Supplier encroachment in competitive supply chains

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Abstract

With the development of e-commerce, wholesale suppliers have opportunities to establish their own direct channels, competing with their retailing channels. It is often referred to as supplier encroachment. While many papers have analyzed encroachment of a monopolistic supplier, none of them study supplier encroachment in competitive supply chains. We first study the case with two non-identical supply chains, and then analyze the case with multiple identical supply chains. We show that, in both cases, the number of the encroaching suppliers in equilibrium decreases monotonically as the operational disadvantages of the suppliers become more significant. We find that there may exist the prisoner's dilemma phenomenon for the suppliers. Furthermore, encroachment may lead to the "lose-lose" outcome for the suppliers and the retailers. We also explore the efficiency of the suppliers, the retailers and the whole system under encroachment relative to the non-encroachment situation.

Keywords: Supplier encroachment, Direct channel, Competitive supply chains, Game theory

1. Introduction

Nowadays, with the development of e-commerce, other than using a single channel (through retailers) to sell products, many manufacturers and wholesale suppliers, e.g., Hewlett Packard, Epson, Lenovo, establish their

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own direct channels (Nasireti (1998); Janah (1999); Xiong et al. (2012), etc.). Thus, there arises competition between the two channels, which is often referred to as "supplier encroachment" (Arya et al. (2007)). Recently, many studies have analyzed encroachment of a monopolistic supplier (e.g., Hendershott and Zhang (2006); Liu and Zhang (2006); Arya et al. (2007); Tsay and Agrawal (2004); Chiang et al. (2003)). Although a monopolistic supplier with complete information always benefits from encroachment, the retailers may suffer from supplier encroachment (e.g., Hendershott and Zhang (2006); Liu and Zhang (2006)) or benefit from it (e.g., Arya et al. (2007); Tsay and Agrawal (2004); Chiang et al. (2003)). To the best of our knowledge, Mizuno (2012) is the only paper studying supplier encroachment with competitive suppliers who wholesale homogenous goods to multiple identical retailers.

In reality, there is often more than one supply chain for most kinds of products. For instance, in the PC (Personal Computer) market, the famous brands include Hewlett Packard, Dell, Lenovo, etc., products of which can be viewed as substitutable. The suppliers often wholesale products to exclusive retailers, through which the products are sold to customers. Thus, the supply chains compete with each other.

Although many papers study supplier encroachment problem, none of them investigate supplier encroachment in competitive supply chains. This paper investigates the supplier encroachment in competitive supply chains. We consider a model of n ($n \ge 2$) vertical supply chains, each composed of a supplier and an exclusive retailer. The exclusive retailers only sell the product of their suppliers. This kind of model fits several industries such as soft drink (McGuire and Staelin (1983)) and clothing, where exclusive dealership is not uncommon. There are also many papers study the chain-tochain competition (McGuire and Staelin (1983); Coughlan (1985); Moorthy (1988); Wu and Chen (2003); Wu et al. (2009); Anderson and Bao (2010); Ai et al. (2012)).

We will first study the case with two non-identical supply chains (referred to as non-identical system for short) and then investigate the case with n $(n \ge 2)$ identical supply chains (referred to as identical system for short). We show that, in both cases, the number of the encroaching suppliers in equilibrium decreases monotonically as the operational disadvantages of the suppliers become more significant. We find that there may exist the prisoner's dilemma phenomenon for the suppliers. Furthermore, encroachment may lead to the "lose-lose" outcome for the suppliers and the retailers. We also explore the efficiency of the suppliers, the retailers and the whole system under encroachment relative to the non-encroachment situation.

The remainder of the paper is organized as follows. Section 2 provides the literature review. Section 3 describes the key elements of the basic model and introduces notation. Section 4 studies supplier encroachment in the nonidentical system. Section 5 investigates supplier encroachment in the identical system. Section 6 concludes the paper. The proofs of the propositions and lemmas are collected in Appendix.

2. Literature Review

This paper focuses on supplier encroachment in competitive supply chains. Thus, related literature includes supplier encroachment and chain-to-chain competition. The literature on supplier encroachment has been dedicated to determine whether a supplier should add a direct channel to its existing retail channel. Chiang et al. (2003) conceptualize customer acceptance of a direct channel, and show that direct marketing may not always be detrimental to the retailer because it brings with a wholesale price reduction. Moreover, direct marketing increases the flow of profit through the retail channel and helps the supplier improve overall profitability by reducing the degree of double marginalization. In the model of Tsay and Agrawal (2004), the supplier and the retailer decide on the sales effort in the respective channels, assuming that the sales effort in one channel exerts a positive externality on the demand in the other channel. They show that the addition of a direct channel alongside a retail channel is not necessarily detrimental to the retailer. In fact, the supplier will reduce the wholesale price to retain some of the retailer's sales effort, and in some cases, this can make both parties better off. Hendershott and Zhang (2006) establish a model in which an upstream firm can sell products to heterogeneous consumers engaging in time-consuming search through direct channel and intermediaries. Direct online channel may be more or less convenient and involves costly returns if the goods fit consumers poorly. They find that dual channel marketing facilitates price discrimination, and encroachment of the upstream firm increases consumer surplus at the expense of the profits of the intermediaries. Liu and Zhang (2006) show that a downstream retailer is worse off when an upstream supplier enters the market. The retailer can deter the supplier from entering the market by acquiring personalized pricing. Arya et al. (2007) identify circumstances under which a retailer benefits from a monopolistic supplier's encroachment. They find that if the operational cost of the direct channel is within a certain range, the supplier encroaches, and the retailers can benefit from encroachment even when encroachment admits no synergies and does not facilitate product differentiation or price discrimination. Li et al. (2013) extend the study of supplier encroachment to the environment where the retailer might be better informed than the supplier. They find that the launch of the supplier's direct channel can result in costly signaling behavior of the retailer, in which the retailer reduces his order quantity when the market size is small. Such a downward order distortion can amplify double marginalization. As a result, in addition to the "win-win" and "win-lose" outcomes for the supplier and the retailer, supplier encroachment can also lead to "lose-lose" and "lose-win" outcomes. In order to investigate the product-market characteristics that influence the optimality of complementing an existing retail channel with a direct online channel, Kumar and Ruan (2006) contemplate a market with a single strategic supplier (focal supplier) selling products through a single strategic retailer. Consumers in the market are either brand loyal or store loyal. The retailer carries products of the focal supplier as well as a competing supplier, and provides retail support which impacts the demand for the suppliers' products. Xiong et al. (2012) study the impact of supplier encroachment in the dual goods market. Besides, many other papers study supplier encroachment (Cattani et al. (2006), Rhee and Park (2000), Yoo and Lee (2011), etc.). All the above papers assume the supplier to be a monopolist in the supply side.

To the best of our knowledge, Mizuno (2012) is the only one studying supplier encroachment with competitive suppliers. S/he studies the encroachment decisions of two identical suppliers wholesaling homogenous goods to multiple identical retailers and examines how the number of the retailers affects the profits of the suppliers and the retailers. An oligopolistic wholesale market and a market with non-homogenous goods are also briefly discussed. S/he shows that as the number of retailers increases, the number of encroaching suppliers decreases. And since an increase in the number of retailers may drive the direct-selling suppliers from the retail market, it may raise the retailers' profits and reduce social welfare. The main differences between our paper and Mizuno (2012) are as follows. First, the structures of the supply chains are different. In Mizuno (2012), two identical suppliers wholesale homogenous goods to multiple retailers, i.e., the retailers carry products of both suppliers. While in our model, the suppliers sell products through exclusive retailers. Second, Mizuno (2012) only investigates the encroachment decisions of two identical suppliers while we not only study the case with two non-identical suppliers but also the case with $n \ (n \ge 2)$ identical suppliers. Third, Mizuno (2012) does not differentiate the retail channels of the retailers from the direct channels of the suppliers while we incorporate a factor which is the operational cost of the direct channel to distinguish the direct channel from the retail channel. Finally, we investigate the impact of encroachment on the profits of the suppliers, the retailers and the whole system while Mizuno (2012) does not consider this problem.

Beyond the above, extensive papers suppose that the suppliers establish their own direct channels and study the dual channel management (Seifert et al. (2006), Chen et al. (2008), Cai et al. (2009), Chiang (2010), Geng and Mallik (2007), Chen et al. (2012), Dumrongsiri et al. (2008), Dan et al. (2012), Xu et al. (2013) etc.).

The literature on chain-to-chain competition dates back to McGuire and Staelin (1983) which investigates the effect of product substitutability on distribution structures in the context of two competing suppliers, each selling his products through an exclusive retailer. The retailers may be either a franchised outlet or a factory store. With a deterministic linear demand function, they find that when the level of competitive substitutability is high, the suppliers will be more likely to use decentralized distribution systems to avoid ruinous price competition between the suppliers. Following McGuire and Staelin (1983), many papers have explored the circumstances in which the above result holds, such as Coughlan (1985) and Moorthy (1988). Anderson and Bao (2010) extend the work of McGuire and Staelin (1983) to a more general situation when there are multiple supply chains with substitutable products competing in the market. Wu et al. (2009) investigate the equilibrium behavior of two competing supply chains in the presence of demand uncertainty. They consider the joint pricing and quantity decisions and competition under three possible supply chain structures: vertical integration, supplier's Stackelberg, and bargaining on the wholesale price over a single or infinitely many periods. Ai et al. (2012) examine decisions of retailers and suppliers in two competing supply chains with substitutable products, where the market demand is uncertain and the suppliers choose whether to offer full returns policies. All these papers assume that the suppliers sell products through the retailers, neither of them consider encroachment of the suppliers which is a fundamental focus of our paper so that we can identify and explore the impact of encroachment on the suppliers and the retailers in competitive supply chains.

3. Problem Description

Our model is somewhat similar to the basic model in Arya et al. (2007), which examines encroachment of a monopolistic supplier. We consider a structure with $n \ (n \ge 2)$ vertical supply chains, each composed of a supplier and an exclusive retailer who only sells the product of its supplier. The suppliers wholesale homogenous goods, i.e., identical and completely substitutable products, to their retailers, and then the retailers compete with each other in the retail market. If a supplier establishes its own direct channel, it can sell the product both through its own direct channel and the retail channel of its counterpart (the retailer). Without loss of generality, we normalize the costs of establishing direct channels and the unit production costs of the suppliers to zero. Market demand is represented by a linear and downward sloping (inverse) function p = a - Q, where a is a strictly positive constant, Q is the total quantity of the product in the market and p is the selling price.

There is an operational cost related to marketing operations. Without loss of generality, we normalize the unit operational costs of the retail channels to zero. Assume the unit operational cost of the *i*th direct channel to be $c_i \in$ (0, a] $(1 \le i \le n)$. These imply that the retailers have a marketing advantage compared to the suppliers. The marketing advantage of the retailers comes from the superior knowledge of the customer preferences, closed relationship with consumers, etc. (Arya et al. (2007)).

The timing in the model is as follows. First of all, the suppliers decide whether to encroach or not simultaneously. Then the suppliers and the retailers play the following game. First, the suppliers determine the wholesale prices w_i $(1 \leq i \leq n)$, respectively. Second, the retailers choose the profitmaximizing order quantities $q_{r_i}(1 \leq i \leq n)$. Third, the suppliers who encroach determine the quantities sold directly to consumers through the direct channels (If the first k suppliers encroach, they determine q_{d_j} $(1 \leq j \leq k)$). We require that only when q_{d_j} is larger than zero, the *j*th supplier encroaches and establishes a direct channel; otherwise, it does not encroach). Backward induction is employed to identify the equilibrium outcomes of the game. We first analyze the case with two non-identical supply chains (the operational costs of the direct channels are different) in Section 4 and then focus on the case with n $(n \geq 2)$ identical supply chains in Section 5. Use notation Π_{s_i} (Π_{r_i}) to represent the profit of the *i*th supplier (retailer).

4. Encroachment in Non-Identical System

4.1. Basic Results

Suppose the operational costs of the two direct channels to be c_1 and c_2 , respectively. Refer to the two suppliers as S_1 and S_2 , respectively, and the two retailers as R_1 and R_2 , respectively, for short. For ease of exposition, assume $c_1 \geq c_2$. Then S_2 is the efficient supplier and S_1 is the less efficient one. In order to obtain the equilibrium outcomes, we just need to compare the suppliers' profits in the following four situations: both S_1 and S_2 encroach; neither S_1 nor S_2 encroaches; only S_1 encroaches and only S_2 encroaches, representing which by superscripts "E", "N", "E1" and "E2", respectively.

Denote the following parameter regions:

$$\begin{split} \Omega &= \left\{ \left(\frac{c_1}{a}, \frac{c_2}{a}\right) : c_1 \ge c_2 \right\}, \\ \Omega_1 &= \left\{ \left(\frac{c_1}{a}, \frac{c_2}{a}\right) : \max\{0, \frac{(77248 + 552\sqrt{2046})c_1}{8565a} - \frac{5}{3}\} \le \frac{c_2}{a} \le \frac{c_1}{a} \right\}, \\ \Omega_2 &= \left\{ \left(\frac{c_1}{a}, \frac{c_2}{a}\right) : \frac{14275}{68683 + 552\sqrt{2046}} \le \frac{c_1}{a} \le \frac{11385 - 80\sqrt{2163}}{36657}, \frac{2855(5 + 3\frac{c_1}{a})}{8(9656 + 69\sqrt{2046})} \le \frac{c_2}{a} \le \frac{c_1}{a} \right\} \\ \Omega_3 &= \left\{ \left(\frac{c_1}{a}, \frac{c_2}{a}\right) : \frac{11385 - 80\sqrt{2163}}{36657} \le \frac{c_1}{a} \le 1, \frac{11385 - 80\sqrt{2163}}{36657} \le \frac{c_2}{a} \le \frac{c_1}{a} \right\}, \\ \Omega_4 &= \Omega - \Omega_1 - \Omega_2 - \Omega_3, \\ \Omega_5 &= \left\{ \left(\frac{c_1}{a}, \frac{c_2}{a}\right) : \frac{c_2}{a} > -\frac{5}{3} + \frac{71\frac{c_1}{a}}{21} + \frac{115\sqrt{17[14 - 81(\frac{c_1}{a})^2]}}{1071} \right\} \bigcap \Omega_1, \\ \Omega_6 &= \left\{ \left(\frac{c_1}{a}, \frac{c_2}{a}\right) : \frac{c_2}{a} \le \frac{6035}{29246} + \frac{3621\frac{c_1}{a}}{29246} - \frac{115\sqrt{-12393(\frac{c_1}{a})^2 - 41310\frac{c_1}{a} + 24067}}{87738} \right\} \bigcap \Omega_1. \end{split}$$

Proposition 1. If $c_1 \geq c_2$, the encroachment decisions of the suppliers in equilibrium are as follows:

(1) Both S_1 and S_2 encroach if $\left(\frac{c_1}{a}, \frac{c_2}{a}\right) \in \Omega_1$ where encroachment is a dominant strategy for both of them.

(2) Either S_1 or S_2 encroaches if $\left(\frac{c_1}{a}, \frac{c_2}{a}\right) \in \Omega_2$. (3) Neither S_1 nor S_2 encroaches if $\left(\frac{c_1}{a}, \frac{c_2}{a}\right) \in \Omega_3$.

(4) Only S_2 encroaches if $\left(\frac{c_1}{a}, \frac{c_2}{a}\right) \in \Omega_4$.

The wholesale prices, the quantities sold through different channels and the profits of the suppliers and the retailers in equilibrium are given in Table 1.

situations	N	E	E1	E2
w_1	$\frac{a}{3}$	$\frac{11a}{69} - \frac{13c_1}{345} + \frac{11c_2}{115}$	$\frac{a}{4} - \frac{c_1}{20}$	$\frac{3a}{16} + \frac{9c_2}{80}$
w_2	$\frac{a}{3}$	$\frac{11a}{69} - \frac{13c_2}{345} + \frac{11c_1}{115}$	$\frac{3a}{16} + \frac{9c_1}{80}$	$\frac{a}{4} - \frac{c_2}{20}$
q_{r_1}	$\frac{2a}{9}$	$\frac{20a+58c_1+12c_2}{20a+58c_1+12c_2}$	$\frac{5a+19c_1}{40}$	$\frac{5a+3c_2}{2}$
q_{r_2}	$\frac{2a}{9}$	$\frac{20a+58c_2+12c_1}{5}$	$\frac{5a+3c_1}{5}$	$\frac{5a+19c_2}{40}$
q_{d_1}	—	$\frac{5a-20c_1+3c_2}{23}$	$\frac{5a-13c_1}{16}$	— 5. 19.
q_{d_2}	-	$\frac{5a-20c_2+3c_1}{23}$	— 	$\frac{5a-13c_2}{16}$
Π_{s_1}	$\frac{2a^2}{27}$	$\Pi_{s_1}^{E^{*}}$	$\frac{33a^2}{256} - \frac{253ac_1}{640} + \frac{4073c_1^2}{6400}$	$\frac{3(5a+3c_2)^2}{1600}$
Π_{s_2}	$\frac{2a^2}{27}$	$\prod_{s_2}^{E^{**}}$	$\frac{3(5a+3c_1)^2}{1600}$	$\frac{33a^2}{256} - \frac{253ac_2}{640} + \frac{4073c_2^2}{6400}$
\prod_{r_1}	$\frac{4a^2}{21}$	$\frac{4(10a+29c_1+6c_2)^2}{200275}$	$\frac{(5a+19c_1)^2}{(5a+2)^2}$	$\frac{(5a+3c_2)^2}{(5a+3c_2)^2}$ 040 0400
П	$\frac{81}{4a^2}$	$\frac{39675}{4(10a+29c_2+6c_1)^2}$	$\frac{3200}{(5a+3c_1)^2}$	$(5a+19c_2)^2$
r_1r_2	81	39675	800	3200

Table 1: Equilibrium outcomes under different encroachment decisions of the suppliers in the non-identical system

$$\label{eq:sigma_state} \begin{array}{l} ^{*} \Pi^{E}_{s_{1}} = \frac{119a^{2}}{1587} - \frac{(2414c_{1} - 714c_{2})a}{7935} + \frac{29246c_{1}^{2} - 7242c_{1}c_{2} + 1071c_{2}^{2}}{39675} \\ ^{**} \Pi^{E}_{s_{2}} = \frac{119a^{2}}{1587} - \frac{(2414c_{2} - 714c_{1})a}{7935} + \frac{29246c_{2}^{2} - 7242c_{1}c_{2} + 1071c_{1}^{2}}{39675} \end{array}$$



Figure 1: Regions corresponding to different encroachment decisions of the suppliers in equilibrium

We depict Regions Ω_i (i = 1, 2, 3, 4) in Figure 1. Proposition 1 is in line with the intuition. c/a (i.e., c_1/a and c_2/a) being small (when c is small or a is large) indicates that the market is optimistic to the suppliers and it may be profitable for them to encroach. On the contrary, c/a being large demonstrates that the market is pessimistic to the suppliers who will have less incentive to encroach. In Figure 1, if S_2 does not encroach, S_1 will encroach if and only if c_1/a falls into the left side of the line with marker " \bullet "; if S_1 does not encroach, S_2 will encroach only if c_2/a falls below the line with marker " \circ ". Moreover, if S_2 encroaches, S_1 will encroach if and only if c_1/a falls into the left side of the dashed line with marker " \bigtriangledown "; if S_1 encroaches, S_2 will encroach if and only if c_2/a falls below the dashed line with marker " Δ ". Thus, the results in Proposition 1 can be easily obtained. Parts (1), (2) and (3) indicate that as the market becomes worse to the suppliers (i.e., c/a getting larger), the number of the encroaching suppliers in equilibrium decreases monotonically. Part (4) indicates that if the efficient supplier has a superior operational advantage compared to the less efficient one, only the efficient one encroaches.

From Table 1, we have the observations in Lemma 1 immediately.

Lemma 1. (1) w_i^E and $q_{d_i}^E$ both decrease with c_i while w_j^E , $q_{r_i}^E$, $q_{r_j}^E$ and $q_{d_j}^E$ all increase with c_i , where i, j = 1, 2 and $i \neq j$. (2) w_1^{E1} and $q_{d_1}^{E1}$ both decrease with c_1 while w_2^{E1} , $q_{r_1}^{E1}$ and $q_{r_2}^{E1}$ all increase with c_1 . (3) w_2^{E2} and $q_{d_2}^{E2}$ both decrease with c_2 while w_1^{E2} , $q_{r_1}^{E2}$ and $q_{r_2}^{E2}$ all increase with c_2 .

Lemma 1 shows that, as the operational disadvantage of an encroaching supplier becomes more significant, it will lower the wholesale price and the quantity to sell through the direct channel, so as to rely more on wholesaling to the retailer to acquire the profit. Due to the mitigated competition from the encroaching supplier, both retailers order more, and the other supplier raises the wholesale price. If the other supplier also encroaches, it also raises the quantity to sell through the direct channel.

The following lemma compares the wholesale prices under different encroachment decisions of the suppliers.

Lemma 2. When the inequality $\max\{0, (20c_1/a-5)/3\} \le c_2/a \le c_1/a$ holds, order relationship between the wholesale prices under different encroachment

decisions of the suppliers are as follows: (1) $w_1^N > w_1^{E1} > w_1^{E2} > w_1^E$, (2) $w_2^N > w_2^{E2} > w_2^{E1} > w_2^E$, (3) $w_1^{E1} > w_2^{E1}$, (4) $w_2^{E2} > w_1^{E2}$.

Only when the inequality $\max\{0, (20c_1/a - 5)/3\} \le c_2/a \le c_1/a$ holds, $q_{d_1}^E, q_{d_2}^E, q_{d_1}^{E_1}$ and $q_{d_2}^{E_2}$ are all larger than zero, then the situations E, E_1 and E_2 are all meaningful. (Recall that we require that only when q_{d_j} is larger than zero, the *j*th supplier encroaches and establishes a direct channel; otherwise, it does not encroach.)

Parts (1) and (2) of Lemma 2 shows that, as the number of the encroaching suppliers increases, the wholesale prices the suppliers propose will get lower (for S_i , the inequalities $w_i^N > w_i^{E1} > w_i^E$ and $w_i^N > w_i^{E2} > w_i^E$ hold). The increase of the number of the encroaching suppliers implies the intensification of the competition in the retail market. From Parts (1) and (3), if only S_i encroaches, it will propose a higher wholesale price than that of S_j (as $w_i^{Ei} > w_j^{Ei}$) and than that when only S_j encroaches (as $w_i^{Ei} > w_i^{Ej}$) to reduce the competition from its retailer.

Recall that Proposition 1 provides the encroachment decisions of the suppliers in equilibrium. Whether encroachment is beneficial to the suppliers is still unknown. If the supplier is a monopolist in the supply side, it encroaches only if encroachment is beneficial to itself. However, in competitive supply chains, the encroachment decision of one supplier is affected by the decisions of other suppliers. Proposition 2 investigates how the profits of the suppliers in equilibrium are affected by encroachment.

Proposition 2. When $c_1 \ge c_2$, compared to the profit before encroachment, the profits of the suppliers in equilibrium are affected by encroachment as follows:

(1) if $(\frac{c_1}{a}, \frac{c_2}{a}) \in \Omega_1$, the profit of S_1 increases if and only if $(\frac{c_1}{a}, \frac{c_2}{a}) \in \Omega_5$ and the profit of S_2 increases if and only if $(\frac{c_1}{a}, \frac{c_2}{a}) \in \Omega_6$;

(2) if $\left(\frac{c_1}{a}, \frac{c_2}{a}\right) \in \Omega_2$, the profit of the encroaching supplier increases while the profit of the non-encroaching one decreases;

(3) if $(\frac{c_1}{a}, \frac{c_2}{a}) \in \Omega_4$, the profit of S_2 increases while the profit of S_1 decreases.

We plot Region Ω_6 in Figure 2 which is the region below the dashed line. Region Ω_5 is too small to present in Figure 2, so we present it solely in Figure



Figure 2: The regions where the suppliers are better off by encroachment



Figure 3: The region where S_1 is better off when both suppliers encroach

3. Proposition 2 indicates that encroachment is not always beneficial to the suppliers. The suppliers' profits include two parts: the profit from wholesaling to the retailer and the profit from the direct channel. From Proposition 1, when $(c_1/a, c_2/a)$ falls into Region Ω_1 , the market is optimistic to the suppliers, so both of the suppliers encroach, which brings intense competition in the retail market. Although the suppliers propose lower wholesale prices in the all-encroachment situation than that in the non-encroachment situation (i.e., $w_i^N > w_i^E$, i = 1, 2), it is not difficult to show that the suppliers' profits from wholesaling to the retailers decrease (i.e., $w_i^E q_{r_i}^E < w_i^N q_{r_i}^N$, i = 1, 2). If the profit from the direct channel cannot offset the reduction of the profit from wholesaling to the retailer, encroachment will be detrimental to the suppliers, especially to the less efficient one. Part (1) of Proposition 2 shows that when the efficient supplier has a superior operational advantage compared to the less efficient one (i.e., $(c_1/a, c_2/a) \in \Omega_6$), it benefits from encroachment while the less efficient supplier benefits from encroachment only in a tiny region Ω_5 .

As $(c_1/a, c_2/a)$ increases and falls into Region Ω_2 , the market is not as good as in Region Ω_1 to the suppliers, so only one supplier encroaches and the degree of competition in the retail market is mitigated compared to the all-encroachment situation. The encroaching supplier benefits from encroachment while the non-encroaching one is worse off due to the addition of a new sales channel. When $(c_1/a, c_2/a) \in \Omega_4$, S_2 has a significant operational advantage compared to S_1 , so only S_2 encroaches and benefits from the encroachment while S_1 is worse off.

Proposition 1 states that encroachment is a dominant strategy for both suppliers when $(c_1/a, c_2/a) \in \Omega_1$. However, Part (1) of Proposition 2 shows that encroachment is detrimental to both suppliers in Region $\Omega_1 - \Omega_5 - \Omega_6$, that is, the prisoner's dilemma phenomenon exists for the suppliers. Therefore, in the environment of competitive supply chains, the suppliers may encroach even if they are worse off. This implies that encroachment may be a business imperative to the suppliers.

The reductions of the wholesale prices under encroachment in Lemma 2 imply that the retailers may benefit from supplier encroachment. But the following proposition shows that the retailers are worse off by encroachment.

Proposition 3. In the non-identical system, both retailers are worse off by encroachment.

Proposition 3 is different from the result in Arya et al. (2007) that the

retailer may benefit from supplier encroachment. Note that the supplier in Arya et al. (2007) is monopolistic while in our model the suppliers are duopolistic. Proposition 3 can be interpreted intuitively as follows. When both suppliers encroach, the retailers are worse off due to intense competition from the direct channels of the two suppliers. If only S_i encroaches, it provides a higher wholesale price than that of S_j (i.e., $w_i^{Ei} > w_j^{Ei}$, Part (4) in Lemma 2), thus its counterpart R_i is at a disadvantage compared to R_j . Therefore, R_i is worse off due to encroachment of S_i . Although S_j provides a lower wholesale price to R_j , R_j faces competition from both R_i and S_i . So R_j is also worse off. Note that i, j = 1, 2 and $i \neq j$.

4.2. Discussions

In the previous analysis, we assume that the products of the two supply chains are identical and completely substitutable. When the products of different supply chains are non-substitutable, the supply chains are independent, so the result is equivalent to that of a single supply chain composed of a monopolistic supplier and an exclusive retailer. Supplier encroachment in a single supply chain is analyzed in Arya et al. (2007) which shows that the supplier encroaches if the market is optimistic to the supplier (i.e., c/afalls below a certain threshold), and the retailer benefits from encroachment if it has a significant operational advantage but the advantage is not so pronounced that the supplier refrains from encroachment. Moreover, the supplier encroaches if and only if encroachment is beneficial to itself.

Note that completely substitutable products and non-substitutable products are the two extreme cases of the substitutable products. Comparing the results of the completely substitutable products with those of the nonsubstitutable products, we could roughly investigate the impact of product substitutability of different supply chains. As the product substitutability of different supply chains increases, the competition between the supply chains intensifies. Proposition 2 and the results in Arya et al. (2007) intuitively indicate that, as the product substitutability of different supply chains increases, supplier encroachment is more likely to result in the prisoner's dilemma phenomenon. Proposition 3 together with the results in Arya et al. (2007) intuitively show that, as the product substitutability of different supply chains increases, the retailers are less likely to benefit from encroachment.

Therefore, to avoid the prisoner's dilemma phenomenon and the "loselose" outcome to the suppliers and the retailers, the supply chains can distinguish each other by differentiated products to mitigate competition.

5. Encroachment in Identical System

In this section, we examine the system composed of $n \ (n \ge 2)$ identical supply chains with homogenous products, and show that the insights obtained in the non-identical system still hold. Assume the unit operational cost of the direct channel of each supplier to be c. Use superscripts "N" and " E_k " to represent the non-encroachment situation and the situation where k $(1 \le k \le n)$ of the n suppliers encroach. The following proposition presents the suppliers' encroachment decisions in equilibrium.

Proposition 4. There exist a series of non-negative numbers $\{b_j, j = 0...n\}$ which are strictly decreasing, and k of the n suppliers encroach in equilibrium if $c/a \in (b_k, b_{k-1}](k = 1...n)$, and none of the suppliers encroach in equilibrium if $c/a \in (b_0, 1]$, where $b_n = 0$.

Proposition 4 shows that as the market is getting worse to the suppliers (i.e., c/a being larger), the number of the encroaching suppliers in equilibrium decreases monotonically, which is intuitive and consistent with the result of the non-identical system in Proposition 1. We provide a numerical example of Proposition 4 in Figure 4 when n varies from 3 to 5. In Figure 4, for fixed n, k is the number of the encroaching suppliers in equilibrium, and the cut-off points for different values of k correspond to $\{b_j, j = 0...n\}$ in Proposition 4. We have the following observations: (a) For fixed n, as c/a increases, k decreases monotonically, i.e., the number of the encroaching suppliers in equilibrium decreases. (b) As n increases, the range of c/a where the suppliers encroach (i.e., k > 0) narrows, which indicates that, as the competition intensifies, the suppliers are less likely to encroach.

To facilitate the presentation, we list the notation of the equilibrium outcomes when k of the n suppliers encroach. Proof of Proposition 4 shows that the equilibrium outcomes of the encroaching suppliers (non-encroaching suppliers, the retailers who face an encroaching supplier, the retailers who face a non-encroaching supplier) are the same. Refer to Proof of Proposition 4 for the explicit expressions of the following notation:

 $w_1(k, n)$: the wholesale price of the encroaching suppliers,

 $w_2(k, n)$: the wholesale price of the non-encroaching suppliers,

 $q_1^r(k, n)$: the order quantity of the retailers who face an encroaching supplier,

 $q_2^r(k,n)$: the order quantity of the retailers who face a non-encroaching supplier,



Figure 4: Ranges of c/a corresponding to different number of the encroaching suppliers in equilibrium when n varies from 3 to 5

 $q^d(k, n)$: the quantity sold through the direct channel of the encroaching suppliers,

 $\Pi_1^s(k,n)$: the profit of the encroaching suppliers,

 $\Pi_2^s(k,n)$: the profit of the non-encroaching suppliers,

 $\Pi_1^r(k,n)$: the profit of the retailers who face an encroaching supplier,

 $\Pi_2^r(k,n)$: the profit of the retailers who face a non-encroaching supplier.

Lemma 3. (1) $w_1(k,n) > w_2(k,n)$ (k = 1, ..., n) always holds. (2) $w_1(1,n)$ decreases with c while $w_1(k,n)$ (k = 2, ..., n) increases with c. (3) $q^d(k,n)$ (k = 1, ..., n) decreases with c. (4) $w_2(k,n)$, $q_1^r(k,n)$ and $q_2^r(k,n)$ (k = 1, ..., n) all increase with c.

Part (1) of Lemma 3 shows that the encroaching suppliers raise the wholesale prices to mitigate the competition from their retailer. Parts (1), (3) and (4) of Lemma 3 are similar to the results of the non-identical system in Lemmas 1 and 2.

We can interpret Part (2) of Lemma 3 intuitively as follows. If only one supplier encroaches, as its operational disadvantage becomes more significant, the supplier lowers its wholesale price. Next, we turn to the situations where

more than one supplier encroaches in equilibrium. In the non-identical system, if both suppliers encroach, the wholesale price of one supplier decreases with its own operational cost and increases with that of the other supplier (Lemma 1). In the identical system, Part (2) of Lemma 3 indicates that the suppliers will raise the wholesale prices as their operational disadvantage gets more significant, which implies that the effect of the operational costs of other suppliers is more significant than that of one's own operational cost. In other words, the wholesale price of one supplier is more sensitive to the operational costs of other suppliers. To see it more clearly, when n = 2 (i.e., in the non-identical system), if both suppliers encroach, w_i^E equals to $11a/69 - 13c_i/345 + 11c_j/115$ $(i, j = 1, 2, i \neq j)$ (Table 1). Obviously, the wholesale price of one supplier is more sensitive to the operational cost of the operational system), if both suppliers encroach, w_i^E equals to $11a/69 - 13c_i/345 + 11c_j/115$ $(i, j = 1, 2, i \neq j)$ (Table 1). Obviously, the wholesale price of one supplier is more sensitive to the operational cost of the other supplier is more sensitive to the operational cost of the other supplier is more sensitive to the operational cost of the other supplier is more sensitive to the operational cost of the other supplier (as 13/345 is smaller than 11/115).

Next, we explore the impact of encroachment on the profits of the suppliers and the retailers.

Proposition 5. For $n \geq 3$,

(1) if only one supplier encroaches (i.e., $c/a \in (b_1, b_0]$), the encroaching supplier is better off by encroachment while the non-encroaching ones are worse off by encroachment; if more than one supplier encroaches (i.e., $c/a \in (0, b_1]$), all suppliers are worse off by encroachment, (2) all retailers are worse off by encroachment.

Proposition 5 shows that there may exist a phenomenon similar to the prisoner's dilemma for the suppliers: when $c/a \in (0, b_{n-1}]$, all the suppliers encroach, but the suppliers' profits after encroachment are lower than those before encroachment. Proposition 5 also shows that encroachment may lead to the "lose-lose" outcome for the suppliers and the retailers. Therefore, the managerial insights of Proposition 5 are consistent with those from Propositions 2 and 3.

At last, we portray the degree of the efficiency losses of the suppliers, the retailers and the whole system due to encroachment. Denote

$$\begin{aligned} \xi_{k} &= \sum_{1 \leq i \leq n} \Pi_{s_{i}}^{E_{k}} / \sum_{1 \leq i \leq n} \Pi_{s_{i}}^{N}, \ k = 1, ..., n, \\ \eta_{k} &= \sum_{1 \leq i \leq n} \Pi_{r_{i}}^{E_{k}} / \sum_{1 \leq i \leq n} \Pi_{r_{i}}^{N}, \ k = 1, ..., n, \\ \zeta_{k} &= \sum_{1 \leq i \leq n} (\Pi_{s_{i}}^{E_{k}} + \Pi_{r_{i}}^{E_{k}}) / \sum_{1 \leq i \leq n} (\Pi_{s_{i}}^{N} + \Pi_{r_{i}}^{N}), \ k = 1, ..., n, \end{aligned}$$
(1)
$$\begin{aligned} \xi &= (\xi_{1}, \xi_{2}, ..., \xi_{n}), \\ \eta &= (\eta_{1}, \eta_{2}, ..., \eta_{n}), \\ \zeta &= (\zeta_{1}, \zeta_{2}, ..., \zeta_{n}). \end{aligned}$$

Define ξ_k , η_k and ζ_k as the efficiency of the suppliers, the retailers and the whole system when k suppliers encroach, respectively; ξ , η and ζ as the efficiency of the suppliers, the retailers and the whole system under encroachment, respectively. Proposition 6 provides a lower bound and an upper bound for the efficiency of the suppliers under encroachment.

Proposition 6. When $n \ge 2$, ξ_k $(1 \le k \le n)$ always falls into the interval $[\xi_{min}, \xi_{max}]$, and both ξ_{min} and ξ_{max} decrease with n, where

$$\xi_{min} = \frac{(n^3 + 2n^2 + n - 1)(n + 1)^4}{n(n^5 + n^4 + 2n^3 + n^2 + 2n - 1)(n^2 + 3n + 3)},$$

$$\xi_{max} = \frac{(8n^5 + 32n^4 + 42n^3 + 10n^2 - 8n - 3)(n + 1)^3}{4n^4(n + 2)^2(2n + 1)^2}.$$
(2)

As the expressions of the lower bounds and the upper bounds of η and ζ are tedious, we present them in a more intuitive way in Figure 5 when n varies from 2 to 8. We observe that the upper bounds of η and ζ are always smaller than 1, which indicates that encroachment is always detrimental to the retailers and the whole system. The bounds of ξ and ζ decrease rapidly as n increases, which indicates that the efficiency losses of the suppliers and the system are more severe as the market becomes more competitive. When n = 3, the efficiency loss of the whole system is more than 20%. The changes of the bounds of η are relatively mild.

In general, when the competition among the supply chains is intense, encroachment may result in a phenomenon similar to the prisoner's dilemma



Figure 5: Lower bounds and upper bounds of ξ , η and ζ when n varies from 2 to 8

to the suppliers and also the "lose-lose" outcome to the suppliers and the retailers. The supply chains can distinguish each other by differentiated products to mitigate the competition.

6. Conclusion

In this paper, we examine supplier encroachment in competitive supply chains. We show that, as the operational disadvantage of the suppliers becomes more significant, the number of the encroaching suppliers in equilibrium decreases monotonically. We find that there may exist the prisoner's dilemma phenomenon for the suppliers, and that encroachment may lead to a "lose-lose" outcome to the suppliers and the retailers. We also investigate the efficiency losses of the suppliers, the retailers and the whole system due to encroachment. The efficiency loss of the whole system under encroachment is severe.

There are several directions deserving future research. First, the supply chains in our model compose of one supplier and one retailer. In reality, one supplier may wholesale products to multiple retailers, and the retailers may sell the products of multiple suppliers. So other structures of supply chains can be considered. Second, our model only analyzes the case when the products of the suppliers are homogenous, i.e., identical and completely substitutable. The case with substitutable products is worth studying although we conjecturer that the main results of our paper still hold. Third, the information is complete and symmetric in our model. It is interesting to investigate the setting with asymmetric information (Li et al. (2013)), e.g., the operational costs of the suppliers are asymmetric information. Fourth, we show that encroachment often leads to the "lose-lose" outcome. Designing mechanisms to prevent the supplier and the retailers from the outcome is a direction for future research.

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Appendix A. Proof of Proposition 1

Both of the suppliers have the option to encroach or not. In order to determine the decisions of the suppliers in equilibrium, we obtain the equilibrium outcomes by backward induction in four situations: non-encroachment, all-encroachment, only S_1 encroaches and only S_2 encroaches.

We first analyze the all-encroachment case. When both of the suppliers encroach, in the third stage, given the wholesale prices w_1 and w_2 and retailers' order quantities q_{r_1} and q_{r_2} , S_1 and S_2 determine the quantities sold through the direct channels. The optimization problem of S_1 is:

$$\max_{q_{d_1}} \Pi_{s_1}^E = (a - q_{r_1} - q_{r_2} - q_{d_1} - q_{d_2} - c_1)q_{d_1} + w_1q_{r_1}.$$
 (A.1)

The optimization problem of S_2 is:

$$\max_{q_{d_2}} \Pi_{s_2}^E = (a - q_{r_1} - q_{r_2} - q_{d_1} - q_{d_2} - c_2)q_{d_2} + w_2 q_{r_2}.$$
 (A.2)

Performing the optimizations in (A.1) and (A.2), q_{d_1} and q_{d_2} should satisfy

$$q_{d_1} = \frac{a - c_1 - q_{r_1} - q_{r_2} - q_{d_2}}{2},\tag{A.3}$$

and

$$q_{d_2} = \frac{a - c_2 - q_{r_1} - q_{r_2} - q_{d_1}}{2},\tag{A.4}$$

respectively.

Solving out q_{d_1} and q_{d_2} from (A.3) and (A.4), we have:

$$q_{d_1}^E = \frac{a - q_{r_1} - q_{r_2} - 2c_1 + c_2}{3}, \ q_{d_2}^E = \frac{a - q_{r_1} - q_{r_2} - 2c_2 + c_1}{3}.$$
 (A.5)

In the second stage, anticipating the suppliers' responses to the quantities they order and given the wholesale prices w_1 and w_2 , the retailers determine their order quantities simultaneously to maximize their profits. The optimization problem of R_1 is:

$$\max_{q_{r_1}} \Pi_{r_1}^E = (a - q_{r_1} - q_{r_2} - q_{d_1}^E - q_{d_2}^E - w_1)q_{r_1}.$$
 (A.6)

Substituting Equation (A.5) into (A.6), the order quantity of R_1 should satisfy the following equation:

$$q_{r_1} = \frac{a + c_1 + c_2 - 3w_1 - q_{r_2}}{2}.$$
 (A.7)

Analogously, the order quantity of R_2 should satisfy the following equation:

$$q_{r_2} = \frac{a + c_1 + c_2 - 3w_2 - q_{r_1}}{2}.$$
 (A.8)

Solve out q_{r_1} and q_{r_2} from (A.7) and (A.8):

$$q_{r_1}^E = \frac{a + c_1 + c_2 - 6w_1 + 3w_2}{3}, \ q_{r_2}^E = \frac{a + c_1 + c_2 - 6w_2 + 3w_1}{3}.$$
 (A.9)

Substituting Equations (A.9) into Equations (A.5), $q_{d_1}^E$ and $q_{d_2}^E$ can be expressed as functions of w_1 and w_2 :

$$q_{d_1}^E = \frac{a - 8c_1 + c_2 + 3w_1 + 3w_2}{9}, \ q_{d_2}^E = \frac{a - 8c_2 + c_1 + 3w_1 + 3w_2}{9}.$$
 (A.10)

In the first stage, the suppliers simultaneously set the wholesale prices w_1 and w_2 to maximize their profits. Substituting Equations (A.9) and Equations (A.10) into (A.1), (A.2) and performing the maximization in (A.1) and (A.2) yield w_1 and w_2 :

$$w_1^E = \frac{11a}{69} - \frac{13c_1}{345} + \frac{11c_2}{115}, \ w_2^E = \frac{11a}{69} - \frac{13c_2}{345} + \frac{11c_1}{115}.$$

 $q_{r_i}^E, q_{d_i}^E, \Pi_{s_i}^E, \Pi_{r_i}^E \ (i = 1, 2)$ can also be easily obtained.

Analyses of other three situations are similar, with all the results given in Table 1. Comparing the suppliers' profits in the four situations, it is not difficult to obtain Proposition 1.

Appendix B. Proof of Proposition 4

Proof of the proposition includes two parts. In Part 1, we apply backward induction to obtain the equilibrium outcomes when k $(1 \le k \le n)$ of the n suppliers encroach. Part 2 turns to the proof of the proposition. Without loss of generality, suppose the first k suppliers encroach.

Part 1 In this part, we apply backward induction to obtain the equilibrium outcomes when k $(1 \le k \le n)$ of the *n* suppliers encroach.

(1) In the third stage, given the order quantities of the retailers q_{r_l} $(1 \le l \le n)$ and the wholesale prices w_l $(1 \le l \le n)$, the encroaching suppliers determine the quantities sold through their direct channels. The optimization problem of $S_i(1 \le i \le k)$ is:

$$\max_{q_{d_i}} \Pi_{s_i}^{E_k} = (a - \sum_{1 \le l \le n} q_{r_l} - \sum_{1 \le j \le k} q_{d_j} - c)q_{d_i} + w_i q_{r_i}, \ 1 \le i \le k.$$
(B.1)

Performing the optimization in (B.1), $q_{d_i}^{E_k}$ should satisfy the following equations:

$$q_{d_i}^{E_k} = \frac{a - c - \sum_{\substack{1 \le j \le k \\ j \ne i}} q_{d_j}^{E_k} - \sum_{1 \le l \le n} q_{r_l}}{2}, \ 1 \le i \le k.$$
(B.2)

It can be observed from (B.2) that $q_{d_i}^{E_k} = q_{d_j}^{E_k}$ $(1 \le i < j \le k)$. Solving out $q_{d_i}^{E_k}$ from (B.2), we obtain:

$$q_{d_i}^{E_k} = \frac{a - c - \sum_{1 \le l \le n} q_{r_l}}{k + 1}, \ 1 \le i \le k.$$
(B.3)

(2) In the second stage, given the wholesale prices w_l $(1 \le l \le n)$ and anticipating the suppliers' responses to the quantities they order, the retailers determine their order quantities simultaneously to maximize their profits. The optimization problem of R_i is:

$$\max_{q_{r_i}} \Pi_{r_i}^{E_k} = \left(a - \sum_{1 \le j \le k} q_{d_j}^{E_k} - \sum_{1 \le l \le n} q_{r_l} - w_i \right) q_{r_i}, \ 1 \le i \le n.$$
(B.4)

Substituting Equations (B.3) into (B.4) and performing the optimization in (B.4), it can be obtained that $q_{r_i}^{E_k}$ should satisfy the following equations:

$$a + kc - (k+1)w_i - \sum_{\substack{1 \le l \le n \\ l \ne i}} q_{r_l}^{E_k}$$
$$q_{r_i}^{E_k} = \frac{1}{2}, \ 1 \le i \le n.$$
(B.5)

Denote $q_r^{E_k} = (q_{r_1}^{E_k}, q_{r_2}^{E_k}, \cdots, q_{r_n}^{E_k})$. Equations (B.5) could be written in matrix form:

$$(I+E)q_r^{E_k} = [a+kc-(k+1)w_i]_{n\times 1},$$
(B.6)

where I is the matrix with all elements being 1 and E is the unit matrix, $[a+kc-(k+1)w_i]_{n\times 1}$ is a n-dimensional vector, the *i*th $(1 \le i \le n)$ element of which is $a+kc-(k+1)w_i$. Solve out $q_{r_i}^{E_k}$ from the system of linear equations (B.6):

$$q_{r_i}^{E_k} = \frac{a+kc-(k+1)\left(nw_i - \sum_{\substack{1 \le l \le n \\ l \ne i}} w_l\right)}{n+1}, \ 1 \le i \le n.$$
(B.7)

Substituting Equations (B.7) into Equations (B.3), $q_{d_i}^{E_k}$ $(1 \le i \le k)$ can be expressed as a function of w_l $(1 \le l \le n)$:

$$q_{d_i}^{E_k} = \frac{a - [n(k+1)+1]c + (k+1)\sum_{1 \le l \le n} w_l}{(k+1)(n+1)}, \ 1 \le i \le k.$$
(B.8)

We observe that, the expression of $q_{d_i}^{E_k}$ is irrelevant to *i*. That is, the quantities sold through the direct channels of each encroaching supplier are equal. Denote $q^d(k,n) = q_{d_i}^{E_k}$.

(3) In the first stage, the suppliers simultaneously set the wholesale prices w_l $(1 \le l \le n)$ to maximize their profits. Substituting Equations (B.7) and (B.8) into (B.1), we obtain that:

$$\Pi_{s_{i}}^{E_{k}} = \frac{1}{(k+1)^{2}(n+1)^{2}} \left\{ \left[a - [n(k+1)+1]c + (k+1)\sum_{1 \le l \le n} w_{l} \right]^{2} + (k+1)^{2}(n+1)w_{i} \left[a + kc - (k+1)(nw_{i} - \sum_{\substack{1 \le l \le n \\ l \ne i}} w_{l}) \right] \right\}, \quad 1 \le i \le k.$$
(B.9)

Performing the maximization in (B.1), it can be shown that the wholesale prices of the encroaching suppliers satisfy the following optimality equations:

$$2\left\{a - [n(k+1)+1]c + (k+1)\sum_{1\leq l\leq n} w_l\right\}$$
$$+ (k+1)(n+1)\left[a + kc - 2n(k+1)w_i + (k+1)\sum_{\substack{1\leq l\leq n\\l\neq i}} w_l\right] = 0, \ 1\leq i\leq k.$$
(B.10)

The optimization problems of the non-encroaching suppliers are:

$$\max_{w_j} \Pi_{s_j}^{E_k} = w_j q_{r_j}^{E_k}, \ k+1 \le j \le n.$$
(B.11)

Substituting (B.7) into (B.11), we obtain that:

$$\Pi_{s_j}^{E_k} = \frac{w_j \left[a + kc - (k+1)(nw_j - \sum_{\substack{1 \le l \le n \\ l \ne i}} w_l) \right]}{n+1}$$
(B.12)

Performing the maximization in (B.11), it can be shown that the wholesale prices of the non-encroaching suppliers satisfy the following optimality equations:

$$a + kc - 2n(k+1)w_j + (k+1)\sum_{\substack{1 \le l \le n \\ l \ne i}} w_l = 0, \ k+1 \le j \le n.$$
(B.13)

From Equations (B.10) and (B.13), it is obvious that the wholesale prices of the encroaching (non-encroaching) suppliers equal to each other. Denote $w_1(k,n)$ and $w_2(k,n)$ as the wholesale prices of the encroaching and nonencroaching suppliers, respectively. From Equations (B.7), the order quantities of the retailers facing an encroaching (non-encroaching) supplier are equal. Denote $q_1^r(k,n) = q_{r_i}^{E_k}$ $(1 \le i \le k)$ and $q_2^r(k,n) = q_{r_j}^{E_k}$ $(k+1 \le j \le n)$. Equations (B.10) and (B.13) can be written as:

$$[(k+1)(n+1)+2]a + \{k(k+1)(n+1) - 2[n(k+1)+1]\}c - (k+1)[(k+1)(n+1)(2n-k+1) - 2k]w_1(k,n)$$
(B.14)
+ (k+1)(n-k)[(k+1)(n+1)+2]w_2(k,n) = 0,

and

$$a + kc + (k+1)[kw_1(k,n) - (k+n+1)w_2(k,n)] = 0.$$
 (B.15)

Solving out $w_1(k, n)$ and $w_2(k, n)$ from (B.14) and (B.15) and substituting them into Equations (B.7) and (B.8), the equilibrium outcomes when k of the n suppliers encroach are as follows:

$$\begin{split} w_1(k,n) &= \frac{(2k^2n^2 + k^2n - 2n^2 - k^2 - 4n + kn - k - 2)c + (2n+1)(kn + n + k + 3)a}{(k+1)(2n+1)(kn^2 + n^2 + 2kn + 2n - k + 1)}, \\ w_2(k,n) &= \frac{(n+1)[(2n-1)kc + (2n+1)a]}{(2n+1)(kn^2 + n^2 + 2kn + 2n - k + 1)}, \\ q_1^r(k,n) &= \frac{(2k^2n^2 + 4kn^2 - k^2n + 2n^2 - kn + 4n + 2)c + (2n+1)(kn + n - 2)a}{(2n+1)(kn^2 + n^2 + 2kn + 2n - k + 1)}, \\ q_2^r(k,n) &= \frac{n(k+1)[(2n-1)kc + (2n+1)a]}{(2n+1)(kn^2 + n^2 + 2kn + 2n - k + 1)}, \\ q^d(k,n) &= \frac{-(kn^2 + n^2 + 2n + 1)c + (2n+1)a}{kn^2 + n^2 + 2kn + 2n - k + 1}, \\ \Pi_1^s(k,n) &= \frac{Dc^2 - Eac + Fa^2}{(k+1)(2n+1)^2(kn^2 + n^2 + 2kn + 2n - k + 1)^2}, \\ \Pi_2^s(k,n) &= \frac{n(n+1)(k+1)[(2n-1)kc + (2n+1)a]^2}{(2n+1)^2(kn^2 + n^2 + 2kn + 2n - k + 1)^2}, \\ \Pi_1^r(k,n) &= \frac{[(2k^2n^2 + 4kn^2 - k^2n + 2n^2 - kn + 4n + 2)c + (2n+1)(kn + n - 2)a]^2}{(k+1)(2n+1)^2(kn^2 + n^2 + 2kn + 2n - k + 1)^2}, \\ \Pi_1^r(k,n) &= \frac{n\Pi_2^s(k,n)}{(n+1)}. \end{split}$$

where,

$$D = 4k^{3}n^{6} + 12k^{2}n^{6} + 4k^{3}n^{5} + 4k^{4}n^{4} + 12kn^{6} + 28k^{2}n^{5} + 9k^{3}n^{4} + 4k^{3}n^{3} - 3k^{4}n^{2} + 4n^{6} + 44kn^{5} + 27k^{2}n^{4} + 59kn^{4} + 20n^{5} + 20k^{2}n^{3} - 8k^{3}n^{2} + k^{4}n + 44kn^{3} + 37n^{4} + 3k^{2}n^{2} + 2k^{3}n + 26kn^{2} + 28n^{3} + k^{2}n + 8kn + 2n^{2} - 2k^{2} - k - 8n - 3,$$
(B.17)
$$E = 2(2n+1)(kn^{2} + n^{2} + kn + n - 1)(-2k^{2}n + 4kn^{2} + 4n^{2} + k^{2} - 2kn + 8n + k + 4),$$

$$F = (k+5)(2n+1)^{2}(kn^{2} + n^{2} + kn + n - 1).$$

If none of the suppliers encroach, it is easy to verify that $w^N = w_2(0, n)$, $q_{r_i}^N = q_2^r(0, n)$, $\Pi_{s_i}^N = \Pi_2^s(0, n)$ and $\Pi_{r_i}^N = \Pi_2^r(0, n)$, where $1 \le i \le n$. Until now, we have obtained the equilibrium outcomes when $k \ (0 \le k \le n)$ of the *n* suppliers encroach.

Part 2 In this part, we prove the proposition. Denote $\pi_i^s(k, n) = a^2 \Pi_i^s(k, n)$ (i = 1, 2) and $c/a = \tilde{c}$.

According to the definition of Nash Equilibrium, k of the n suppliers encroach in equilibrium if and only if the following two inequalities hold:

$$\pi_1^s(k,n) \ge \pi_2^s(k-1,n), \ k=1,...,n,$$
 (B.18)

$$\pi_1^s(k+1,n) \leq \pi_2^s(k,n), \qquad k=0,...,n-1.$$
 (B.19)

Denote $\tilde{c} \geq f_1(k, n)$ or $\tilde{c} \leq g_1(k, n)$ as the solution of Inequality (B.18) and $g_2(k, n) \leq \tilde{c} \leq f_2(k, n)$ as the solution of Inequality (B.19). As the expressions of $f_1(k, n)$, $g_1(k, n)$, $f_2(k, n)$ and $g_2(k, n)$ are complicated, we do not present them here. Comparing Inequality (B.18) with (B.19), it is obvious that the following equations hold:

$$f_1(k,n) = f_2(k-1,n), \ g_1(k,n) = g_2(k-1,n).$$
 (B.20)

If k equals to n (i.e., all suppliers encroach) in equilibrium, we only need to make sure that Inequality (B.18) holds, i.e., $\tilde{c} \ge f_1(n,n)$ or $\tilde{c} \le g_1(n,n)$. If k equals to 0 (i.e., none of the suppliers encroach) in equilibrium, we only need to make sure that Inequality (B.19) holds, i.e., $g_2(0,n) \le \tilde{c} \le f_2(0,n)$.

Next, we show that $f_i(k, n)$ and $g_i(k, n)$ (i = 1, 2) decrease with k. Referring to Inequality (B.20), the monotonicity of $f_1(k, n)$ $(g_1(k, n))$ and $f_2(k, n)$ $(g_2(k, n))$ are the same. The proof of the monotonicity of $g_i(k, n)$ is similar to that of $f_i(k, n)$. So we only show that $f_i(k, n)$ deceases with k.

Applying the reduction to absurdity, if $f_i(k, n)$ does not decrease with k, then there exists j $(1 \le j \le n-1)$ such that $f_2(j, n) < f_2(j+1, n)$. Denote $l = \min\{j | f_2(j, n) < f_2(j+1, n), 1 \le j \le n-1\}$. We discuss the problem in the following two cases:

(1) If $1 \leq l \leq n-2$, referring to Equations (B.20), the inequality $f_1(l+1,n) < f_2(l+1,n)$ and the equation $f_1(l+2,n) = f_2(l+1,n)$ hold. According to the definition of $f_i(k,n)$, when $\tilde{c} \in (f_1(l+1,n), f_2(l+1,n))$, the inequality $\pi_1^s(l+2,n) < \pi_2^s(l+1,n)$ holds. When $\tilde{c} > f_1(l+2,n)$, the inequality $\pi_1^s(l+2,n) > \pi_2^s(l+1,n)$ holds.

(2) If l = n - 1, referring to Equations (B.20), the inequality $f_2(l, n) < f_1(l, n)$ and the equation $f_2(l-1, n) = f_1(l, n)$ hold. When $\tilde{c} \in (f_2(l, n), f_1(l, n))$, the inequality $\pi_1^s(l, n) < \pi_2^s(l-1, n)$ holds. When $\tilde{c} > f_2(l-1, n)$, the inequality $\pi_1^s(l, n) > \pi_2^s(l-1, n)$ holds.

However, it can be shown that $\pi_1^s(l+1,n) - \pi_2^s(l,n)$ $(1 \leq l \leq n-1)$ decreases with \tilde{c} . Specifically, we can show that $\pi_1^s(l+1,n) - \pi_2^s(l,n)$ is a convex function of \tilde{c} . Denote the \tilde{c} which minimizes $\pi_1^s(l+1,n) - \pi_2^s(l,n)$ as c_0 . The suppliers encroach only if the quantities sold through the direct channels are larger than zero. If l of the n suppliers encroach, $q^d(k,n)$ is larger than zero if and only if $\tilde{c} < (1+2n)/(n^2l+n^2+2n+1) \triangleq h(l,n)$. It can be shown that $c_0 \geq h(l+1,n)$. Thus, $\pi_1^s(l+1,n) - \pi_2^s(l,n)$ decreases with \tilde{c} when $\tilde{c} \leq h(l+1,n)$. Therefore, $f_i(k,n)$ and $g_i(k,n)$ (i = 1,2) decrease with k.

Applying the reduction to absurdity and the definition of Nash Equilibrium, it can be easily shown that $f_2(k, n) \ge g_1(k, n)$.

In the following, we show that $f_1(k,n) < f_1(k+1,n)$ and $g_1(k,n) < g_1(k+1,n)$, where $1 \le k \le n-1$. The proof of the two inequalities are similar, so we only show that $f_1(k,n) < f_1(k+1,n)$. Consider the continuous and differentiable function $f_1(x,n)$ ($x \in [1,n]$) which equals to $f_1(k,n)$ when x = k. Applying the reduction to absurdity, if there exists j ($1 \le j \le n-1$) such that $f_1(j,n) = f_1(j+1,n)$, then $f_1(x,n) = f_1(j,n)$ when $x \in [j, j+1]$. Note that, $f_1(x,n)$ is differentiable and is not a constant. This is a contradiction. Therefore, $f_1(k,n)$ is strictly larger than $f_2(k,n)$ and $g_1(k,n)$ is strictly larger than $g_2(k,n)$. So the following inequalities hold:

$$f_1(k,n) > f_2(k,n) \ge g_1(k,n) > g_2(k,n), \ k = 1,...,n.$$
 (B.21)

As a result, k of the n suppliers encroach in equilibrium if $\tilde{c} \in (g_2(k, n), g_1(k, n)]$ (k = 1, ..., n - 1), and all suppliers encroach in equilibrium if $\tilde{c} \leq g_2(n - 1, n)$. When k (k = 1, ..., n) of the n suppliers encroach, $q^d(k, n)$ is larger than zero if and only if $\tilde{c} < (1 + 2n)/(n^2k + n^2 + 2n + 1) \triangleq h(k, n)$. It is not difficult of show that $h(k, n) \geq g_1(k, n)$.

Referring to Inequalities (B.18) and (B.19), none of the suppliers encroach in equilibrium if and only if $g_2(0,n) \leq \tilde{c} \leq f_2(0,n)$. It can be shown that h(k,n) decreases with k and $h(1,n) \leq f_2(0,n)$ always holds. Recall that we require that the suppliers encroach and establish direct channels only if the quantities sold through the direct channels are larger than zero. Thus, when $g_2(0,n) < \tilde{c} \leq 1$, none of the suppliers encroaches in equilibrium.

Denoting $b_i = g_2(i, n)$ and referring to Equations (B.20), we complete the proof of Proposition 4.

Appendix C. Proof of Lemma 3

The proof of Parts (3) and (4) are obvious, so we only present the proof of Parts (1) and (2).

(1) The inequality $w_1(k,n) \ge w_2(k,n)$ holds if and only if $c/a < (1 + 2n)/(n^2k + n^2 + 2n + 1) \triangleq h(k,n)$ According to Proposition 4, k of the n suppliers encroach in equilibrium if $c/a \in (b_k, b_{k-1}]$. As mentioned in Proof of Proposition 4, $h(k,n) \ge g_1(k,n)$. According to Equations (B.20), $h(k,n) \ge g_2(k-1,n)$. As $b_{k-1} = g_2(k-1,n)$, the inequality $h(k,n) \ge b_{k-1}$ holds.

(2) Denote $R(k,n) = 2k^2n^2 + k^2n - 2n^2 - k^2 - 4n + kn - k - 2$. Then,

$$w_1(k,n) = \frac{R(k,n)c + (2n+1)(kn+k+n+3)a}{(k+1)(2n+1)(kn^2+n^2+2kn+2n-k+1)}$$

It suffices to show that for fixed n, R(1,n) < 0 and R(k,n) > 0 for $k \ge 2$. As R(1,n) = -2n - 4, R(1,n) < 0 always holds. We only need to show that R(k,n) > 0 for $k \ge 2$.

Denote $A = 16n^4 + 40n^3 + 25n^2 - 10n - 7$. The zero points of R(k, n) are as follows:

$$k_1(n) = -\frac{n-1+\sqrt{A}}{2(2n^2+n-1)},$$

$$k_2(n) = -\frac{n-1-\sqrt{A}}{2(2n^2+n-1)}.$$

 $k_1(n)$ is always smaller than zero. In order to show that R(k,n) > 0 for $k \ge 2$, it suffices to prove that $k_2(n) < 2$. When n = 2, $k_2(2) = \frac{\sqrt{649}-1}{18} < 2$. In the following, we show that $k_2(n)$ decreases with n.

Taking derivative of $k_2(n)$ with respect to n, we have:

$$\frac{\partial k_2(n)}{\partial n} = -\frac{12n^4 + 31n^3 + 15n^2 - 4n - 6 - n(n-2)\sqrt{A}}{(n+1)^2(2n-1)^2\sqrt{A}}$$

 $\partial k_2(n)/\partial n$ is always smaller than zero as

$$(12n^4 + 31n^3 + 15n^2 - 4n - 6)^2 - n^2(n-2)^2 A$$

=4(2n-1)²(n+1)³(8n³ + 32n² + 21n + 9) > 0.

Therefore, $k_2(n)$ decreases with n.

Appendix D. Proof of Proposition 5

(1) Denote $\pi_i^s(k,n) = a^2 \Pi_i^s(k,n)$ (i = 1, 2) and $c/a = \tilde{c}$. When none of the suppliers encroaches, the profit of each supplier equals to $\Pi_2^s(0,n)$. Thus, we only need to show that the following inequalities hold:

$$\pi_1^s(1,n) \ge \pi_2^s(0,n),$$
 (D.1)

$$\pi_1^s(k,n) \leq \pi_2^s(0,n), \quad k=2,...,n,$$
 (D.2)

$$\pi_2^s(k,n) \leq \pi_2^s(0,n), \quad k = 1, ..., n.$$
 (D.3)

According to the definition of Nash Equilibrium, Inequality (D.1) always holds. It is not difficult to show that $\pi_2^s(k, n)$ decreases with k. Thus, Inequality (D.3) always holds.

Denote $g_3(k,n) \leq \tilde{c} \leq f_3(k,n)$ as the solution of Inequality (D.2). As the expressions of $g_3(k,n)$ and $f_3(k,n)$ are complicated, we do not present them here. Referring to Proof of Proposition 4, the solution of the inequality $\pi_1^s(k,n) \geq \pi_2^s(k-1,n)$ is $\tilde{c} \geq f_1(k,n)$ or $\tilde{c} \leq g_1(k,n)$. As $\pi_2^s(k,n)$ decreases with k, the inequalities $g_3(k,n) \leq g_1(k,n) \leq f_1(k,n) \leq f_3(k,n)$ always hold. Furthermore, it is easy to show that $g_3(k,n)$ decreases with k and $g_3(k,n) \leq 0$ when $k \geq 2$. According to Proof of Proposition 4, k of the n suppliers encroach in equilibrium if and only if $\tilde{c} \in (g_1(k-1,n), g_1(k,n)]$. Therefore, Inequality (D.2) always holds.

(2) If none of the suppliers encroach, the profits of the retailers equal to $\Pi_2^r(0,n)$. It is not difficult to show that $\Pi_2^r(k,n)$ decreases with k. Thus, $\Pi_2^r(k,n) \leq \Pi_2^r(0,n)$. In the following, we show that $\Pi_1^r(k,n) \leq \Pi_2^r(0,n)$. It suffices to show that $\Pi_1^r(k,n) \leq \Pi_2^r(k,n)$. It can be easily shown that when $c/a < (1+2n)/(n^2k+n^2+2n+1) \triangleq h(k,n)$, the inequality $\Pi_1^r(k,n) \leq \Pi_2^r(k,n)$ holds. Note that k of the n suppliers encroach in equilibrium if and only if $c/a \in (b_k, b_{k-1}]$. As $h(k, n) \geq b_{k-1}$, the inequality $\Pi_1^r(k, n) \leq \Pi_2^r(k, n)$ always holds.

Appendix E. Proof of Proposition 6

Through tedious calculations, we could show that the total profit of the suppliers decreases with k. Thus, for fixed value of c/a, the inequalities $\xi_k \leq \xi_{k-1}$ ($2 \leq k \leq n$) always hold. Denote $\tilde{c} = c/a$. The expressions of ξ_1 and ξ_n are as follows:

$$\xi_1 = \frac{(n+1)^3 (A_1 \tilde{c}^2 - 2B_1 \tilde{c} + C_1)}{4n^4 (n+2)^2 (2n+1)^2},$$

$$\xi_n = \frac{(n+1)^2 (A_n \tilde{c}^2 - B_n \tilde{c} + C_n)}{n(n^3 + 3n^2 + n + 1)^2},$$

where,

$$\begin{split} A_1 &= 16n^6 + 56n^5 + 60n^4 + 42n^3 + 18n^2 - 3, \\ B_1 &= 8n^5 + 32n^4 + 34n^3 + 14n^2 - 4n - 3, \\ C_1 &= 8n^5 + 32n^4 + 42n^3 + 10n^2 - 8n - 3, \\ A_n &= n^7 + 4n^6 + 8n^5 + 10n^4 + 11n^3 + 8n^2 + 3n - 3, \\ B_n &= 2n^5 + 6n^4 + 14n^3 + 16n^2 + 6n - 8, \\ C_n &= n^4 + 7n^3 + 11n^2 + 4n - 5. \end{split}$$

Denote

$$\xi_{max} = \frac{(n+1)^3 C_1}{4n^4 (n+2)^2 (2n+1)^2},$$

$$\xi_{min} = \frac{(n^3+2n^2+n-1)(n+1)^4}{n(n^5+n^4+2n^3+n^2+2n-1)(n^2+3n+3)}.$$
(E.1)

It is obvious that ξ_1 attains its maximum ξ_{max} at $\tilde{c} = 0$, and ξ_n attains its minimum ξ_{min} at $\tilde{c} = \frac{B_n}{(2A_n)}$. Therefore, ξ_{min} is a lower bound of ξ_k $(1 \le k \le n)$ and ξ_{max} is an upper bound of $\xi_k (1 \le k \le n)$. Besides, it is easy to show that both ξ_{min} and ξ_{max} decrease with n.

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