditions the effect of deterioration cost will dominate the mixed effects of deterioration and holding cost, i.e., the mixed effects of deterioration and holding cost in RW are more than that of OW.

(ii) On the contrary, if H is significantly greater than F, then the best choice is to use FIFO model. This can be explained by the argu-

Table 4
Comparison of policies by varying holding costs when $(\alpha, \beta) = (0.05, 0.06)$

H	TC_P^*	TC_L^*	TC_F^*	Policy choice
1	6388.823	6388.570	6429.575	LIFO_L
1.5	6533.183	6533.548	6474.339	FIFO
2	6676.519	6677.481	6519.032	FIFO
4	7239.511	7242.643	6697.090	FIFO
6	7785.575	7790.521	6874.016	FIFO
8	8314.089	8320.476	7049.820	FIFO

Table 5
Comparison of policies by varying holding costs when $(\alpha, \beta) = (0.03, 0.06)$

H	TC_P^*	TC_L^*	TC_F^*	Policy choice
1	6270.450	6269.986	6393.808	LIFO_L
1.5	6416.220	6416.139	6438.694	LIFO_L
2	6560.952	6561.240	6483.507	FIFO
4	7129.380	7131.014	6662.046	FIFO
6	7680.667	7683.432	6839.445	FIFO
8	8214.216	8217.888	7015.714	FIFO

Table 6
Comparison of policies by varying holding costs when $(\alpha, \beta) = (0.01, 0.06)$

H	TC_P^*	TC_L^*	TC_F^*	Policy choice
1	6150.422	6150.160	6357.892	LIFO_L
1.5	6297.633	6297.503	6402.900	LIFO_L
2	6443.791	6443.787	6447.835	LIFO_L
4	7017.772	7018.232	6626.856	FIFO
6	7574.390	7575.241	6804.731	FIFO
8	8113.072	8114.239	6981.469	FIFO

ments that under these conditions the effect of holding cost will dominate the mixed effects of deterioration and holding cost, i.e., the mixed effects of deterioration and holding cost in RW are less than that of OW.

In a word, the results in these numerical examples show that for the two-warehouse system, the selection for dispatching policies depends critically on the relative importance of the mixed effects of deterioration and holding cost between RW and OW. Besides, it is interesting to notice that the LIFO_P model is never the best choice under all the ranges of parameters we have illustrated. These observations are helpful for the practitioners to operate the two-warehouse inventory system with deterioration items.

Acknowledgements

References

[1] C.C. Lee, Two-warehouse inventory model with deterioration under FIFO dispatching policy, European Journal of Operational Research 174 (2006) 861–873.
[2] T.P.M. Pakkala, K.K. Achary, A deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate, European Journal of Operational Research 57 (1992) 71–76.