Chapter 45
Mathematical Modelling Skills and Creative Thinking Levels: An Experimental Study

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Abstract In the last decade, extensive experimental studies were carried out to assess university students' modelling skills in various countries. However, such studies have not been done in China. This chapter tries to fill the gap by introducing the findings from a simple experimental study in a Chinese university. We evaluated 33 engineering students in a class and obtained the distributions of the students' mathematical modelling skills and their creative thinking levels. The data from the experiments show that there is a strong positive correlation between these two kinds of competencies. We also examined the relationship between the students' mathematical modelling skills and the scores they achieved in basic mathematical courses, and found that the correlation between them is insignificant, although some patterns of relationships do exit.

1 Introduction

In the last 30 years China's national economy has achieved much through its Reform-and-Open Policy. However, the reform in China's education system has not progressed rapidly. As a result, the education system is often, especially in recent years, attacked and criticized in China by pointing out that the students graduating from colleges and universities lack creative abilities and are not open to innovative practices and procedures.

For example, in 1999, the Ministry of Education of China and the China Youth League co-sponsored a survey of Chinese students' creative thinking abilities among 19,000 students in 31 provinces (Ban 2001). The survey revealed that only

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4.7% of students considered themselves to have curiosity, confidence, perseverance, and imagination. Only 14.9% of students hoped to cultivate their exploring spirits for new things and to enhance their abilities of information collection and imagination. Only 33% of students participated in practical activities during their study life in schools. The proportions of the students with the initial creativity personality and creativity characteristics are as low as being 4.7% and 14.9%, respectively. In addition to this, if a student raised an objection to his/her teacher in the class, 48.1% of students thought that most students would keep silent, and 16.5% of students even thought that most students would criticize the objector.

The above figures from the survey clearly show that most students in China are not open to innovation. The reasons for that are surely complicated. As to our understanding, one of the most important reasons might be that the schools and teachers in China put their attention on teaching the students only about the knowledge and skills, but neglect cultivating the creative thinking ability of students. As a result of this kind of teaching style, the knowledge and skills of the subject concerned are the only focus for the students. The second reason lies in that the evaluation criteria in Chinese schools neglect the student's individuality and personality development. For a long time, we think a good student is the only one who gets very high grades in his class courses, and the students with lower grades but more creative ideas are not valued at all.

In more recent years, it has been widely recognized in China that in order for the country to develop in a sustainable manner, it is crucial to embrace innovation. Since the education system shoulders the special mission of cultivating a national spirit of innovation and fostering creative talents, reforming the teaching styles and the evaluation criteria for students has attracted increasing attention in China. The primary objective of the reform is to regard the cultivation of the innovation spirit and practical ability as the key of the education system.

As one component of the reform in China tertiary education, mathematical modelling courses and related activities are highlighted as the breakthrough of reforming mathematical education in Chinese universities (Jiang et al. 2007a, b; Xiao 2000). The reason behind this is that more and more mathematical teachers in China have recognized the importance and value of the mathematical modelling teaching process and related activities. The Chinese teachers now think that the key to the mathematical modelling teaching process is to create an environment that arouses students' desire to learn and develop their ability of self-study and to enhance their application and innovation ability. In order to improve the students' quality in mathematics, the emphasis is put on the students' ability of acquiring new knowledge and the processes of problem solving, rather than only on knowledge and skills in pure mathematics. Therefore, mathematical modelling is gradually becoming the best bonding point to enhance students' mathematical knowledge and application ability.

In this chapter, we are concerned with a primary question as follows: What is the current status of mathematical modelling skills of the students in Chinese universities? As we know, in the last decade, extensive experimental studies have been carried out to assess students' modelling skills in various countries (e.g., Houston and Neill 2003a, b; Izard et al. 2003; Lingefjärd 2004). Recently, Xu and Ludwig (2007, 2008) and Dan et al. (2007) also carried out experimental analyses on the mathematical modelling ability levels for high school students in China. However, from the authors' knowledge, this kind of experimental studies has never been done in Chinese universities. This chapter tries to fill the gap by introducing the results of a simple experimental study in a Chinese university.

Furthermore, we are more concerned with two relevant questions as follows: What is the relationship between the students' mathematical modelling skills and their creative thinking levels? What is the relationship between their mathematical modelling skills and their basic knowledge of mathematics? The answers to these questions will enhance our understanding about the relationships among the students' mathematical modelling skills, their creative thinking levels, and their achievements in basic mathematical courses, and thus provide evidence why now in China people think mathematical modelling is a vehicle to improve the students' innovation ability. This chapter presents some key findings concerning these questions from our experimental study.

2 Mathematical Modelling Skills

2.1 Test Questions

Multiple-choice questions have been widely used to test students' mathematical modelling skills in the last decade in various countries (Haines and Crouch 2001, 2005; Haines et al. 2001; Houston and Neill 2003a, b; Izard et al. 2003; Lingefjärd 2004; Lingefjärd and Holmquist 2005). It is also reported that the validity, reliability, and stability of the test are very good, thus the approach is adopted as the test instrument in our experiment. Specifically, we used all the 22 multiple-choice questions in our test (the details of the 22 questions may be found in Lingefjärd 2004 and accessed directly on the web from the given reference). The correct answer for each question gains two marks, and a partially correct answer gains one mark. Thus the maximum possible score is 44. Students scoring 29 or more are regarded to have 'strong' mathematical modelling ability; those scoring 21 or less are regarded to have 'poor' mathematical modelling ability; others are considered as 'medium' (Lingefjärd 2004).

2.2 Implementation

We chose to perform the test at Logistics Engineering College (LEC), one of the engineering universities in China with average-level students. All 33 students from Class 2005171, who entered into LEC in September 2005 with a major in engineering (automation), were tested with the previously mentioned questions (in a Chinese version we translated from the original English version) on March 15, 2007. At that time, the students had just completed their courses in higher mathematics (calculus), linear algebra, probability and statistics, and mathematical modelling. The test lasted 40 min, and during the test, the students answered the questions independently without any interruptions from the teacher.

Table 45.1 Basic test results for mathematical modelling skills

	Poor (21– marks)	Medium (21–29 marks)	Strong (29+ marks)
Number of students	4	14	15
Percentage	12.12	42.42	45.45

Table 45.2 The test results by question groups

		% Agreeing with experts' solutions		% Partially correct	
Modelling skills	Questions	LEC	Ulster	LEC	Ulster
Type 1 (simplifying assumptions)	1,2,3	53	47	22	25
Type 2 (clarifying the goal)	4,5,6	37	25	20	35
Type 3 (formulating the problem)	7,8,9	46	64	26	14
Type 4 (assigning variables, parameters, and constants)	10,11,12	52	78	24	12
Type 5 (formulating mathematical statements)	13,14,15	87	77	4	13
Type 6 (selecting a model)	16,17,18	47	28	14	28
Type 7 (graphical representations)	19,20	37	46	56	29
Type 8 (relating back to the real world)	21,22	63	42	22	28

2.3 *Results*

The basic results are summarized in Table 45.1. The students achieved an average score of 28, with a standard deviation of 6.03 and a range from 17 to 37. The median of the scores is 27, and lower- and upper-quartiles are 23 and 33, respectively. These figures are similar to the test results previously obtained at the University of Ulster, UK (Houston and Neill 2003b, p.160), showing that these students' mathematical modelling skills were satisfactory.

Table 45.2 compares our test results from LEC with those from Ulster by question groups (Houston and Neill 2003b). The students from LEC performed better than the students from Ulster in clarifying the goal, formulating mathematical statements, and relating back to the real world, while they performed worse in formulating the problem; assigning variables, parameters, and constants; and graphical representations.

3 Creative Thinking Levels

Creativity is the sum of a person's mental ability and personality quality in creative activities, and it is the displayed special ability in his or her creative activities (Li and Zhang 1999). When solving problems, a person with a strong creative ability always tends to use a unique way of connecting different concepts and knowledge and makes creative solutions. Students and teachers usually think that mathematical modelling is difficult and it is a creative activity. This section motivates to investigate how the students' creative thinking levels are affected by their mathematical modelling skills.

3.1 Test Questions

Torrance Tests of Creative Thinking (TTCT) is a widely used test instrument for testing one's creative thinking, focusing on one's abilities such as fluency, flexibility, originality, and elaboration (Curtis and Rick 2006; Li and Zhang 1999). Therefore, we adopted TTCT as the test instrument in this study. Specifically, the test questions were from a Chinese book written by Li and Zhang (1999). The test included 20 multiple-choice questions, and each one has a unique correct answer that contributes one mark (the other answers contribute nothing). The test time is 30 min. If one can finish the test in 5 min, he/she can obtain an additional five marks; for 10 min, three marks will be added; and for 20 min, two points will be added. Therefore, the maximum score was 25. Students scoring 14 or more are regarded to be 'strong' in creativity, and those scoring ten or less are regarded as 'poor'. Others are considered as 'medium' (Li and Zhang 1999).

3.2 Implementation

The students tested were from Class 2005171 at LEC, the same as those mentioned previously. The test was carried out for the 33 students on March 20, 2007, just five days later than the test given for their mathematical modelling skills.

3.3 Results

The results are summarized in Table 45.3. The average score of the students' creative thinking levels was 12.3, with a standard deviation of 2.1 and a range from 8 to 16. Overall, the students' creative thinking levels were 'medium'. The students with 'poor' creative thinking levels accounted for 12.12%, which is the same figure we have just observed for the students with 'poor' mathematical modelling skills in Sect. 2. However, the students with 'strong' creative thinking levels accounted for

Table 45.3 Test results for creative thinking levels

	Poor (10– marks)	Medium (10–14 marks)	Strong (14+ marks)
Number of students	4	22	7
Percentage	12.12	66.67	21.21

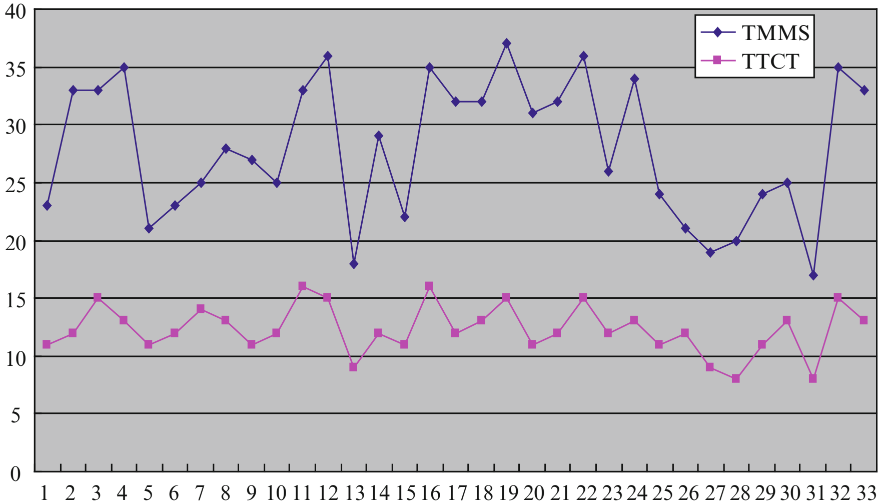

Fig. 45.1 TMMS vs. TTCT

only 21.21%, which is much less than what we have just observed for the students with 'strong' mathematical modelling skills in Sect. 2.

3.4 Relationship with Mathematical Modelling Skills

The detailed data from these two tests (i.e., test for mathematical modelling skills (TMMS) in Sect. 2 and TTCT in this section) are plotted in Fig. 45.1, for all the 33 students tested. The four students who scored less than 9 in TTCT were the same four students who scored less than 21 in TMMS. The seven students who scored over 14 in TTCT were among the fifteen students who scored over 29 in TMMS. This observation clearly indicates that students with 'strong' creative thinking levels were also 'strong' in mathematical modelling skills, and those with 'poor' creative thinking levels were also 'poor' in mathematical modelling skills. The correlation coefficient of mathematical modelling skills and creative thinking levels was 0.815, which indicated that there was a strong positive correlation between these two types of competencies. This observation validated the belief that 'mathematical modelling is necessary for creativity' (D'Ambrosio 1989).

In fact, a careful examination of the TTCT test questions suggests to us that some of the questions were designed to test similar ability as the TMMS questions did. Two example questions of TTCT are given as follows.

Question 12: In Fig. 45.2a, which image is another one's reflection seen in a mirror?

Question 14: For the table in Fig. 45.2b, can you fill a number in the place of "?" by using an arithmetic operation horizontally or vertically?

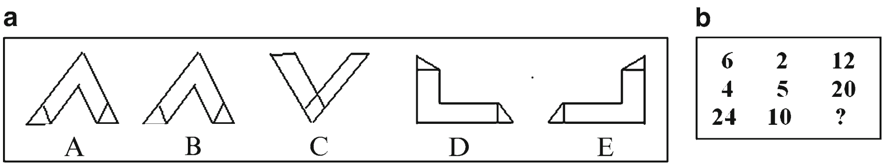

Fig. 45.2 Two example questions of TTCT

Table 45.4 Test results for basic mathematics courses

	Fail (60− marks)	Pass (60–90 marks)	Excellent (90+ marks)
Number of students	2	21	10
Percentage	6.1	63.6	30.3

Clearly, Question 12 tests the students' ability in understanding graphics and their symmetry, which has some links with the ability of graphical representation (Type 7 modelling skills in Table 45.2). Question 14 tests the students' ability in constructing a mathematical relationship between two variables, which has strong links with Type 5 modelling skills in Table 45.2 (formulating mathematical statements). Therefore, it is not surprising that a strong positive correlation exists between these two types of competencies.

4 Knowledge in Basic Mathematics

4.1 Score in Basic Mathematical Courses

The motivation of this section is to investigate how the students' knowledge in basic mathematics is affected by their mathematical modelling skills. The 33 students from Class 2005171 at LEC already had a basic mathematics course test (BMCT) at the end of their first year as freshman (i.e., in July 2006), just after their basic mathematics courses had been completed. The test questions came from a test database developed by Xi'an Jiaotong University (a famous engineering university in China) and published by the Higher Education Press of China. Specifically, the test includes 22 problems in the formats of 'fill-in-the-blank', 'multiple-choice', and 'calculation'. The mathematical content of the test included calculus and linear algebra, and the perfect score was 100. Students with more than 90 marks were regarded as 'excellent'. Those with less than 60 marks were regarded as having 'failed' and 'poor'. The results are summarized in Table 45.4. The average score of the test was 80, with a standard deviation of 16.7 and a range from 25 to 99. Overall, the students performed very well in BMCT.

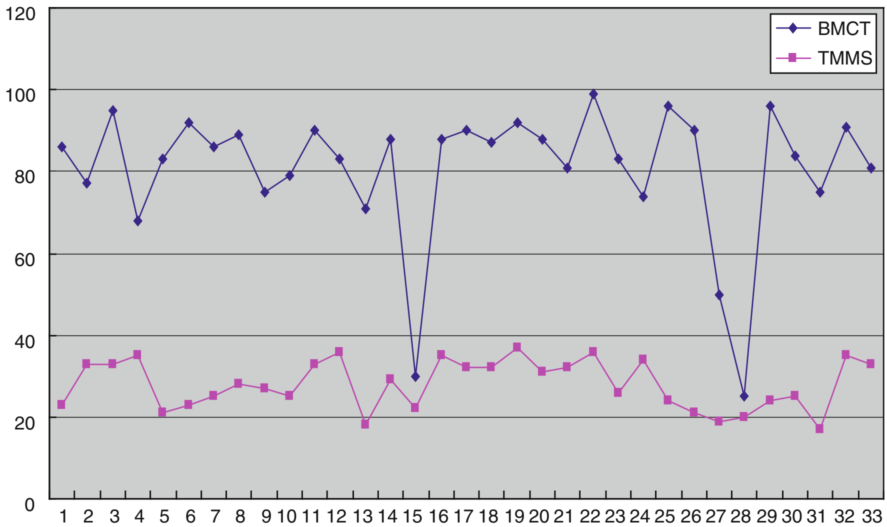

Fig. 45.3 TMMS vs. BMCT

4.2 Relationship with Mathematical Modelling Skills

The detailed data from the two tests (i.e., TMMS and BMCT) are plotted in Fig. 45.3, for all the 33 students tested. The correlation coefficient between the scores of TMMS and BMCT was 0.381, which was very small and thus indicated that the correlation between them was insignificant.

However, some patterns of relationships do exist between these two types of competencies. In more details, their relationship could be summarized as follows:

1. Students with very low scores in BMCT did not have 'strong' mathematical modelling skills (e.g., the students failed in BMCT scored less than 21 in TMMS). In other words, students with 'strong' mathematical modelling skills did not score very low in BMCT.
2. Students with very high scores in BMCT might not have 'strong' mathematical modelling skills. Among the ten excellent students in BMCT, only seven of them scored more than 29 marks in TMMS, and the other three scored only 21, 23, and 24, respectively, in TMMS.
3. Students with strong mathematical modelling skills might not score very high in BMCT. For example, three students who scored over 30 in TMMS scored only 68, 74, and 77, respectively, in BMCT.

For a long time, the teachers and students in China usually think that only those with very high scores in basic mathematics courses can learn and do mathematical modelling well. However, the earlier findings conflict with the traditional thoughts that exist among many teachers and students in China, but they are very consistent with their teaching experience. For example, in mathematical modelling activities such as China's undergraduates mathematical contests in modelling (Jiang et al. 2007b), the students who outperformed the others and were awarded the best achievements were usually not those with very high scores in their basic mathematics courses.

One of the reasons for these observations might be the nature of the TMMS test questions. The mathematical knowledge and skills needed for the test are basically very simple, and advanced mathematical knowledge and skills are not needed for the test. Therefore, mathematical knowledge and skills did not become an obstacle for most of the students when they completed the test. Thus the advantages of these students in mathematical knowledge cannot be reflected in the test.

Another possible reason for these observations might be that if a student gains a very high BMCT score it does not mean that the student really grasps the basic mathematics very well. In other words, the assumptions we make about students regarding their knowledge base and successful completion of earlier modules and/or examinations cannot be relied upon (Anderson et al. 1998; Haines and Crouch 2001).

5 Summary

In this chapter, we introduced some findings from an experimental study concerning mathematical modelling for some engineering students in an average-level university in China. The findings provide strong evidence to support the mathematical education reform in China with regards to mathematical modelling courses and related activities as a vehicle to improve the students' innovation ability. We have a plan to do more experimental studies in China to figure out whether the findings could be extended to other Chinese students as well, since the current study is based only on 33 students. We are also going to investigate how mathematical modelling courses should be taught and how mathematical-modelling-related activities influence the students' creative thinking modes.

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