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Short Communication

A note on "Price discount based on early order commitment in a single manufacturer-multiple retailer supply chain"

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ABSTRACT

In a recent paper by Xie et al. [Xie, J., Zhou, D., Wei, J.C., Zhao, X., 2010. Price discount based on early order commitment in a single manufacturer-multiple retailer supply chain. European Journal of Operational Research 200, 368–376], the authors have studied the early order commitment (EOC) strategy for a decentralized, two-level supply chain consisting of a single manufacturer and multiple retailers. They fail to provide an algorithm to determine the optimal EOC periods to minimize the total supply chain cost. This note proposes a polynomial-time algorithm to find the optimal solutions, and provides a new set of sufficient conditions under which the wholesale price discount scheme coordinates the whole supply chain.

1. Introduction

In a recent paper, Xie et al. (2010) provided an analytical model to quantify the effects of early order commitment (EOC) strategy on the performance of a two-level supply chain consisting of a single manufacturer and N independent retailers. Under EOC strategy, retailer i ($i = 1,2,\ldots,N$) places her order x_i periods in advance, where x_i is called the EOC period for retailer i. In order to minimize the expected holding and shortage cost per period for the whole supply chain, Xie et al. (2010) proposed the following optimization problem:

$$\min_{0\leqslant x_i\leqslant L_0+1} SC(x) = r_0\sqrt{\sum_{i=1}^{N}\left(\frac{\sigma_i}{1-\rho_i}\right)^2\sum_{j=L_i+x_i+2}^{L_i+L_0+2}(1-\rho_i^j)^2}+\sum_{i=1}^{N} r_i\frac{\sigma_i}{1-\rho_i}\sqrt{\sum_{j=1}^{L_i+x_i+1}(1-\rho_i^j)^2}, \quad (1)$$

where $x = (x_1,x_2,\ldots,x_N)$ are decision variables, and $L_0 > 0$, $r_0 > 0$, $L_i > 0$, $d_i > 0$, $\sigma_i > 0$, $0 < \rho_i < 1$ and $r_i > 0$ ($i = 1,2,\ldots,N$) are known parameters (please refer to Xie et al., 2010 for details). They failed to provide an algorithm to find an optimal solution to Problem (1). In Section 2 of this note, we propose a polynomial-time algorithm to find the optimal solutions.

Xie et al. (2010) also proposed a wholesale price discount scheme to induce the retailers to practice EOC strategy and identified a set of sufficient conditions under which the scheme coordinates the whole supply chain. In Section 3 of this note, we provide a new set of sufficient conditions which also leads to supply chain coordination.

2. An optimal algorithm

In Theorem 1 of Xie et al. (2010), they identified an amazing characteristic for the optimal solutions of Problem (1): the EOC period x_i for each retailer i should be either 0 or $L_0 + 1$. Therefore, we can define $y_i = (L_0 + 1 - x_i)/(L_0 + 1)$, where $y_i \in \{0,1\}$ ($i = 1,2,\ldots,N$), and $y_i = 0$ means that retailer i uses EOC policy, and $y_i = 1$ means that retailer i does not use EOC policy. After the variable redefinition, the objective function $SC(x)$ in Problem (1) can be expressed as a function of $y = (y_1,y_2,\ldots,y_N)$:

$$\overline{SC}(y) = \left(\sum_{i=1}^{N} a_i y_i\right)^{\frac{1}{2}} - \sum_{i=1}^{N} b_i y_i + c, \quad (2)$$

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where

$$a_i = \left(\frac{\sigma_i r_0}{1-\rho_i}\right)^2 \sum_{j=L_i+2}^{L_i+L_0+2} (1-\rho_i^j)^2, \tag{3}$$

$$b_i = \frac{r_i \sigma_i}{1-\rho_i}\left(\sqrt{\sum_{j=1}^{L_0+L_i+2} (1-\rho_i^j)^2} - \sqrt{\sum_{j=1}^{L_i+1} (1-\rho_i^j)^2}\right), \tag{4}$$

$$c = \sum_{i=1}^{N} r_i \frac{\sigma_i}{1-\rho_i} \sqrt{\sum_{j=1}^{L_0+L_i+2} (1-\rho_i^j)^2}. \tag{5}$$

Since a_i, b_i, c are constants independent of the decision variables, Problem (1) is equivalent to the following 0–1 programming problem:

$$\min_{y_i \in \{0,1\}} \quad f(y) = \left(\sum_{i=1}^{N} a_i y_i\right)^{\frac{1}{2}} - \sum_{i=1}^{N} b_i y_i. \tag{6}$$

Now consider the following class of 0–1 programming problems:

$$\min_{y_i \in \{0,1\}} \quad f(y) = \left(\sum_{i=1}^{N} a_i y_i\right)^{p} - \left(\sum_{i=1}^{N} b_i y_i\right)^{q}, \tag{7}$$

where $a_i > 0$, $b_i > 0$, $0 \leqslant p \leqslant 1$ and $q \geqslant 1$. Obviously, Problem (6) is a special case of Problem (7) with $p = 1/2$, $q = 1$. For Problem (7), we have the following theorem.

Theorem 1. *Suppose that N pairs of positive numbers* (a_i, b_i), $i = 1, 2, \ldots, N$, *satisfy* $a_1/b_1 \geqslant a_2/b_2 \geqslant a_3/b_3 \geqslant \cdots \geqslant a_N/b_N$.

(*a*) *If* $p = q = 1$, *then there exists a binary vector* $y = (y_1, y_2, \ldots, y_N)$ *minimizing (7) and satisfying the following property: If* $y_j = 0$ *for some j* $(1 \leqslant j \leqslant N)$, *then* $y_i = 0$ *for any* $1 \leqslant i < j$.
(*b*) *If* $0 \leqslant p < 1$ *and* $q \geqslant 1$, *or* $0 \leqslant p \leqslant 1$ *and* $q > 1$, *then the binary vector* $y = (y_1, y_2, \ldots, y_N)$ *minimizing (7) should satisfy the following property: If* $y_j = 0$ *for some j* $(1 \leqslant j \leqslant N)$, *then* $y_i = 0$ *for any* $1 \leqslant i < j$.

Proof. Part (a) is obviously true. For Part (b), we only provide a proof for the case of $0 \leqslant p < 1$ and $q \geqslant 1$, since the proof for the other case is similar.

Suppose y is a binary vector minimizing (7) with $y_j = 0$ for some j $(1 \leqslant j \leqslant N)$ and $y_i = 1$ for some $1 \leqslant i < j$. Denote y' as a binary vector where $y'_i = 0$ and $y'_k = y_k$ for all $k \neq i$. By contradiction, we only need to prove that $f(y') < f(y)$.

Denote $A = \sum_{k \neq i,j} a_k y_k$ and $B = \sum_{k \neq i,j} b_k y_k$. By definition of $f(y)$, $f(y') < f(y)$ is equivalent to

$$(A + a_i)^p - A^p > (B + b_i)^q - B^q. \tag{8}$$

To prove Inequality (8), we choose y'' such that $y''_i = y''_j = 1$ and $y''_k = y_k$ for all $k \neq i, j$. Since y is an optimal solution of (7), we have $f(y) \leqslant f(y'')$, which is equivalent to

$$(A + a_i + a_j)^p - (A + a_i)^p \geqslant (B + b_i + b_j)^q - (B + b_i)^q. \tag{9}$$

Since $a_i > 0$ and $b_i > 0$ for all $i = 1, 2, \ldots, N$, Inequality (9) implies Inequality (8) if the following inequality holds:

$$\frac{(A + a_i)^p - A^p}{(A + a_i + a_j)^p - (A + a_i)^p} > \frac{(B + b_i)^q - B^q}{(B + b_i + b_j)^q - (B + b_i)^q}. \tag{10}$$

Now we prove Inequality (10). Consider a function $g(u) = u^p$. By Mean Value Theorem, there exists a $\xi \in (A + a_i, A + a_i + a_j)$ such that

$$p\xi^{p-1} = g'(\xi) = \frac{(A + a_i + a_j)^p - (A + a_i)^p}{a_j}. \tag{11}$$

Similarly, there exists a $\eta \in (A, A + a_i)$ such that

$$p\eta^{p-1} = g'(\eta) = \frac{(A + a_i)^p - A^p}{a_i}. \tag{12}$$

Clearly, $0 < \eta < \xi$. This, together with the fact that $g'(u) = pu^{p-1}$ $(0 \leqslant p < 1)$ is strictly decreasing with respect to u $(u > 0)$, implies that $p\eta^{p-1} > p\xi^{p-1}$. Therefore, by Eqs. (11) and (12), we have

$$\frac{(A + a_i)^p - A^p}{a_i} > \frac{(A + a_i + a_j)^p - (A + a_i)^p}{a_j},$$

which is equivalent to

$$\frac{(A + a_i)^p - A^p}{(A + a_i + a_j)^p - (A + a_i)^p} > \frac{a_i}{a_j}. \tag{13}$$

With similar arguments, for $q \geqslant 1$, one can prove that

$$\frac{(B+b_i)^q - B^q}{(B+b_i+b_j)^q-(B+b_i)^q} \leqslant \frac{b_i}{b_j}. \tag{14}$$

Since $a_i/b_i \geqslant a_j/b_j$, inequalities (13) and (14) imply (10). Therefore, $f(y') < f(y)$. This completes the proof. □

Theorem 1 allows us to design an algorithm as follows, which finds an optimal solution for Problem (7) and obviously runs in polynomial time $O(N\log N)$.

Algorithm

Step 1: Sort the N pairs of numbers (a_i, b_i), $i = 1,2,\ldots,N$, in descending order of a_i/b_i.
Step 2: Let $y^{(i)} = (y_1^{(i)}, y_2^{(i)}, \ldots, y_N^{(i)})$ be a binary vector with $y_j^{(i)} = 0$ for all $j \leqslant i$, and $y_j^{(i)} = 1$ for all $j > i$. For $i = 0,1,2,\ldots,N$, evaluate the value of $f(y^{(i)})$.
Step 3: The optimal value of (7) is $f^* = \mathrm{Min}_{0\leqslant i\leqslant N} f(y^{(i)})$, and an optimal solution of Problem (7) is $y^* = y^{(i)}$ such that $f(y^{(i)}) = f^*$.

This algorithm can be directly applied to find an optimal solution y^* for Problem (6). Then the optimal EOC periods for Problem (1) can be determined as

$$x_i^* = \begin{cases} 0, & \text{if} \quad y_i^* = 1, \\ L_0+1, & \text{if} \quad y_i^* = 0, \end{cases} \quad i = 1,2,\ldots,N.$$

3. Coordination based on wholesale price discount scheme

Under the wholesale price discount scheme proposed in Xie et al. (2010), the manufacturer announces a discount parameter α, and then the retailers determine whether to adopt EOC or not. It has been shown in Xie et al. (2010) that retailer i, $i = 1,2,\ldots,N$, will adopt EOC policy (i.e., $x_i = L_0 + 1$) iff $\alpha \geqslant \alpha_i$, where

$$\alpha_i = \frac{r_i\sigma_i}{d_i}\left(\sqrt{\sum_{j=1}^{L_0+L_i+2}(1-\rho_i^j)^2} - \sqrt{\sum_{j=1}^{L_i+1}(1-\rho_i^j)^2}\right). \tag{15}$$

Furthermore, for a given discount parameter α and given decisions of the retailers $x = (x_1,x_2,\ldots,x_N)$, the expected cost of the manufacturer per period is

$$C_0(\alpha) = r_0\sqrt{\sum_{i=1}^{N}\left(\frac{\sigma_i}{1-\rho_i}\right)^2\sum_{j=L_i+2}^{L_i+L_0+2}(1-\rho_i^j)^2 y_i} + \alpha\sum_{i=1}^{N}\frac{d_i}{1-\rho_i}(1-y_i), \tag{16}$$

where $y_i = (L_0 + 1 - x_i)/(L_0 + 1)$, the same as defined in Section 2.

Without loss of generality, we assume $\alpha_i \leqslant 1$ for all $i = 1,2,\ldots,N$. Denote $\alpha_0 = 0$. In order to minimize its cost (16), the manufacturer only needs to choose an optimal discount parameter α^* from the set $\{\alpha_0,\alpha_1,\ldots,\alpha_N\}$. Given this optimal discount parameter α^*, only retailer i with $\alpha_i \leqslant \alpha^*$ will adopt EOC policy (i.e., $y_i = 0$). In general situations, this optimal discount parameter α^* for the manufacturer does not lead to the minimal cost described as (1) for the whole supply chain. When both the manufacturer's minimal cost (16) under the wholesale price discount scheme and the supply chain's minimal cost (1) can be reached at the same time, we say that this scheme coordinates the supply chain. Proposition 4 of Xie et al. (2010) provides a set of sufficient conditions under which the price discount scheme can coordinate the supply chain. In the rest of this section, we provide a new set of sufficient conditions for supply chain coordination under the wholesale price discount scheme.

We focus on the situation where all the retailers face the same type of demand (*i.e.*, $d_i = d$, $\sigma_i = \sigma$ and $\rho_i = \rho$ for $i = 1,2,\ldots,N$) and have the same delivery lead time (*i.e.*, $L_i = L$ for $i = 1,2,\ldots,N$). Suppose the retailers are indexed according to their cost parameters as $r_1 < r_2 < \cdots < r_N$. Denote

$$a = a_i = \left(\frac{\sigma r_0}{1-\rho}\right)^2\sum_{j=L+2}^{L+L_0+2}(1-\rho^j)^2, \tag{17}$$

$$b = \frac{b_i}{r_i} = \frac{\sigma}{1-\rho}\left(\sqrt{\sum_{j=1}^{L_0+L+2}(1-\rho^j)^2} - \sqrt{\sum_{j=1}^{L+1}(1-\rho^j)^2}\right). \tag{18}$$

Under this situation, by Definition (15), we have $\alpha_0 \leqslant \alpha_1 < \alpha_2 < \cdots < \alpha_N$, which means that if the manufacturer chooses $\alpha = \alpha_i$ for some i, then retailer j will use EOC policy (i.e., $y_j = 0$) for all $j \leqslant i$ and retailer j will not use EOC policy (i.e., $y_j = 1$) for all $j > i$. According to (16), the manufacturer's cost under the discount parameter $\alpha = \alpha_i$ is

$$C_0(\alpha_i) = \sqrt{a(N-i)} + bir_i. \tag{19}$$

According to (2), for $\alpha = \alpha_i$ and the corresponding binary vector y, the cost for the whole supply chain can be expressed as

$$\overline{\overline{SC}}(\alpha_i) = \overline{SC}(y) = \sqrt{a(N-i)} + b\sum_{j=1}^{i} r_j + c. \tag{20}$$

Lemma 1. *Suppose all the retailers face the same type of demand and have the same delivery lead time (i.e., $d_i = d$, $\sigma_i = \sigma$, $\rho_i = \rho$, and $L_i = L$ for all retailer i, $i = 1,2,\ldots,N$), and the retailers are indexed as $r_1 < r_2 < \cdots < r_N$. For any $0 \leqslant t < s \leqslant N$,*

(a) *If $C_0(\alpha_s) \leqslant C_0(\alpha_t)$, then $\overline{\overline{SC}}(\alpha_s) \leqslant \overline{\overline{SC}}(\alpha_t)$;*
(b) *If $\overline{\overline{SC}}(\alpha_s) \geqslant \overline{\overline{SC}}(\alpha_t)$, then $C_0(\alpha_s) \geqslant C_0(\alpha_t)$.*

Proof. We only provide a proof for Part (a), since the proof for Part (b) is similar. The condition $C_0(\alpha_s) \leqslant C_0(\alpha_t)$ implies

$$\sqrt{a(N-s)} - \sqrt{a(N-t)} \leqslant -b(sr_s - tr_t). \tag{21}$$

According to Eq. (20) and Inequality (21),

$$\overline{\overline{SC}}(\alpha_s) - \overline{\overline{SC}}(\alpha_t) = \left(\sqrt{a(N-s)} - \sqrt{a(N-t)}\right) + b\sum_{j=t+1}^{s} r_j \leqslant b\left(-sr_s + tr_t + \sum_{j=t+1}^{s} r_j\right) = b\left(t(r_t - r_s) - (s-t)r_s + \sum_{j=t+1}^{s} r_j\right).$$

Noticing that $t(r_t - r_s) \leqslant 0$ and $r_j \leqslant r_s$ for all j with $1 \leqslant j \leqslant s$, we have

$$\overline{\overline{SC}}(\alpha_s) - \overline{\overline{SC}}(\alpha_t) \leqslant b\left(-(s-t)r_s + \sum_{j=t+1}^{s} r_j\right) \leqslant b(-(s-t)r_s + (s-t)r_s) = 0.$$

This completes the proof. □

Theorem 2. *Suppose all the retailers face the same type of demand and have the same delivery lead time (i.e., $d_i = d$, $\sigma_i = \sigma$, $\rho_i = \rho$, and $L_i = L$ for all retailer i, $i = 1,2,\ldots,N$, and the retailers are indexed as $r_1 < r_2 < \cdots < r_N$. If there is a retailer k ($1 \leqslant k < N$) such that*

$$kr_k\left(\sqrt{N-k+1} + \sqrt{N-k}\right) \leqslant \frac{\sqrt{a}}{b} \leqslant r_{k+1}\sqrt{N-k}, \tag{22}$$

then both the manufacturer's cost (16) under its optimal price discount scheme and the supply chain's minimal cost (1) can be reached at the same time with the first k retailers using EOC and the others not using EOC.

Proof. From the condition $r_1 < r_2 < \cdots < r_N$, we have

$$\frac{a}{b_1} > \frac{a}{b_2} > \ldots > \frac{a}{b_N}. \tag{23}$$

According to Theorem 1, Problem (1) is equivalent to

$$\mathrm{Min}_{0\leqslant i\leqslant N}\overline{SC}(y^{(i)}) = \overline{\overline{SC}}(\alpha_i), \tag{24}$$

where $y^{(i)} = (y_1^{(i)}, y_2^{(i)}, \ldots, y_N^{(i)})$ be a binary vector with $y_j^{(i)}$=0 for all $j \leqslant i$, and $y_j^{(i)} = 1$ for all $j > i$. Thus we only need to prove that

(a) $C_0(\alpha_k) \leqslant C_0(\alpha_n)$, $\overline{\overline{SC}}(\alpha_k) \leqslant \overline{\overline{SC}}(\alpha_n)$, for any n with $k < n \leqslant N$;
(b) $C_0(\alpha_k) \leqslant C_0(\alpha_n)$, $\overline{\overline{SC}}(\alpha_k) \leqslant \overline{\overline{SC}}(\alpha_n)$, for any n with $0 \leqslant n < k$.

We prove Part (a) first. For $k < n \leqslant N$, noticing the second inequality of (22), we have

$$\frac{\sqrt{a}}{b} \leqslant r_{k+1}\left(\sqrt{N-k} + \sqrt{N-n}\right) = \frac{(n-k)r_{k+1}}{\sqrt{N-k} - \sqrt{N-n}} \leqslant \frac{\sum_{i=k+1}^{n} r_i}{\sqrt{N-k} - \sqrt{N-n}},$$

which is equivalent to

$$\sqrt{a(N-n)} + b\sum_{i=1}^{n} r_i + c \geqslant \sqrt{a(N-k)} + b\sum_{i=1}^{k} r_i + c.$$

By Eq. (20), one has $\overline{\overline{SC}}(\alpha_n) \geqslant \overline{\overline{SC}}(\alpha_k)$, which implies $C(\alpha_n) \geqslant C(\alpha_k)$ by Part (b) of Lemma 1.

Now we prove Part (b). For $0 \leqslant n < k$, from the first inequality of (22), we have

$$\frac{\sqrt{a}}{b} \geqslant \frac{kr_k}{\sqrt{N-k+1} - \sqrt{N-k}} \geqslant \frac{kr_k - nr_n}{\sqrt{N-n} - \sqrt{N-k}},$$

which is equivalent to

$$\sqrt{a(N-k)} + bkr_k \leqslant \sqrt{a(N-n)} + bnr_n.$$

By Eq. (19), one has $C_0(\alpha_k) \leqslant C_0(\alpha_n)$, which implies $\overline{\overline{SC}}(\alpha_k) \leqslant \overline{\overline{SC}}(\alpha_n)$ by Part (a) of Lemma 1. This ends the proof. □

Theorem 2 states that under certain conditions, the wholesale price discount scheme can coordinate the whole supply chain. In a similar way with the proof of Theorem 2, it can be proved that if $r_N N \leqslant \sqrt{a}/b$, the whole supply chain is coordinated with all of the retailers using EOC policy; While if $r_1\sqrt{N} \geqslant \sqrt{a}/b$, the whole supply chain is coordinated with none of the retailers using EOC policy.

Finally, we provide an example for Theorem 2.

Example. Suppose $N = 5$, $r_0 = 0.95$, $L_0 = 1$, $L_i = L = 8$, and $d_i = d$, $\sigma_i = \sigma$, $\rho_i = \rho = 0$ for all retailer i (here d and σ can be any positive numbers). The cost parameters for the retailers are as follows: $r_1 = 1$, $r_2 = 2.2$, $r_3 = 2.3$, $r_4 = 2.4$, $r_5 = 2.5$. For $k = 1$, it can be easily calculated that $kr_k\left(\sqrt{N-k+1}+\sqrt{N-k}\right) = 4.236$, $r_{k+1}\sqrt{N-k} = 4.4$ and $\sqrt{a}/b = 4.244$, which implies that the conditions of Theorem 2 are satisfied with $k = 1$. Therefore, the wholesale price discount scheme coordinates the supply chain with only the first retailer using EOC policy.

It can be easily verified that this example does not satisfy the conditions of Proposition 4 in Xie et al. (2010). Furthermore, Example 6 in Xie et al. (2010), which satisfies the conditions of Proposition 4 in Xie et al. (2010), does not satisfy the conditions of Theorem 2 in this note. This indicates that the two sets of sufficient conditions for supply chain coordination, respectively provided in this note and Xie et al. (2010), are not implied by each other. Besides, in comparison with Proposition 4 in Xie et al. (2010), the conditions of Theorem 2 in this note are much easier to check.

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Reference

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