Manufacturer-retailer contracting with asymmetric information on retailer's degree of loss aversion

Xiaoxue Deng

Science and Technology on Information System Engineering Laboratory

National University of Defense Technology

Changsha 410073, China

Tel: +8613487319541

Email: littlesnow18@gmail.com

Jinxing Xie*

Department of Mathematical Sciences

Tsinghua University

Beijing 100084, China

Tel: (+86-10) 6278-7812

Email: jxie@math.tsinghua.edu.cn

Huachun Xiong

Department of Mathematical Sciences

Tsinghua University

Beijing 100084, China

Tel: +8615110079661

Email: xionghc06@gmail.com

* Corresponding Author.

Accepted by Journal IJPE, Nov. 2012

Manufacturer-retailer contracting with asymmetric information on retailer's degree of loss aversion

Abstract

In a recent paper [Wang C., Webster S., 2007. Channel coordination for a supply chain with a risk-neutral manufacturer and a loss-averse retailer. Decision Sciences 38 (3), 361-389.], the authors study a supply chain consisting of a risk neutral manufacturer and a loss averse retailer and show that the supply chain can be coordinated by three contracts: buy back (BB), gain/loss sharing (GL) and gain/loss sharing and buy back (GLB). They assume that the retailer's degree of loss aversion is common knowledge. However, this assumption does not reflect real situations, since in the real industry one party's degree of loss aversion is always unknown by other parties. To reflect more realistic situations, we propose a principal-agent model, assuming the retailer's degree of loss aversion to be asymmetric information. Within the principal-agent framework, we obtain the following results: (1) The optimal contract menu is derived for the manufacturer (the principal) by mechanism design theory; (2) Under the optimal contract menu, information asymmetry lowers the production quantity, decreases the manufacturer's profit and deteriorates supply chain performance, while increasing the retailer's utility; (3) While the contracts proposed by Wang and Webster cannot coordinate the supply chain with asymmetric information in an implementable way, the optimal contract menu proposed in the present paper can, if the wholesale price is endogenously determined by the manufacturer and its lower bound is 0.

Keywords: loss aversion; information asymmetry; principal-agent model; revelation principle

1. Introduction

Traditional literature mainly focuses on supply chains with risk neutral members, who maximize expected profits or minimize expected costs. However, many evidences point out that most decision makers do not practice as the models with the risk neutrality assumption predict (e.g., Kahn, 1992; Fisher & Raman, 1996, and Patsuris, 2001). In view of this, some researchers have advocated studying supply chains without the assumption of risk neutrality to represent more realistic situations (e.g. Anupindi, 1999; Tsay et al., 1999, and Wu et al., 1999).

A few studies have deviated from the risk neutrality assumption and incorporated other objective functions rather than profit maximization. One stream of these studies is the models incorporating loss aversion, which is a critical feature of the prospect theory (Kahneman & Tversky, 1979). Loss aversion means that people are more averse to losses than they are attracted to same-sized gains. It is well supported in finance, economics, marketing, and organizational behavior (Rabin, 1998; Camerer, 2001). In a recent paper, Wang & Webster (2007) propose a model to analyze a supply chain consisting of a risk neutral supplier and a loss averse retailer. They show that such a supply chain can be coordinated by three different kinds of contracts: buy back, gain/loss sharing (GL), gain/loss sharing and buy back (GLB) contracts. In that paper, it is assumed that the retailer's degree of loss aversion is common knowledge. However, this assumption does not reflect real situations since in the real industry one party's degree of loss aversion is always unknown by other parties. In order to reflect more realistic situations of a manually operated supply chain, it is worthy to study the case that the degree of loss aversion is asymmetric information to supply chain members, i.e., the retailer's degree of loss aversion is private information, unknown by the manufacturer.

In the present paper, we propose a principal-agent model to study a similar supply chain, which is consisting of a risk neutral manufacturer (the principal) and a loss averse retailer (the agent), but the retailer's degree of loss aversion is asymmetric information among the supply chain members. The existence of information asymmetry on the degree of loss aversion gives rise to several interesting questions:

(1) Do the contracts coordinating the supply chain without asymmetric information continue to coordinate the supply chain with the asymmetric information in an implementable way? (2) How does information asymmetry affect the production quantity of the supply chain, total supply chain profit, manufacturer's profit and the retailer's utility?

Generally, in a principal-agent model, an optimal contract menu should be found standing at the point of the principal. For the agent, there is a participation constraint: the agent should get a profit or utility that is greater than its reservation one. Along this principle, we identify an optimal truth-telling contract menu (truth-telling means the contract menu can induce the retailer to report the truthful degree of loss aversion) by assembling some modified GL contracts. Furthermore, we find that under the optimal contract menu, the supply chain production quantity is larger, total supply chain profit is higher, the manufacturer's profit is lower and the retailer's utility is higher with asymmetric information than without it. We also show that the coordinating contracts provided in Wang & Webster (2007) cannot coordinate the supply chain with the asymmetric information in an implementable way. However, under the optimal contract menu, the supply chain can be coordinated if the wholesale price is endogenously determined by the manufacturer and its lower bound is 0.

Next we review the literature related to the present paper. First, the present paper is related to the supply chain models which deviate from the assumption of risk neutrality. There are mainly two streams of this literature: models incorporating risk aversion and models incorporating loss aversion.

The literature considering risk averse decision makers is rich (e.g., Lau, 1980; Eeckhoudt et al., 1995; Agrawal & Seshadri, 2000; Gan et al. 2004, 2005; Choi, 2007; Choi et al., 2008a, 2008b; Wei & Choi, 2010; Chiu et al. 2011; Choi & Chiu, 2012). Lau (1980), Eeckhoudt et al. (1995), Choi et al. (2008a) and Choi & Chiu (2012) investigate the optimal decisions of risk averse newsvendors under various risk measures (e.g., mean-variance, mean-downside-risk, etc.). Gan et al. (2004) provide the definition of coordination of supply chains consisting of risk averse members. Choi et al. (2008b), Wei & Choi (2010) and Chiu et al. (2011) consider the issues of supply chain coordination based on well-known contracts such as the buy back contract, the wholesale pricing and profit sharing contract and the target sales rebate contract. Agrawal & Seshadri (2000) and Gan et al. (2005) propose new contracts to improve supply chain performance and achieve supply chain coordination. Choi (2007) investigate fashion retailers' pre-stocking and pricing decisions with risk considerations. Unlike these papers which assume supply chain members are risk averse, our paper employs a loss aversion framework.

The studies on loss aversion are relatively limited. Sorger (1988), Greenleaf (1995), Kopalle et al. (1996), Fibich et al. (2003) and Popescu & Wu (2007) discuss the optimal pricing strategies of firms when considering customers' reference and loss aversion effects on historical prices. Schweitzer & Cachon (2000) and Wang & Webster (2009) discuss the optimal decisions for loss averse newsvendors. The above papers only analyze the decision making of a single enterprise rather than a supply chain. Ho & Zhang (2008) conduct a laboratory study to investigate how the use of the fixed fee in pricing contracts affects market outcomes of a manufacturer-retailer channel. To account for the experimental results, they propose a model in which loss aversion is embedded in to a quantal response framework. Wang (2010) propose a model where multiple newsvendors with loss aversion compete for inventory from a risk neutral supplier and identify two kinds of effects: demand stealing effect and loss aversion effect. It is shown that while the demand stealing effect increases total order quantity, the loss aversion effect decreases total order quantity. All the literature above assumes complete information, including the supply chain (channel) members' degrees of loss aversion. In contrast, the present paper assumes that the retailer's degree of loss aversion is asymmetric information.

Second, the present paper is also related to the principal-agent models which consider asymmetric information. Although the publications in this area are rich, they mainly focus on two kinds of asymmetric information: production cost information and market demand information. Examples for the first kind of asymmetric information include Corbett & de Groote (2000), Ha (2001) etc., and examples for the second kind include Cachon & Lariviere (2001), Ozer & Wei (2008) etc. More recently, new types of asymmetric information have emerged into literature, for example, quality (e.g., Kaya & Ozer, 2003), and risk sensitivity (Wei & Choi, 2010 and Xiao & Yang, 2010). Different from their papers, our work considers asymmetric information on supply chain member's degree of loss aversion.

Finally, the present paper is related to the literature on supply chain contracts and supply chain coordination. For this literature, see the comprehensive review by Cachon (2003).

The rest of the paper proceeds as follows. In Section 2, we describe our principal-agent model. In Section 3, assuming the wholesale price exogenously determined, we provide the optimal contract menu for the manufacturer and analyze the effects of information asymmetry. Section 4 discusses the cases of endogenous wholesale price and Section 5 investigates the issues of supply chain coordination. Finally, in Section 6, we come to the concluding remarks and future research directions.

2. Principal-agent model

Consider a supply chain consisting of one manufacturer (she) and one retailer (he). There is one selling season with stochastic demand for a single product and a single opportunity for the retailer to order inventory from the manufacturer before the selling season begins. The product is produced by the manufacturer at a unit cost of c and sold by the retailer to the customer at an exogenous retail price p. Leftover products at the end of the selling season are salvaged with a value of s per unit ($s<c<p$). Without loss of generality, s is normalized to 0. The manufacturer sells products to the retailer at a wholesale price w. The market demand D possesses a cumulative distribution function (CDF) $F(x)$ and a probability density function (PDF) $f(x)$. The CDF $F(x)$ is defined over an interval $I \subset [0,+\infty)$ (we normalize the lower bound of I to 0 without loss of generality). As in most contract literature based on the newsvendor framework (e.g. Tsay, 1999; Cachon, 2003), we assume $F(x)$ is differentiable and strictly increasing on I. All parameters above are assumed to be common knowledge.

According to Wang & Webster (2007), we further assume that the manufacturer, which is the principal, is risk neutral, and the retailer (the agent) is loss averse, since the manufacturer can diversify her assets across multiple firms, while the retailer's income is tied to the manufacturer. Specifically, we assume that the retailer has a kinked piece wise linear utility function as

$$U(x)=\begin{cases} x-x_0, & \text{if } x>x_0, \\ \lambda(x-x_0), & \text{if } x\le x_0, \end{cases}$$

where x_0 is the reference point of the profit and λ ($\lambda \ge 1$) is a coefficient that measures the degree of loss aversion (larger λ represents higher degree of loss aversion) . Although this piecewise linear form of loss aversion utility does not count on diminishing sensitivity property in prospect theory, it is used widely due to its simplicity in existing literature (e.g., Kahneman & Tversky, 1979; Schweitzer & Cachon, 2000; Wang & Webster, 2007). The reference point x_0 is assumed to be common knowledge and normalized to 0 without loss of generality. The degree of loss aversion λ is assumed to be private information of the retailer. Therefore, the manufacturer doesn't know the exact value of λ, but she has a prior belief on it. Specifically, we assume the manufacturer's belief on the retailer's degree of loss aversion is distributed on $[\underline{\lambda},\overline{\lambda}]\subseteq[1,+\infty)$ with a CDF $H(\lambda)$ and a PDF $h(\lambda)$.

Next we provide the definition of supply chain coordination. To do this, we analyze the vertical integrated supply chain, in which the manufacturer (the principal) act as a central planner and make decisions for the whole supply chain. If q units are produced before the selling season, the profit of the whole supply chain is

$$\Pi_C(q)=\begin{cases} (p-c)q, & \text{if } q<D, \\ pD-cq, & \text{if } q\ge D, \end{cases}$$

and the corresponding expected profit is

$$\pi_C(q)=E_D\Pi_C(q)=\int_0^q [px-cq]f(x)dx+\int_q^{+\infty}(p-c)qf(x)dx,$$

where $E_D(\cdot)$ is the expectation operator with respect to D. The problem for the central planner is choosing a production quantity to maximize $\pi_C(q)$. Clearly, it is a standard newsvendor problem and the optimal production quantity is

$$q_C^* = F^{-1}\left(\frac{p-c}{p}\right).$$

As in Wang & Webster (2007), the supply chain coordination is defined as follows: if the production (order) quantity equals to q_C^*, in which case total supply chain profit achieves its maximum, we say the supply chain is coordinated.

3. Optimal contract menu and analysis

In Wang & Webster (2007), the authors propose a gain/loss (GL) sharing contract (w, γ, β), which specifies that, in addition to paying the manufacturer the wholesale price *w*, the retailer shares a fraction $\beta \in [0,1)$ of his gain with the manufacturer or is reimbursed a fraction $\gamma \in [0,1)$ of its loss by the manufacturer.

Next we use some modified GL contracts to propose the contract menu (we call it MGL contract menu for short in the rest of this paper) which will be discussed in the present paper. An MGL contract menu consists of some MGL contracts (w, β, q). Each MGL contract (w, β, q) designates a wholesale price *w*, a gain sharing percentage β ($\beta < 1$) and an order quantity *q*. That is, in comparison to GL, an MGL fixes γ at 0, allows $\beta < 0$ and adds on an order quantity.

Under the MGL contract menu, the events proceed as follows. (1) Before the selling season, the manufacturer offers a menu consisting of some MGL contracts (w, β, q). (2) At the beginning of the selling season, the retailer decides to participate in the game or not. If he decides to leave the game, he gets his reservation utility U_0 (we normalize U_0 to 0 without loss of generality). If the retailer decides to participate in the game, he chooses one MGL contract (w, β, q) from this menu and orders *q* units from the manufacturer at the wholesale price *w* according to the chosen contract. (3) During the selling season, the retailer sells the products to the customer. (4) At the end of the selling season, the retailer shares a percentage β of his gain to the manufacturer. Here we don't restrict $\beta > 0$. If $\beta < 0$, the manufacturer reimburse $-\beta \times G$ to the retailer, where G stands for the retailer's gain.

Throughout this section, we assume the wholesale price is exogenously determined by the market. In Section 4, we will extend the results in this section to the case that the wholesale price is endogenously determined by the manufacturer.

3.1 Optimal MGL contract menu

The manufacturer's challenge is to maximize her expected profit by designing a contract menu $\{w(\lambda), \beta(\lambda), q(\lambda) \mid \lambda \in [\underline{\lambda}, \overline{\lambda}]\}$ while ensuring the retailer's participation. The parameter λ denotes that the specific contract $(w(\lambda), \beta(\lambda), q(\lambda))$ is intended for the retailer whose degree of loss aversion is λ. Let

$$L(w,q) = \int_0^{B(w,q)} (px - wq) f(x)dx\,,$$

$$G(w,q) = \int_{B(w,q)}^{q} (px - wq) f(x)dx + (p-w)q(1-F(q)),$$

where $B(w,q) = wq/p$. It is easy to know that, if an MGL contract $(w(\lambda'), \beta(\lambda'), q(\lambda'))$ is chosen, the expected utility of the retailer with degree of loss aversion λ (hereafter we call it "the retailer with type λ") is

$$u_R(\lambda' \mid \lambda) := \lambda L(w(\lambda'), q(\lambda')) + (1-\beta(\lambda'))G(w(\lambda'), q(\lambda')).$$

Hence $u_R(\lambda) := u_R(\lambda \mid \lambda)$ denotes the utility of the retailer with type λ when he chooses the truth-telling contract, i.e., the contract $(w(\lambda), \beta(\lambda), q(\lambda))$. Furthermore, let

$$\pi_M(\lambda) = (w(\lambda) - c)q(\lambda) + \beta(\lambda)G(w(\lambda), q(\lambda))$$

be the expected profit of the manufacturer when the retailer chooses $(w(\lambda), \beta(\lambda), q(\lambda))$. By revelation principle (Fudenberg & Tirole, 1991), the manufacturer can limit the search for the optimal contract menu to the class of truth-telling contracts. Therefore, the optimal truth-telling contract menu can be identified by solving the following problem, denoted by (AI):

$$\max_{w(\lambda),\beta(\lambda),q(\lambda)} \quad E_{\lambda}\pi_M(\lambda) = \int_{\underline{\lambda}}^{\bar{\lambda}} \left[(w(\lambda)-c)q(\lambda) + \beta(\lambda)G\left(w(\lambda), q(\lambda)\right) \right] h(\lambda)d\lambda \tag{9}$$

$$\text{s.t. (IC)} \quad u_R(\lambda' \mid \lambda) \le u_R(\lambda), \quad \text{for any } \lambda \in [\underline{\lambda}, \bar{\lambda}] \text{ and } \lambda' \in [\underline{\lambda}, \bar{\lambda}], \tag{10}$$

$$\text{(PC)} \quad u_R(\lambda) \ge 0 \text{, for any } \lambda \in [\underline{\lambda}, \bar{\lambda}]. \tag{11}$$

The first set of constraints is the incentive compatibility constraints (IC). These constraints ensure that the retailer maximizes his expected utility by telling the truthful information on his degree of loss aversion. The second set of constraints is the participation constraints (PC). These constraints ensure the retailer to at least get his reservation utility 0 regardless of λ.

A solution to Problem (AI) is feasible if it satisfies constraints (10) and (11). Recall that we assume the wholesale price w is exogenously determined by the market. Thus, here we fix $w(\lambda) = w$, $\lambda \in [\underline{\lambda}, \bar{\lambda}]$ and only solve $(\beta(\lambda), q(\lambda))$ for Problem (AI) in the rest of this section. We now provide a lemma that will be useful for deriving the optimal contract menu.

Lemma 1. *A solution* $(\beta(\lambda), q(\lambda))$ *is feasible for Problem (AI) if and only if*

(a) $u_R(\lambda) = -\int_{\lambda}^{\bar{\lambda}} L\left(w, q(x)\right)dx + a$, *where* $a \ge 0$ *is a constant;*

(b) $q(\lambda)$ *is decreasing with respect to* λ.

From Part (a) of Lemma 1 and the definition of $u_R(\lambda)$, we have

$$\begin{aligned} \beta(\lambda) &= 1 - \frac{u_R(\lambda) - \lambda L\left(w, q(\lambda)\right)}{G\left(w, q(\lambda)\right)} \\ &= 1 - \frac{a - \int_{\lambda}^{\bar{\lambda}} L\left(w, q(x)\right)dx - \lambda L\left(w, q(\lambda)\right)}{G\left(w, q(\lambda)\right)}. \end{aligned} \tag{12}$$

With Eqn. (12), we can rewrite our objective function in (9) as follows:

$$\begin{aligned}E_{\lambda}\pi_M(\lambda) &= \int_{\underline{\lambda}}^{\bar{\lambda}}\Big[(w-c)q(\lambda)+G\big(w,q(\lambda)\big)+\lambda L\big(w,q(\lambda)\big) \\ &\quad + \int_{\lambda}^{\bar{\lambda}} L\big(w,q(x)\big)dx\Big]dH(\lambda)-a \\ &= \int_{\underline{\lambda}}^{\bar{\lambda}}\Big[(w-c)q(\lambda)+G\big(w,q(\lambda)\big) \\ &\quad + \left(\frac{H(\lambda)}{h(\lambda)}+\lambda\right)L\big(w,q(\lambda)\big)\Big]dH(\lambda)-a.\end{aligned} \tag{13}$$

Apparently, to maximize $E_{\lambda}\pi_M(\lambda)$ in (13), the constant *a* must be 0. The following theorem prescribes the optimal MGL contract menu and characterizes its properties.

Theorem 1. *Assume* $H(\lambda)/h(\lambda)+\lambda$ *is an increasing function of* λ.

(a) The optimal MGL contract menu to solve problem (AI) is $\left\{\big(\beta_{AI}^*(\lambda), q_{AI}^*(\lambda)\big) \mid \lambda \in [\underline{\lambda}, \bar{\lambda}]\right\}$, *where* $q_{AI}^*(\lambda)$ *is the unique solution of the equation*

$$\frac{(p-c)-pF(q)}{wF\left(\frac{wq}{p}\right)} = \frac{H(\lambda)}{h(\lambda)}+\lambda-1, \tag{14}$$

and

$$\beta_{AI}^*(\lambda) = 1+\frac{\int_{\lambda}^{\bar{\lambda}} L\big(w,q_{AI}^*(x)\big)dx+\lambda L\big(w,q_{AI}^*(\lambda)\big)}{G\big(w,q_{AI}^*(\lambda)\big)}; \tag{15}$$

(b) $q_{AI}^*(\lambda)$ *is a decreasing function of* λ.

(c) Under the optimal MGL contract menu, the expected utility of the retailer with type λ *is*

$$\begin{aligned}u_{R,AI}^*(\lambda) &= \lambda L\big(w,q_{AI}^*(\lambda)\big)+\big(1-\beta_{AI}^*(\lambda)\big)G\big(w,q_{AI}^*(\lambda)\big) \\ &= -\int_{\lambda}^{\bar{\lambda}} L\big(w,q_{AI}^*(x)\big)dx,\end{aligned} \tag{16}$$

and $u_{R,AI}^*(\lambda)$ *is decreasing with respect to* λ;

(d) When the retailer's type is λ, *the manufacturer's profit under the optimal MGL contract menu is*

$$\begin{aligned}\pi^*_{M,AI}(\lambda) &= (w-c)q^*_{AI}(\lambda) + \beta^*_{AI}(\lambda)G\left(w, q^*_{AI}(\lambda)\right) \\ &= (w-c)q^*_{AI}(\lambda) + G\left(w, q^*_{AI}(\lambda)\right) \\ &\quad + \left(\frac{H(\lambda)}{h(\lambda)} + \lambda\right)L\left(w, q^*_{AI}(\lambda)\right),\end{aligned} \tag{17}$$

and $\pi^*_{M,AI}(\lambda)$ *is decreasing with respect to* λ.

Note that the assumption $H(\lambda)/h(\lambda)+\lambda$ is an increasing function is a sufficient condition but not necessary. This assumption is a regular one that often appears in asymmetric information context. The assumption is clearly true if $H(\lambda)/h(\lambda)$ is increasing, or equivalently, $H(\lambda)$ is log-concave. Many CDFs are log-concave, including uniform distribution, beta distribution and truncated normal distribution over a finite interval (see Rosling, 2002)

Part (a) of Theorem 1 prescribes the optimal contract menu for the manufacturer. Part (b) states that under this optimal contract menu, the retailer will order less if he is more loss averse. This is rather intuitive and consistent with the case of complete information in Wang & Webster (2007). Furthermore, the monotonicity of $q(\lambda)$ enables the manufacturer to infer the retailer's degree of loss aversion from the contract that the retailer chooses. Parts (c) and (d) indicate that under the optimal MGL contract menu, the retailer gets a lower utility and the manufacturer gets a lower profit if the retailer is more loss averse.

3.2 Illustrative example for uniform distributions

In order to make the ideas more clear and demonstrate how the optimal MGL contract menu is used in practice, we provide an illustrative example in this subsection. We assume $H(\lambda)$ be the CDF of a uniform distribution over [1, *B*] and *F*(*x*) be the CDF of a uniform distribution over [0, *A*] (without loss of generality, here we normalize the lower bound of the loss aversion degree to 1, and the lower bound of the demand to 0). For this kind of special distributions, we will first give some analytical solutions according to Theorem 1, and then provide the numerical example.

From Theorem 1, we can note that $q^*_{AI}(\lambda)$, $\beta^*_{AI}(\lambda)$, $u^*_{R,AI}(\lambda)$ and $\pi^*_{M,AI}(\lambda)$

depend on w, thus in this example, while solving for the analytical solutions, we rewrite them as $q_{AI}^*(w,\lambda)$, $\beta_{AI}^*(w,\lambda)$, $u_{R,AI}^*(w,\lambda)$ and $\pi_{M,AI}^*(w,\lambda)$, respectively. In specific, according to Eqn. (14),

$$q_{AI}^*(w,\lambda)=\frac{Ap(p-c)}{p^2+2(\lambda-1)w^2}, \tag{18}$$

thus we have

$$L(w,q_{AI}^*(w,\lambda))=-\frac{w^2 q_{AI}^{*\,2}(w,\lambda)}{2Ap},$$

$$G(w,q_{AI}^*(w,\lambda))=(p-w)q_{AI}^*(w,\lambda)(1-\frac{q_{AI}^*(w,\lambda)(p+w)}{2Ap}).$$

According to Eqns. (15), (16) and (17),

$$\beta_{AI}^*(w,\lambda)=1+\frac{-\frac{A^2p(p-c)^2w^2(B-\lambda)}{2(B-1)(2w^2\lambda+p^2-2w^2)(2w^2B+p^2-2w^2)}+\lambda L(w,q_{AI}^*(w,\lambda))}{G(w,q_{AI}^*(w,\lambda))}, \tag{19}$$

$$u_{R,AI}^*(w,\lambda)=\frac{A^2p(p-c)^2w^2(B-\lambda)}{2(B-1)(2w^2\lambda+p^2-2w^2)(2w^2B+p^2-2w^2)},$$

$$\pi_{M,AI}^*(w,\lambda)=(w-c)q_{AI}^*(w,\lambda)+G(w,q_{AI}^*(w,\lambda))+(2\lambda-1)L(w,q_{AI}^*(w,\lambda)).$$

The corresponding expected profit for the whole supply chain can be expressed as

$$\pi_{C,AI}^*(q_{AI}^*(w,\lambda))=(p-c)q_{AI}^*(w,\lambda)-\frac{pq_{AI}^{*\,2}(w,\lambda)}{2A}. \tag{20}$$

In particular, the numerical results for p=10, c=4, w=6, $A=200$ and $B=3$ are listed in Table 1, in which the first column lists the retailer's degree of loss aversion, the second column lists the optimal MGL contract menu, and the last three columns list the performances of the whole supply chain and its members. In practice, the manufacturer provides a contract menu like the second column. Note that only parts of the contract menu are listed in Table 1 for the sack of saving space. More MGL contracts can be added on according to Eqns. (18) and (19) if necessary. That is to say, although theoretically a complete contract menu should include MGL contracts for all possible values of λ, but in practice, it usually suffices to list MGL contracts for certain values of λ by discretizing λ with small enough gaps.

Retailer's degree of loss aversion λ	Optimal MGL contract menu $\left(\beta^*_{AI}(\lambda), q^*_{AI}(\lambda)\right)$	Manufacturer's profit $\pi^*_{M,AI}(\lambda)$	Retailer's utility $u^*_{R,AI}(\lambda)$	Total supply chain profit $\pi_C\left(q^*_{AI}(\lambda)\right)$
1	(0.0552,120)	253.7705	106.2295	360
1.2857	(0.2065,99.5261)	248.5307	75.5186	349.5205
1.5714	(0.3049,85.0202)	238.4757	53.7599	329.4104
1.8571	(0.3792,74.2050)	227.5557	37.5369	307.5703
2.1429	(0.4404,65.8307)	217.0916	24.9756	286.6422
2.4286	(0.4937,59.1549)	207.494	14.9619	267.4469
2.7143	(0.5420,53.7084)	198.8384	6.7922	250.1357
3	(0.5867,49.1803)	191.0777	0	234.6143

Table 1. Optimal MGL contract menu and performances of the whole supply chain and its members

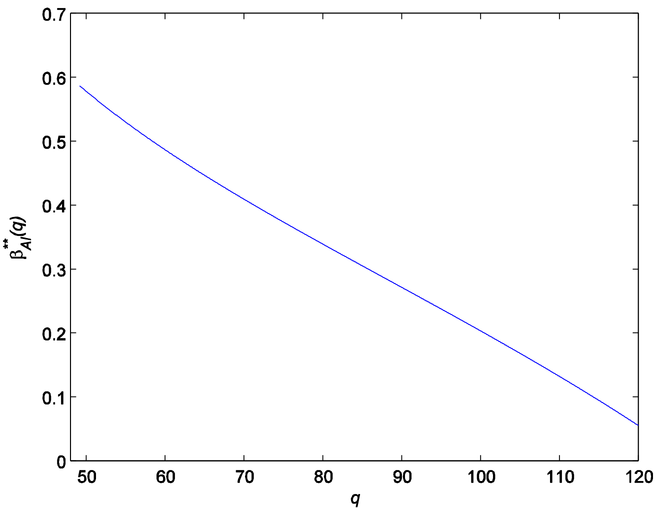

Fig. 1. Gain sharing percentage v.s. order quantity

Clearly, the optimal MGL contract menu is specified by Eqns. (18) and (19), and these equations determine a nonlinear relationship between the profit sharing percentage β and the retailer's order quantity q, with λ being the parameter. Denoting the relationship as a function $\beta_{AI}^{**}(q)$, we can visualize it as in Fig. 1 (with the same parameters as in Table 1). Therefore, as an alternative way to implement the optimal MGL contract menu, the manufacturer can just announce $\beta_{AI}^{**}(q)$ other than listing many MGL contracts as in the second column of Table 1. From Fig. 1, we can see that the gain sharing percentage $\beta_{AI}^{**}(q)$ decreases as the retailer's order quantity increases. This is consistent with our intuition, since the manufacturer tends to encourage the retailer to order more products by a lower gain sharing percentage.

3.3 Influence of information asymmetry

In this subsection, we investigate the influence of information asymmetry over the equilibrium results. To this end, we first characterize the optimal MGL contract under the case of symmetric information, i.e., the manufacturer knows the exact value of λ. For a given $\lambda \in [\underline{\lambda}, \overline{\lambda}]$, the manufacturer faces the following problem, which is denoted as (SI):

$$\max_{w(\lambda),\beta(\lambda),q(\lambda)} \pi_M(\lambda) = \big(w(\lambda) - c\big)q(\lambda) + \beta(\lambda)G\big(w(\lambda), q(\lambda)\big) \tag{21}$$

$$\text{s.t.} \quad u_R(\lambda) = \lambda L\big(w(\lambda), q(\lambda)\big) + \big(1 - \beta(\lambda)\big)G\big(w(\lambda), q(\lambda)\big) \geq 0\,. \tag{22}$$

Again, we only consider the case that the wholesale price is exogenously determined by the market, so we let $w(\lambda) = w$.

Without violating Constraint (22), we let $u_R(\lambda) = a \geq 0$. By simple calculation, we have

$$\beta(\lambda) = 1 - \frac{u_R(\lambda) - \lambda L\big(w, q(\lambda)\big)}{G\big(w, q(\lambda)\big)} = 1 - \frac{a - \lambda L\big(w, q(\lambda)\big)}{G\big(w, q(\lambda)\big)}.$$

Thus, one can rewrite the objective in (21) as

$$\pi_M(\lambda) = (w - c)q(\lambda) + G\big(w, q(\lambda)\big) + \lambda L\big(w, q(\lambda)\big) - a\,. \tag{23}$$

To maximize the objective in (23), obviously one should let $a = 0$. The following lemma characterizes the optimal MGL contract and its properties with symmetric information.

Lemma 2. *Given* $\lambda \in [\underline{\lambda}, \overline{\lambda}]$ *and assume the manufacturer knows the exact value of* λ.

(a) The optimal MGL contract to solve problem (SI) is $\left(\beta_{SI}^*(\lambda), q_{SI}^*(\lambda)\right)$, *where* $q_{SI}^*(\lambda)$ *is the unique solution of the equation*

$$\frac{(p-c)-pF(q)}{wF\left(\frac{wq}{p}\right)} = \lambda - 1, \tag{24}$$

and

$$\beta_{SI}^*(\lambda) = 1 + \frac{\lambda L\left(w, q_{SI}^*(\lambda)\right)}{G\left(w, q_{SI}^*(\lambda)\right)}; \tag{25}$$

(b) $q_{SI}^*(\lambda)$ *is a decreasing function of* λ.

(c) Under the optimal MGL contract, the expected utility of the retailer is always 0 regardless of his type λ, *i.e.,*

$$u_{R,SI}^*(\lambda) = \lambda L\left(w, q_{SI}^*(\lambda)\right) + \left(1 - \beta_{SI}^*(\lambda)\right)G\left(w, q_{SI}^*(\lambda)\right) = 0, \tag{26}$$

for any $\lambda \in [\underline{\lambda}, \overline{\lambda}]$.

(d) The manufacturer's profit under the optimal MGL contract $\left(\beta_{SI}^*(\lambda), q_{SI}^*(\lambda)\right)$ *is*

$$\begin{aligned} \pi_{M,SI}^*(\lambda) &= (w-c)q_{SI}^*(\lambda) + \beta_{SI}^*(\lambda)G\left(w, q_{SI}^*(\lambda)\right) \\ &= (w-c)q_{SI}^*(\lambda) + G\left(w, q_{SI}^*(\lambda)\right) + \lambda L\left(w, q_{SI}^*(\lambda)\right), \end{aligned} \tag{27}$$

and $\pi_{M,SI}^*(\lambda)$ *is decreasing with respect to* λ.

Comparing Theorem 1 and Lemma 2, we can come to the following results:

Theorem 2. *For any* $\lambda \in [\underline{\lambda}, \overline{\lambda}]$, *let* $z(\lambda) = H(\lambda)/h(\lambda) + \lambda$. *Then*

(a) $\beta_{AI}^*(\lambda) = \beta_{SI}^*\left(z(\lambda)\right)$

(b) $q_{AI}^*(\lambda) = q_{SI}^*\left(z(\lambda)\right) \le q_{SI}^*(\lambda) \le q_C^*$;

(c) $u_{R,AI}^*(\lambda) - u_{R,SI}^*(\lambda) = -\int_{\lambda}^{\overline{\lambda}} L\left(w, q_{AI}^*(x)\right)dx \ge 0$;

(d) $\pi^*_{M,AI}(\lambda) = \pi^*_{M,SI}\big(z(\lambda)\big) \le \pi^*_{M,SI}(\lambda)$.

From Theorem 2, we have several observations. First, from the equations $\beta^*_{AI}(\lambda) = \beta^*_{SI}\big(z(\lambda)\big)$, $q^*_{AI}(\lambda) = q^*_{SI}\big(z(\lambda)\big)$, and $\pi^*_{M,AI}(\lambda) = \pi^*_{M,SI}\big(z(\lambda)\big)$, we can see that information asymmetry causes an effect equivalent to that of loss aversion. In particular, $z(\lambda) - \lambda = H(\lambda)/h(\lambda) > 0$ reflects the fictitious degree of loss aversion caused by information asymmetry.

Second, the first inequality in Part (b) indicates that the existence of information asymmetry lowers the producing quantity, thus lowers the total profit of the supply chain. The second inequality suggests that even though no information asymmetry exists, the production quantity is still less than the optimal production quantity for the vertically integrated supply chain except for the case $\lambda = 1$ (by Eqn. (24) $q^*_{SI}(1) = q^*_C$).

Third, Parts (c) and (d) show that information asymmetry on the retailer's degree of loss aversion works to the detriment of the manufacturer, but to the advantage of the retailer. Therefore, the retailer will obviously withhold his private information and the difference $u^*_{R,AI}(\lambda) - u^*_{R,SI}(\lambda)$ reflects the retailer's accessional utility gained by withholding this private information. As we can see, this accessional utility is higher when the retailer is less loss averse. The quantity $\pi^*_{M,SI}(\lambda) - \pi^*_{M,AI}(\lambda)$ stands for the highest cost that the manufacturer is willing to pay to investigate the exact value of λ.

Next we quantify the influence of information asymmetry under an extreme case (uniform demand distribution).

Proposition 1. *If F(x) is the CDF of a uniform distribution over* $[0, A]$, *then the following properties hold.*

(a) $0 \le \pi^*_{M,SI}(\lambda) - \pi^*_{M,AI}(\lambda) \le k_1(\lambda) \cdot \dfrac{A(p-c)^2}{2p}$,

(b) $0 \le u^*_{R,AI}(\lambda) - u^*_{R,SI}(\lambda) \le k_2(\lambda, \overline{\lambda}) \cdot \dfrac{A(p-c)^2}{2p}$,

(c) If $\frac{\lambda}{z(\lambda)} \le \frac{\pi^*_{M,AI}(\lambda)}{\pi^*_{M,SI}(\lambda)} \le 1$,

(d) $\frac{\lambda^2[z(\lambda)-1]}{z(\lambda)^2(\lambda-1)} \le \frac{\pi_C\left(q^*_{AI}(\lambda)\right)}{\pi_C\left(q^*_{SI}(\lambda)\right)} \le 1$,

where $z(\lambda) = \frac{H(\lambda)}{h(\lambda)} + \lambda$,

$$k_1(\lambda) = \begin{cases} \frac{\sqrt{z(\lambda)-1} - \sqrt{\lambda-1}}{\sqrt{z(\lambda)-1} + \sqrt{\lambda-1}}, \text{ if } \left(z(\lambda)-1\right)\left(\lambda-1\right) > 1, \\ \frac{z(\lambda)-\lambda}{z(\lambda)\lambda}, \text{ if } \left(z(\lambda)-1\right)\left(\lambda-1\right) \le 1, \end{cases}$$

$$k_2(\lambda, \bar{\lambda}) = \begin{cases} \frac{\sqrt{\bar{\lambda}-1} - \sqrt{\lambda-1}}{\sqrt{\bar{\lambda}-1} + \sqrt{\lambda-1}}, \text{ if } \left(\bar{\lambda}-1\right)\left(\lambda-1\right) > 1, \\ \frac{\bar{\lambda}-\lambda}{\bar{\lambda}\lambda}, \text{ if } \left(\bar{\lambda}-1\right)\left(\lambda-1\right) \le 1. \end{cases}$$

To demonstrate Proposition 1 in a better way, we provide the following example, which indicates that the information asymmetry may have substantial effects.

Example. Suppose that the demand is uniformly distributed over [0, A], $H(\lambda)$ is the CDF of uniform distribution on [1, 3]. By Proposition 1, we have $0 \le \pi^*_{M,SI}(2) - \pi^*_{M,AI}(2) \le 0.171 \cdot \frac{A(p-c)^2}{2p}$, $0 \le u^*_{R,AI}(2) - u^*_{R,SI}(2) \le 0.171 \cdot \frac{A(p-c)^2}{2p}$, $\frac{2}{3} \le \frac{\pi^*_{M,AI}(2)}{\pi^*_{M,SI}(2)} \le 1$, $\frac{8}{9} \le \frac{\pi_C\left(q^*_{AI}(2)\right)}{\pi_C\left(q^*_{SI}(2)\right)} \le 1$.

4. Endogenous wholesale price

In Section 3, we assume the wholesale price is determined exogenously by the market. In this section, we generalize the results of Sections 3 to the case that the wholesale price is endogenously determined by the manufacturer. In particular, we assume the wholesale price can be chosen within a certain range $[\underline{w}, \bar{w}]$ where $\bar{w} > \underline{w} \ge s = 0$ (see for example, Wang & Webster, 2007; Xiong et al., 2011).

We will show that later (in Theorem 3), with the MGL contract menu, the manufacturer will always fix the wholesale price at the lower bound $\underline{w}$ under both cases of asymmetry information and symmetric information, although she is allowed to choose it within the range $[\underline{w},\overline{w}]$. Here we only discuss the optimal MGL contract menu (with wholesale price decision) under the case of asymmetry information, since the discussion for the case of symmetric information is similar.

Recall Problem (AI) defined by (9), (10) and (11). As similar as the case that $w(\lambda)$ is fixed, the objective in (11) can also be rewritten as

$$E_{\lambda}\pi_M(\lambda)=\int_{\underline{\lambda}}^{\overline{\lambda}}\Big[(w(\lambda)-c)q(\lambda)+G\big(w(\lambda),q(\lambda)\big) + \left(\frac{H(\lambda)}{h(\lambda)}+\lambda\right)L\big(w(\lambda),q(\lambda)\big)\Big]dH(\lambda)-U_0. \tag{28}$$

To maximize the objective in (28), one only needs to choose *w* and *q* to maximize

$$(w-c)q+G\big(w,q\big)+\left(\frac{H(\lambda)}{h(\lambda)}+\lambda\right)L\big(w,q\big), \tag{29}$$

for each $\lambda\in[\underline{\lambda},\overline{\lambda}]$. Differentiating (29) with respect to *w*, one has

$$-qF\left(\frac{wq}{p}\right)\left(\frac{H(\lambda)}{h(\lambda)}+\lambda-1\right)<0,$$

which indicates for any *q*>0, $w\in[\underline{w},\overline{w}]$, it holds that

$$\begin{aligned}&(w-c)q+G\big(w,q\big)+\left(\frac{H(\lambda)}{h(\lambda)}+\lambda\right)L\big(w,q\big)\\ &\le(\underline{w}-c)q+G\big(\underline{w},q\big)+\left(\frac{H(\lambda)}{h(\lambda)}+\lambda\right)L\big(\underline{w},q\big).\end{aligned}$$

Therefore, it follows that the optimal wholesale price should be $w_{AI}^*(\lambda)=\underline{w}$. Thus, by similar arguments as in Section 3, when the wholesale price is determined by the manufacturer within the range $[\underline{w},\overline{w}]$, Theorem 1 remains true by replacing *w* with $\underline{w}$. Similar results also hold for Lemma 2 and Theorem 2.

Theorem 3. *If the wholesale price is determined by the manufacturer within the range* $[\underline{w},\overline{w}]$, *then* $w_{AI}^*(\lambda)=w_{SI}^*(\lambda)=\underline{w}$ *and Theorems 1, 2, and Lemma 2 remain true by replacing w with* $\underline{w}$.

According to Theorem 3 we come to the conclusion that the optimal contract menu can still be given even if we only have the wholesale price range $[\underline{w}, \overline{w}]$ instead of the exact wholesale price *w*. For example, similarly as in the example in Subsection 3.2, here we only need to substitute "w=6" there with "$w=\underline{w}$", then we get the optimal contract menu.

Theorem 3 implies a counter-intuitive insight that the manufacturer tends to set the wholesale price as low as possible in the optimal MGL contract menu. Intuitively, the manufacturer prefers a higher wholesale price, which leads to a higher profit margin. This is indeed true for a given profit sharing percentage β and order quantity *q*. However, if the profit sharing percentage β and the order quantity *q* are adjusted to satisfy the optimality conditions (i.e., Eqns. (14) and (15)), things are totally different. In fact, a lower wholesale price encourages the retailer to choose an option with larger *q*, which leads to a higher profit shared from the retailer to the manufacturer. As a result, the manufacturer gets a higher profit by providing a lower wholesale price and meanwhile adjusting β and *q* to satisfy Eqns. (14) and (15).

5. Supply chain coordination

Cachon (2003) suggests three criteria to evaluate the effectiveness of supply chain contracts: (1) Is the contract capable of coordinating the supply chain? (2) Is the contract flexible enough to allow for arbitrary profit allocation among the supply chain members? (3) Is the contract economical to implement? In this section, we will compare the optimal MGL contract menu proposed in the present paper with the contracts proposed by Wang & Webster (2007) (i.e., BB, GL and GLB, which coordinate the supply chain considered in Wang & Webster, 2007) from the perspectives of the above three criteria.

In Wang & Webster (2007), the authors prove that under some conditions, BB, GL and GLB are capable of coordinating the supply chain with complete information (Corollaries 1-3 in Wang & Webster, 2007). Can these contracts still coordinate the supply chain with asymmetric information on λ in an implementable way? The answer is "no". The reason is as follows: the contract parameters that coordinate the supply chain considered in Wang & Webster (2007) are related to the value of λ. With asymmetric information on λ, the manufacturer doesn't know the exact value of λ when she makes the contract. Therefore, the contracts that coordinate the supply chain in Wang & Webster (2007) cannot coordinate the supply chain in the present paper in an implementable way.

Now we investigate whether the optimal MGL contract menu (with the wholesale price endogenously determined by the manufacturer) can coordinate the supply chain with asymmetric information on λ. From Theorem 3 and Eqns. (14) and (24), we can note that both $q_{AI}^*(\lambda)$ and $q_{SI}^*(\lambda)$ depend on $\underline{w}$. Thus in this section we rewrite $q_{AI}^*(\lambda)$ as $q_{AI}^*(\underline{w},\lambda)$, and $q_{SI}^*(\lambda)$ as $q_{SI}^*(\underline{w},\lambda)$. Then the following property follows.

Theorem 4. *For any* $\lambda \in [\underline{\lambda},\overline{\lambda}]$, $\lim_{\underline{w}\to 0} q_{AI}^*(\underline{w},\lambda) = \lim_{\underline{w}\to 0} q_{SI}^*(\underline{w},\lambda) = q_C^*$, *and the optimal MGL contract menu coordinates the supply chain.*

The intuition of Theorem 4 is as follows. The total supply chain profit suffers from both the loss aversion effect (see Wang & Webster, 2007) and the information asymmetry effect (see Theorem 2). First, when the wholesale price is zero, the retailer will not suffer any loss even if the demand is very low. Therefore, without any possibility of losses for the retailer, the loss aversion effect is eliminated. Second, note that the information asymmetry is on the degree of loss aversion λ. Now that, regardless of the value of λ, the corresponding loss aversion effect can be totally eliminated by setting the wholesale price at zero, then the information asymmetry effect is cleared up at the same time.

In particular, when $H(\lambda)$ is the CDF of a uniform distribution over [1, B] and $F(x)$ is the CDF of a uniform distribution over [0, A], and the lower bound of the wholesale price $\underline{w} = 0$, according to Eqns. (18), (20), we have

$$q_{AI}^*(0,\lambda)=\frac{A(p-c)}{p}=q_C^*,$$

$$\pi_{C,AI}^*(q_{AI}^*(0,\lambda))=\frac{A(p-c)^2}{2p}=\pi_C^*(q_C^*).$$

These results certainly accord with the conclusion in Theorem 4.

In fact, when the manufacturer sets the wholesale price at 0, the optimal MGL contract menu which coordinates the supply chain reduces to an MGL contract $(w,\beta,q)=(0,\beta,q_C^*)$ (hereafter we refer to it as the coordinating MGL contract), where β depends on the retailer's reservation utility U_0. For example, if U_0=0, then the corresponding coordinating MGL contract is $(0,1,q_C^*)$. Contracts similar to the coordinating MGL contract are adopted in the Vendor Managed Inventory (VMI) programs, in which the vendors make inventory decisions on behalf of the retailers and also bear the risks and costs associated with these decisions (Andel, 1996).

Since we have argued that the supply chain considered in our paper cannot be coordinated by the contracts which coordinate the supply chain in Wang & Webster (2007), for these contracts, it is meaningless to discuss the issue of profit allocation. Hence, we only consider the flexibility of profit allocation for the coordinating MGL contract. For any reservation utility $U_0 \le \pi_C(q_C^*)$ of the retailer, there always exists a coordinating contract $(0,\beta^*(U_0),q_C^*)$, under which the retailer gets his reservation utility U_0 (note that the retailer's utility equals to his profit in this case), and the manufacturer extracts all the supply chain's profit except the retailer's reservation profit (utility) U_0. Here

$$\beta^*(U_0)=1-\frac{U_0}{G(0,q_C^*)}.$$

Therefore, the coordinating MGL contract is flexible enough to allow for arbitrary profit allocation among supply chain members.

We have compared the optimal MGL contract menu with the contracts proposed in Wang & Webster (2007) (i.e., BB, GL and GLB) from the perspective of Criteria (1) and (2) suggested in the beginning of this section. Next, we compare them based on Criterion (3). Under BB, the retailer needs to ship the unsold products to the manufacturer, which incurs a shipment cost. Under GL, the manufacturer has to know the retailer's net profit by investigating his revenues and costs, which also incurs a detecting cost. Implementing GLB is more costly since it incurs both a shipment cost and a detecting cost. Implementing the coordinating MGL contract also requires the manufacturer to detect the retailer's net profit. Therefore, its implementing cost is the same as GL.

Finally, we should emphasize that if the manufacturer makes mistakes in acquiring demand information $F(x)$ or price information p, or in estimating cost c, she will put forward an MGL contract which cannot lead to supply chain coordination. However, as long as the information on demand, price and cost is correct, the MGL contract put forward by the manufacturer will coordinate the supply chain, even if she doesn't possess any prior belief over the retailer's degree of loss aversion. To this extent, the coordinating MGL contract is robust for supply chain coordination.

6. Conclusions and future directions

In a recent paper, Wang & Webster (2007) study a supply chain consisting of a risk neutral manufacturer and a loss averse retailer, and propose three contracts to coordinate the supply chain: buy back (BB), gain/loss sharing (GL) and gain/loss sharing and buy back (GLB). They assume that the retailer's degree of loss aversion is complete information. However, this assumption does not reflect real situations, since in the real industry one party's degree of loss aversion is always unknown by other parties. To reflect more realistic situations, we propose a principal-agent model to study a similar supply chain, in which the retailer's degree of loss aversion is assumed to be asymmetric information. Within the principal-agent framework, we obtain the following results:

(1) We prescribe the optimal contract menu for the manufacturer.

(2) Under the optimal contract menu, the information asymmetry lowers the

production quantity, decreases the manufacturer's profit and deteriorates supply chain performance, while increasing the retailer's utility. Furthermore, under a special case, we quantify the effects of information asymmetry by estimating some bounds on them.

(3) While the contracts proposed in Wang & Webster (2007) cannot coordinate the supply chain with the asymmetric information in an implementable way, the optimal contract menu proposed in the present paper can if the wholesale price is endogenously determined by the manufacturer and its lower bound is 0. Issues on flexibility of profit allocation, costs of implementing these contracts and robustness in achieving supply chain coordination are also discussed.

The distribution of the random demand faced by the retailer is assumed to be exogenously given in this research. It is also interesting to study similar problems when the demand is influenced by the retailer's pricing or advertising strategies. It is more attractive for one to study a supply chain consisting of a risk neutral manufacturer and multiple competitive loss averse retailers. Furthermore, it is a challenging task for future research to discuss similar problems when information asymmetry exists for more parameters (e.g., the reference point of the retailer, the demand parameters, the cost parameters, etc.).

Acknowledgements

This work is supported by the NSFC Projects Nos. 70971072 and 71031005. The authors sincerely thank the editor and the anonymous referees, whose insightful and constructive comments led to a substantial improvement of the paper.

Appendix

Proof of Lemma 1. *Only if*: By definitions of $u_R(\lambda^{'} \mid \lambda)$ and $u_R(\lambda)$, we have

$$u_R(\lambda^{'}) - u_R(\lambda^{'} \mid \lambda) = (\lambda^{'} - \lambda) L(w, q(\lambda^{'})).$$

Thus, by Inequality (10), we have $u_R(\lambda^{'}) - u_R(\lambda) \le (\lambda^{'} - \lambda) L(w, q(\lambda^{'}))$. Similarly, we have $u_R(\lambda)-u_R(\lambda^{'})\le(\lambda-\lambda^{'})L(w,q(\lambda))$. Combining these two inequalities, we have

$$(\lambda^{'}-\lambda)L(w,q(\lambda))\le u_R(\lambda^{'})-u_R(\lambda)\le(\lambda^{'}-\lambda)L(w,q(\lambda^{'})). \qquad \text{(A.1)}$$

Dividing (A.1) by $\lambda^{'}-\lambda$ and let $\lambda^{'}\to\lambda$, we have $u_R^{'}(\lambda)=L(w,q(\lambda))<0$, which indicates $u_R(\lambda)$ decreases in λ. To meet Constraints (11), it must hold that $u_R(\overline{\lambda})=a\ge 0$. Therefore, by integration, we have $u_R(\lambda)=-\int_{\lambda}^{\overline{\lambda}}L(w,q(x))dx+a$.

For any $\lambda^{'}>\lambda$, from (A.1), we have $(\lambda^{'}-\lambda)L(w,q(\lambda))\le(\lambda^{'}-\lambda)L(w,q(\lambda^{'}))$, which implies $L(w,q(\lambda))\le L(w,q(\lambda^{'}))$. Thus, $q(\lambda)\ge q(\lambda^{'})$ since $L(w,q)$ is decreasing in *q*.

If: Since $u_R(\lambda)=-\int_{\lambda}^{\overline{\lambda}}L(w,q(x))dx+a$, then Inequality (11) holds obviously. In addition, since $L(w,q)$ is decreasing in *q* and $q(\lambda)$ is decreasing in λ, we have

$$\begin{aligned}
&u_R(\lambda^{'}\mid\lambda)-u_R(\lambda)=(\lambda-\lambda^{'})L(w,q(\lambda^{'}))+u_R(\lambda^{'})-u_R(\lambda)\\
&=(\lambda-\lambda^{'})L(w,q(\lambda^{'}))+\int_{\lambda}^{\lambda^{'}}L(w,q(x))dx\\
&\le(\lambda-\lambda^{'})L(w,q(\lambda^{'}))+(\lambda^{'}-\lambda)L(w,q(\lambda^{'}))=0.
\end{aligned}$$

Thus, Inequality (10) holds. □

Proof of Theorem 1. *Part (a).* By Lemma 1 and the analysis following Lemma 1, to maximize the manufacturer's expected profit (9), we only need to maximize (13). Furthermore, we only need to choose $q_{AI}^{*}(\lambda)$ to maximize

$$\overline{\Pi}(q,\lambda)=(w-c)q+G(w,q)+\left(\frac{H(\lambda)}{h(\lambda)}+\lambda\right)L(w,q)$$

for any $\lambda\in[\underline{\lambda},\overline{\lambda}]$. Taking first-order and second-order derivatives of $\overline{\Pi}(q,\lambda)$ with respect to *q*, we have

$$\frac{\partial\overline{\Pi}(q,\lambda)}{\partial q}=(p-c)-pF(q)-\left(\frac{H(\lambda)}{h(\lambda)}+\lambda-1\right)\left[(wF\left(\frac{w}{p}q\right)\right], \qquad \text{(A.2)}$$

$$\frac{\partial^2\overline{\Pi}(q,\lambda)}{\partial q^2}=-pf(q)-\left(\frac{H(\lambda)}{h(\lambda)}+\lambda-1\right)\frac{w^2}{p}f\left(\frac{w}{p}q\right)<0,$$

which means that $\overline{\Pi}(a,\lambda)$ is concave in q. Hence, $q^*_{AI}(\lambda)$ satisfies the equation $\partial\overline{\Pi}(q,\lambda)/\partial q = 0$, which is equivalent to Eqn. (14). Moreover, by Eqn. (12), we have (15) is true.

Parts (b) and (c) is obvious from Lemma 1.

Part (d). By definition, we have $\pi^*_{M,AI}(\lambda) = \overline{\Pi}(q^*_{AI}(\lambda),\lambda)$. Suppose that $\lambda_1 < \lambda_2$. Since it is easy to verify that $\overline{\Pi}(q,\lambda)$ is decreasing with respect to λ, then $\overline{\Pi}(q^*_{AI}(\lambda_1),\lambda_1) \geq \overline{\Pi}(q^*_{AI}(\lambda_2),\lambda_1) \geq \overline{\Pi}(q^*_{AI}(\lambda_2),\lambda_2)$. □

Proof of Lemma 2. The proof of this theorem is similar as Theorem 1. □

Proof of Thereom 2. *Part (b).* By definitions of $q^*_{AI}(\lambda)$ and $q^*_{SI}(\lambda)$, we can easily come to $q^*_{AI}(\lambda) = q^*_{SI}(z(\lambda))$. By Part (b) of Theorem 2, we have $q^*_{SI}(z(\lambda)) \leq q^*_{SI}(\lambda)$. Moreover, since the left hand side of (24) is decreasing with respect to q and

$$\frac{(p-c) - pF(q^*_C)}{wF\left(\frac{wq^*_C}{p}\right)} = 0 < \lambda - 1,$$

then $q^*_{SI}(\lambda) \leq q^*_C$.

Parts (a), (c) and (d). Part (a) can be obtained by comparing Eqns. (15) and (25), and using Part (b) of this theorem. Part (c) can be obtained by Eqns. (16) and (26). Part (d) follows from Eqns. (17) and (27) and Part (d) of Theorem 1 and Part (d) of Theorem 2. □

Proof of Proposition 1. *Part (a).* With demand uniformly distributed on [0, A], we have $F(x) = x/A$ and $f(x) = 1/A$. Since

$$\begin{aligned}
&\pi^*_{M,SI}(\lambda) - \pi^*_{M,AI}(\lambda) \\
&= \frac{Ap(p-c)^2}{2}\left[\frac{1}{(\lambda-1)w^2 + p^2} - \frac{1}{(z(\lambda)-1)w^2 + p^2}\right] \\
&= \frac{A(p-c)^2}{2p} \cdot \frac{(z(\lambda)-\lambda)(p/w)^2}{\left(\lambda - 1 + (p/w)^2\right)\left(z(\lambda) - 1 + (p/w)^2\right)},
\end{aligned}$$

then we only need estimate the bounds of

$$\bar{g}(x) = \frac{(z(\lambda)-\lambda)x}{(\lambda-1+x)(z(\lambda)-1+x)},$$

where $x \in [1,+\infty)$. Differentiating $g(x)$, we have

$$\bar{g}'(x) = \frac{(z(\lambda)-\lambda)\left[(z(\lambda)-1)(\lambda-1)-x^2\right]}{(\lambda-1+x)^2(z(\lambda)-1+x)^2}.$$

Clearly, $g(x)$ approaches to 0 as x goes to infinity. Therefore, $\bar{g}(x) \geq 0$. If $(z(\lambda)-1)(\lambda\text{-}1) \leq 1$, then $\bar{g}'(x) \leq 0$ for any $x \in [1,+\infty)$. Thus, $\bar{g}(x) \leq (z(\lambda)-\lambda)/(\lambda-1)(z(\lambda)-1)$. If $(z(\lambda)-1)(\lambda\text{-}1) > 1$, then the upper bound of $\bar{g}(x)$ is $\bar{g}(x^*) = \left(\sqrt{z(\lambda)-1}-\sqrt{\lambda-1}\right)/\left(\sqrt{z(\lambda)-1}+\sqrt{\lambda-1}\right)$ where $x^* = \sqrt{(z(\lambda)-1)(\lambda\text{-}1)}$. Then the result in Part (b) follows.

Part (b). By calculation,

$$u^*_{R,AI}(\lambda) - u^*_{R,SI}(\lambda) = \frac{p(p-c)^2 A}{2}\left[\frac{1}{(\lambda-1)w^2+p^2} - \frac{1}{(\bar{\lambda}-1)w^2+p^2}\right].$$

With a similar arguments as in Part (b), we can prove this result.

Part (c). By calculation, we have

$$\pi^*_{M,SI}(\lambda) = \frac{p(p-c)^2 A}{2\left[(\lambda-1)w^2+p^2\right]},\quad \pi^*_{M,AI}(\lambda) = \frac{p(p-c)^2 A}{2\left[(z(\lambda)-1)w^2+p^2\right]}$$

Thus,

$$\frac{\pi^*_{M,AI}(\lambda)}{\pi^*_{M,SI}(\lambda)} = \frac{\lambda-1+(p/w)^2}{z(\lambda)-1+(p/w)^2}.$$

Since $0 < w \leq p$, then $1 \leq (p/w)^2 < +\infty$, which, together with the equation above, implies the Part (a) of this proposition.

Part (d). By calculation, we have $\pi_C\left(q^*_{AI}(\lambda)\right) = \pi_C\left(q^*_{SI}(z(\lambda))\right)$ and

$$\pi\left(q^*_{SI}(\lambda)\right) = \frac{(\lambda-1)w^2 p(p-c)^2 A}{\left[(\lambda-1)w^2+p^2\right]^2}.$$

By similar proof as in Part (c), we can come to the result. □

Proof of Theorem 4. Note that $q^*_{AI}(\lambda) \leq q^*_{SI}(\lambda) \leq q^*_C$ (Part (a) of Theorem 3). We only need to prove $\lim_{\underline{w}\to 0} q^*_{AI}(\underline{w},\lambda) = q^*_C$. Denote

$$K(\underline{w},q) = \left(\frac{H(\lambda)}{h(\lambda)} + \lambda - 1\right)\left[\underline{w}F\left(\frac{\underline{w}}{p}q\right)\right].$$

For any $\varepsilon > 0$, we have $(p-c)-(p-s)F(q^*_C - \varepsilon) > 0$. This fact, joint with $\lim_{\underline{w}\to 0} K(\underline{w}, q^*_C - \varepsilon) = 0$, implies that there always exists a $\delta > 0$ such that ($\partial\overline{\Pi}/\partial q$ is defined in (A.2))

$$\frac{\partial\overline{\Pi}}{\partial q}(\underline{w}, q^* - \varepsilon, \lambda) = (p-c) - pF(q^*_C - \varepsilon) - K(\underline{w}, q^*_C - \varepsilon) > 0,$$

for any $0 < \underline{w} < \delta$. In addition, we have

$$\frac{\partial\overline{\Pi}}{\partial q}(\underline{w}, q^*_C, \lambda) = (p-c) - pF(q^*_C) - K(\underline{w}, q^*_C) < 0.$$

Thus, by the definition of $q^*_{AI}(\underline{w},\lambda)$, we have $q^*_C - \varepsilon < q^*_{AI}(\underline{w},\lambda) \le q^*_C$ for any $\underline{w} \in (s, s+\delta)$, which implies that $\lim_{\underline{w}\to 0} q^*_{AI}(\underline{w},\lambda) = q^*_C$. □

References

Agrawal, V., Seshadri, S., 2000. Risk intermediation in supply chains. IIE Transactions 32 (9), 819-831.

Andel, T., 1996. Manage inventory, own information. Transportation and Distribution 37 (5), 54-56.

Anupindi, R., 1999. Supply contracts with quantity commitments and stochastic demand. In: Tayur, S., Ganeshan, R., Magazine, M., (Eds.). Quantitative models for supply chain management. Boston: Kluwer Academic, 199-232.

Cachon, G.P., 2003. Supply chain coordination with contracts. In: Kok, de A.G., Graves, S.C., (Eds). Handbooks in operations research and management science, vol. 11, Supply chain management: design, coordination and operation. Elsevier: Amsterdam, 229-339.

Cachon, G.P., Lariviere, M.A., 2001. Contracting to assure supply: how to share demand forecasts in a supply chain. Management Science 47 (5), 629-646.

Camerer, C., 2001. Prospect theory in the wild: Evidence from the field. In: Kahneman, D., Tversky, A. (Eds.). Choices, values, and frames. Cambridge: Cambridge University Press, 288-300.

Chiu, C.H., Choi, T.M., Li, X., 2011. Supply chain coordination with risk sensitive retailer under target sales rebate. Automatica 47 (8), 1617-1625.

Choi, T.M., 2007. Pre-season stocking and pricing decisions for fashion retailers with multiple information updating. International Journal of Production Economics 106 (1), 146-170.

Choi, T.M., Chiu, C.H., 2012. Mean-downside-risk and mean-variance newsvendor models: implications for sustainable fashion retailing. International Journal of Production Economics 135, 552-560.

Choi, T.M., Li, D., Yan, H., 2008a. Mean-variance analysis for the newsvendor problem. IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans 38 (5), 1169-1180.

Choi, T.M., Li, D., Yan, H., 2008b. Mean-variance analysis of a single supplier and retailer supply chain under a returns policy. European Journal of Operational Research 184 (1), 356-376.

Corbett, C., de Groote, X., 2000. A supplier's optimal quantity discount policy under asymmetric information. Management Science 46 (3), 444–450.

Eeckhoudt, L., Gollier, C., Schlesinger, H., 1995. The risk-averse (and prudent) newsboy. Management Science 41 (5), 786-794.

Fibich, G., Gavious, A., Lowengrart, O., 2003. Explicit solutions of optimization models and differential games with nonsmooth (asymmetric) reference-price effect. Operations Research 51 (5), 721-734.

Fisher, M., Raman, A., 1996. Reducing the cost of demand uncertainty through accurate response to early sales. Operations Research 44 (1)**,** 87-99.

Fudenberg , D., Tirole, J., 1991. Game Theory, Cambridge: MIT Press.

Gan, X., Sethi, S., Yan, H., 2004. Coordination of supply chains with risk-averse agents. Production and Operations Management 13 (2), 135-149.

Gan, X., Sethi, S., Yan, H., 2005. Channel coordination with a risk-neutral supplier and a downside-risk-averse retailer. Production and Operations Management 14 (1), 80-89.

Greenleaf, E.A., 1995. The impact of reference price effects on the probability of price promotions. Marketing Science 14 (1), 82-104.

Ha, A., 2001. Supplier-buyer contracting: asymmetric cost information and cutoff level policy for buyer participation. Naval Research Logistics 48 (1), 41-64.

Ho, T., Zhang, J., 2008. Designing pricing contracts for boundedly rational customers: does the framing of the fixed fee matter? Management Science 54 (4), 686-700.

Kahn, J.A., 1992. Why is production more volatile than sales? Theory and evidence on the stockout-avoidance motive for inventory holding. Quarterly Journal of Economics 107 (2), 481–510.

Kahneman, D., Tversky, A., 1979. Prospect theory: An analysis of decisions under risk. Econometrica 47 (2), 263–291.

Kaya, M., Ozer, O., 2003. Quality risk in outsourcing: noncontractible product quality and private quality cost information. Working paper, Stanford University, CA.

Kopalle, P.K., Rao, A.G., Assuncao, J.L., 1996. Asymmetric reference price effects and dynamic pricing policies. Marketing Science 15 (1), 60-85.

Lau, H.S., 1980. The newsboy problem under alternative optimization objectives. Journal of the Operational Research Society 31 (6), 525-535.

Ozer, O., Wei, W., 2006. Strategic commitments for an optimal capacity decision under asymmetric forecast information. Management Science 52 (8), 1238-1257.

Patsuris, P., 2001. Christmas sales: the worst growth in 33 years. Available at http://www.forbes.com/2001/10/30/1030retail.html, accessed on May 11, 2007.

Popescu, L., Wu, Y., 2007. Dynamic pricing strategies with reference effects. Operations Research 55 (3), 413-429.

Rabin, M., 1998. Psychology and economics. Journal of Economic Literature 36 (1), 11-46.

Rosling, K., 2002. Inventory cost rate functions with nonlinear shortage costs. Operations Research 50 (6), 1007 – 1017.

Schweitzer, M.E., Cachon, G.P., 2000. Decision bias in the newsvendor problem with a known demand distribution: experimental evidence. Management Science 46 (3), 404–420.

Sorger, G., 1998. Reference price formation and optimal marketing strategies. In: Feichtinger, G. (Ed.). Optimal control theory and economic analysis, vol. 3. Amsterdam: Elsevier Science, 97-120.

Spengler, J.J., 1950. Vertical integration and antitrust policy. Journal of Political Economy 58 (4), 347–352.

Tsay, A.A., Nahmias, S., Agrawal, N., 1999. Modeling supply chain contracts: a review. In: Tayur, S., Ganeshan, R., Magazine, M. (Eds.). Quantitative models for supply chain management. Boston: Kluwer Academic, 299-336.

Wang, C., 2010. The loss-averse newsvendor game. International Journal of Production Economics 124 (2), 448–452.

Wang, C., Webster, S., 2007. Channel coordination for a supply chain with a risk-neutral manufacturer and a loss-averse retailer. Decision Sciences 38 (3), 361-389.

Wang, C., Webster, S., 2009. The loss-averse newsvendor problem. Omega 37 (1), 93-105.

Wei, Y., Choi, T.M., 2010. Mean-variance analysis of supply chains under wholesale pricing and profit sharing schemes. European Journal of Operational Research 204 (2), 255-262.

Wu, S.D., Roundy, R., Storer, R.H., Martin-Vega, L.A., 1999. Manufacturing logistics research: taxonomy and directions. Working paper, Lehigh University, PA.

Xiao, T., Yang, D., 2009. Risk sharing and information revelation mechanism of a one-manufacturer and one-retailer supply chain facing an integrated competitor. European Journal of Operational Research 196 (3), 1076-1085.

Xiong, H., Chen, B., Xie, J., 2011. A composite contract based on buy back and quantity flexibility contracts. European Journal of Operational Research 210 (3), 559-567.