

A note on “Cooperative advertising, game theory and manufacturer–retailer supply chains”

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Abstract

This note extends the results in the manufacturer-dominated game model of the paper by Li et al. (Omega 30 (2002) 347) to the case where the manufacturer's marginal profit is not large enough. In such situations, the profit of the entire supply chain under the co-op advertising mode is higher than the one under the Stackelberg game, which is consistent with the results of the original paper. However, the advertising expenditures of the manufacturer and the retailer under the co-op advertising model are not always larger than those under the Stackelberg game, which is different from the results of the original paper. Furthermore, the results are also compared with the simultaneous move game of the paper by Huang and Li (Eur. J. Oper. Res. 135 (2001) 527). The manufacturer always prefers the leader–follower structure rather than the simultaneous move structure, which is consistent with the results of the original paper. However, the retailer always prefers the simultaneous move structure rather than the leader–follower structure, which differs from the results of the original paper.

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1. Introduction

A recent paper by Li et al. [1] develops three strategic models for determining equilibrium marketing and investment effort levels for a manufacturer and a retailer in a two-member supply chain. The first model offers a formal normative approach for analyzing the traditional cooperative (co-op) advertising program where the manufacturer is the leader and the retailer is a follower. The second model provides a further analysis on this manufacturer-dominated relationship, by making use of the concept of “higher order Stackelberg equilibrium”. The third model incorporates the recent market trend of retailing power shifts from manufacturers to retailers to analyze efficiencies of co-op

advertising programs. The first and the third models can also be found in other two papers by the same set of authors [2,3], while Ref. [2] presents discussions on another model where the manufacturer and the retailer simultaneously and noncooperatively maximize their own profits with respect to any possible strategies set by the other member in the system.

All the discussions in Refs. [1–3] for the sequential move Stackelberg game (the first model mentioned above) are based on a primary assumption of $\rho_m/\rho_r \geq \gamma + 1$, and the part of $\rho_m/\rho_r < \gamma + 1$ is missing. (Here ρ_m and ρ_r are the marginal profits for the manufacturer and the retailer, respectively, and γ is the quasi-advertising elasticity [3] related to the one-period sales response volume function.) In the leader–follower structure, it is assumed that the manufacturer as the leader holds the extreme power and has almost complete control over the retailer, thus it seems reasonable to assume that the manufacturer's marginal profit is higher

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than the retailer's, and that is why $\rho_m/\rho_r \geq \gamma + 1$ is implicitly assumed in Refs. [1–3]. However, in recent severely competitive market, the marginal profits for both the manufacturer and the retailer tend to be low, thus the situations where the manufacturer's marginal profit is smaller than the retailer's are also possible. Even for the situations where the manufacturer's marginal profit is higher than the retailer's, the manufacturer's marginal profit may not be large enough to make the inequality $\rho_m/\rho_r \geq \gamma + 1$ hold.

In this note, we extend the results for the Stackelberg game to the case of $\rho_m/\rho_r < \gamma + 1$, and compare the results with those under the co-op advertising model [1] and those under simultaneous move noncooperative game [2]. This extension can enrich the decision makers' understandings about the whole picture of the manufacturer–retailer relationship in the market.

2. Stackelberg equilibrium under $\rho_m/\rho_r < \gamma + 1$

Based on the assumptions and notations in Li et al. [1], the manufacturer's, retailer's and system's profit functions are described as follows:

$$\pi_m = \rho_m(1 - a^{-\gamma}q^{-\delta}) - ta - q, \quad (1)$$

$$\pi_r = \rho_r(1 - a^{-\gamma}q^{-\delta}) - (1 - t)a, \quad (2)$$

$$\pi = \pi_m + \pi_r = (\rho_m + \rho_r)(1 - a^{-\gamma}q^{-\delta}) - a - q, \quad (3)$$

where ρ_m and ρ_r are the marginal profits (positive constants) for the manufacturer and the retailer, respectively, a and q are the retailer's local advertising level and the manufacturer's national brand name investments, respectively, t is the fraction of total local advertising expenditures which the manufacturer agrees to share with the retailer, and γ and δ are positive constants related to the one-period sales response volume function $S(a, q)$ (this function is assumed to be in the form of $S(a, q) = 1 - a^{-\gamma}q^{-\delta}$).

In the first model of Li et al. [1], the authors discuss an interactive two-stage game with the manufacturer as the leader and the retailer as the follower. It can be easily shown that the manufacturer will not allow $t = 1$. Otherwise, since the manufacturer pays for all of the local advertising investment, the retailer prefers $a \rightarrow +\infty$ in order to maximize her own profit, which will result in negative profit for the manufacturer. Thus we can focus our discussion on $0 \leq t < 1$ only and then the retailer's optimal response function can be easily obtained as

$$a = \left(\frac{\gamma\rho_r}{(1-t)q^\delta} \right)^{1/(\gamma+1)}. \quad (4)$$

The solution of the game is called Stackelberg equilibrium and can be obtained easily by making use of (4). According to Ref. [1], when $\rho_m/\rho_r > \gamma + 1$, the solution is

$$a^* = [\delta^{-\delta}\gamma^{\delta+1}(\rho_m - \gamma\rho_r)]^{1/(\delta+\gamma+1)}, \quad (5)$$

$$t^* = \frac{\rho_m - (\gamma + 1)\rho_r}{\rho_m - \gamma\rho_r}, \quad (6)$$

$$q^* = [\delta^{\gamma+1}\gamma^{-\gamma}(\rho_m - \gamma\rho_r)]^{1/(\delta+\gamma+1)} \quad (7)$$

and thus the manufacturer will offer positive advertising allowance to the retailer as indicated in Eq. (6). When $\rho_m/\rho_r < \gamma + 1$, the solution can be similarly obtained as

$$a^* = \left[\left(\frac{\delta\rho_m}{\gamma + 1} \right)^{-\delta} (\gamma\rho_r)^{\delta+1} \right]^{1/(\delta+\gamma+1)}, \quad (8)$$

$$t^* = 0, \quad (9)$$

$$q^* = \left[\left(\frac{\delta\rho_m}{\gamma + 1} \right)^{\gamma+1} (\gamma\rho_r)^{-\gamma} \right]^{1/(\delta+\gamma+1)}. \quad (10)$$

That is to say, the manufacturer will offer no allowance to the retailer, which coincides with the intuition that the higher marginal profit of the retailer will give him incentive to do local advertising without the manufacturer's financial support.

By the way, we point out that for the special case where $\rho_m/\rho_r = \gamma + 1$, both expressions (5)–(7) and (8)–(10) are valid and they degenerate to the same expressions.

3. Comparison with the co-op advertising model

In this section we compare the results of the Stackelberg game for the case of $\rho_m/\rho_r < \gamma + 1$ with those under the co-op advertising model (the third model in [1]), and identify whether it is consistent with the case of $\rho_m/\rho_r \geq \gamma + 1$.

According to [1], the collection of Pareto efficient schemes $(\bar{a}^*, \bar{t}^*, \bar{q}^*)$ for the co-op advertising model is described by

$$\bar{a}^* = [\delta^{-\delta}\gamma^{\delta+1}(\rho_m + \rho_r)]^{1/(\delta+\gamma+1)}, \quad (11)$$

$$0 \leq \bar{t}^* \leq 1, \quad (12)$$

$$\bar{q}^* = [\delta^{\gamma+1}\gamma^{-\gamma}(\rho_m + \rho_r)]^{1/(\delta+\gamma+1)}. \quad (13)$$

Recalling (8)–(10), we have

$$\begin{aligned} \bar{a}^* - a^* &= [\delta^{-\delta}\gamma^{\delta+1}(\rho_m + \rho_r)]^{1/(\delta+\gamma+1)} \\ &\quad - \left[\left(\frac{\delta\rho_m}{\gamma + 1} \right)^{-\delta} (\gamma\rho_r)^{\delta+1} \right]^{1/(\delta+\gamma+1)} \\ &= \begin{cases} \geq 0, & \text{if } 1 + \frac{\rho_m}{\rho_r} \geq \left(\frac{(\gamma + 1)\rho_r}{\rho_m} \right)^\delta \\ < 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (14)$$

$$\begin{aligned} \bar{q}^* - q^* &= [\delta^{\gamma+1}\gamma^{-\gamma}(\rho_m + \rho_r)]^{1/(\delta+\gamma+1)} \\ &\quad - \left[\left(\frac{\delta\rho_m}{\gamma + 1} \right)^{\gamma+1} (\gamma\rho_r)^{-\gamma} \right]^{1/(\delta+\gamma+1)} \\ &< 0. \end{aligned} \quad (15)$$

That is to say, when $1 + (\rho_m/\rho_r) \geq ((\gamma + 1)\rho_r/\rho_m)^\delta$, the retailer's local advertising investment is higher at any Pareto efficient scheme than at Stackelberg; Otherwise, it is lower at any Pareto efficient scheme than at Stackelberg. This result differs from the one under the case of $\rho_m/\rho_r \geq \gamma + 1$, where the retailer's local advertising investment is always higher at any Pareto efficient scheme than at Stackelberg. As to the manufacturer's brand name investment, it is always lower at the co-op advertising structure than at Stackelberg. This observation also differs from the case of $\rho_m/\rho_r \geq \gamma + 1$, where it is always higher at the co-op advertising structure than at Stackelberg [1].

Furthermore, it is critical for the decision makers to compare the system profits under these two different games, and to see whether the co-op advertising model still generates more profit than the leader–follower structure when $\rho_m/\rho_r < \gamma + 1$. Substituting (8)–(10) into (1) and (2), the manufacturer's and the retailer's profits under the leader–follower structure are, respectively,

$$\pi_m^* = \rho_m \left[1 - \left(\frac{\delta \rho_m}{\gamma + 1} \right)^{-\delta/(\delta+\gamma+1)} (\gamma \rho_r)^{-\gamma/(\delta+\gamma+1)} \right] - \left(\frac{\delta \rho_m}{\gamma + 1} \right)^{(\gamma+1)/(\delta+\gamma+1)} (\gamma \rho_r)^{-\gamma/(\delta+\gamma+1)}, \quad (16)$$

$$\pi_r^* = \rho_r \left[1 - \left(\frac{\delta \rho_m}{\gamma + 1} \right)^{-\delta/(\delta+\gamma+1)} (\gamma \rho_r)^{-\gamma/(\delta+\gamma+1)} \right] - \left(\frac{\delta \rho_m}{\gamma + 1} \right)^{-\delta/(\delta+\gamma+1)} (\gamma \rho_r)^{(\delta+1)/(\delta+\gamma+1)}. \quad (17)$$

Summing up (16) and (17), the total profit of the supply chain under leader–follower structure when $\rho_m/\rho_r < \gamma + 1$ is

$$\begin{aligned} \pi^* &= \pi_m^* + \pi_r^* \\ &= (\rho_m + \rho_r) - \left[\left(\frac{\delta \rho_m}{\gamma + 1} \right)^{-\delta} (\gamma \rho_r)^{-\gamma} \right]^{1/(\delta+\gamma+1)} \\ &\quad \times \left[\frac{\delta + \gamma + 1}{\gamma + 1} \rho_m + (\gamma + 1) \rho_r \right]. \end{aligned} \quad (18)$$

Similarly, the total profit under co-op advertising model can be calculated as

$$\begin{aligned} \bar{\pi}^* &= \bar{\pi}_m^* + \bar{\pi}_r^* \\ &= (\rho_m + \rho_r)(1 - (\bar{a}^*)^{-\gamma}(\bar{q}^*)^{-\delta}) - \bar{a}^* - \bar{q}^* \\ &= (\rho_m + \rho_r) - [\delta^{-\delta} \gamma^{-\gamma} (\rho_m + \rho_r)]^{1/(\delta+\gamma+1)} \\ &\quad \times (1 + \delta + \gamma). \end{aligned} \quad (19)$$

From (18) and (19),

$$\begin{aligned} \bar{\pi}^* - \pi_m^* &= (\delta + \gamma + 1) \left[\left(\frac{\delta \rho_m}{\gamma + 1} \right)^{-\delta} (\gamma \rho_r)^{-\gamma} \right]^{1/(\delta+\gamma+1)} \\ &\quad \times (\rho_m + \rho_r) f\left(\frac{\rho_m}{\rho_m + \rho_r}\right), \end{aligned} \quad (20)$$

where

$$f(x) = \left\{ \frac{1}{\delta + \gamma + 1} \left[\delta \left(\frac{1}{\gamma + 1} x \right) + \gamma(1 - x) + 1 \right] - \left(\frac{1}{\gamma + 1} x \right)^{\delta/(\delta+\gamma+1)} (1 - x)^{\gamma/(\delta+\gamma+1)} \right\}. \quad (21)$$

It is easy to see that $f(x) \geq 0$ for any $0 \leq x \leq 1$. Therefore $0 < \rho_m/\rho_m + \rho_r < 1$ implies $\bar{\pi}^* - \pi_m^* \geq 0$. That is to say, the system profit under co-op advertising model is higher than the system profit under leader–follower structure. This result is consistent with that under the case of $\rho_m/\rho_r \geq \gamma + 1$. Therefore a bargaining model similar to that described in [1] can also be used to share the benefit achieved from the advertising cooperation.

We close this section by summarizing the results for the case of $\rho_m/\rho_r < \gamma + 1$ as the following proposition.

Proposition 1. (1) When $1 + \rho_m/\rho_r \geq ((\gamma + 1)\rho_r/\rho_m)^\delta$, the retailer's local advertising investment is higher at any Pareto efficient scheme than at Stackelberg; otherwise, it is lower at any Pareto efficient scheme than at Stackelberg.

(2) The manufacturer's brand name investment is always lower at any Pareto efficient scheme than at Stackelberg.

(3) The system profit under any Pareto efficient scheme is higher than the system profit under the leader–follower structure.

4. Comparison with the Nash equilibrium

In this section we compare the results of the Stackelberg game for the case of $\rho_m/\rho_r < \gamma + 1$ with those under simultaneous move noncooperative game [2], and identify whether it is consistent with the case of $\rho_m/\rho_r \geq \gamma + 1$.

According to Huang and Li [2], the Nash equilibrium (a^{**}, t^{**}, q^{**}) for the simultaneous move noncooperative game is

$$a^{**} = [(\delta \rho_m)^{-\delta} (\gamma \rho_r)^{\delta+1}]^{1/(\delta+\gamma+1)}, \quad (22)$$

$$t^{**} = 0, \quad (23)$$

$$q^{**} = [(\delta \rho_m)^{\gamma+1} (\gamma \rho_r)^{-\gamma}]^{1/(\delta+\gamma+1)}. \quad (24)$$

It can be easily shown that $q^* < q^{**}$ always holds, no matter $\rho_m/\rho_r \geq \gamma + 1$ or $\rho_m/\rho_r < \gamma + 1$. That is to say, the manufacturer's brand name investment is always higher at Nash than at Stackelberg. As to the relationship between a^* and a^{**} , we can also easily prove that $a^* < a^{**}$ holds under $\rho_m/\rho_r < \gamma + 1$. That is to say, the retailer's local advertising level is always higher at Nash than at Stackelberg. This observation differs from the result under the case of $\rho_m/\rho_r \geq \gamma + 1$, where both $a^* < a^{**}$ and $a^* \geq a^{**}$ are possible, depending on the comparison result of magnitude of $\gamma \rho_r/\rho_m + (\rho_r/\rho_m)^{\delta+1}$ with one (cf. [2, Proposition 2]).

Since both the advertising investments from the manufacturer and the retailer are always higher at Nash than at Stackelberg when $\rho_m/\rho_r < \gamma + 1$, it is critical for the decision makers to compare the manufacturer's and the retailer's profits under these two different games, and then to decide on which game they prefer. Substituting (22)–(24) into (1) and (2), the manufacturer's and the retailer's profits under simultaneous move structure are, respectively,

$$\pi_m^{**} = \rho_m [1 - (\delta\rho_m)^{-\delta/(\delta+\gamma+1)} (\gamma\rho_r)^{-\gamma/(\delta+\gamma+1)}] - (\delta\rho_m)^{(\gamma+1)/(\delta+\gamma+1)} (\gamma\rho_r)^{-\gamma/(\delta+\gamma+1)}, \quad (25)$$

$$\pi_r^{**} = \rho_r [1 - (\delta\rho_m)^{-\delta/(\delta+\gamma+1)} (\gamma\rho_r)^{-\gamma/(\delta+\gamma+1)}] - (\delta\rho_m)^{-\delta/(\delta+\gamma+1)} (\gamma\rho_r)^{(\delta+1)/(\delta+\gamma+1)}. \quad (26)$$

Therefore, expressions (16)–(17) and (25)–(26) imply

$$\begin{aligned} \pi_m^{**} - \pi_m^* &= [\delta + \gamma + 1 - (\delta + 1)(\gamma + 1)^{(\gamma+1)/(\delta+\gamma+1)}] \\ &\quad \times (\gamma + 1)^{-(\gamma+1)/(\delta+\gamma+1)} \rho_m \\ &\quad \times (\delta\rho_m)^{-\delta/(\delta+\gamma+1)} (\gamma\rho_r)^{-\gamma/(\delta+\gamma+1)} < 0, \end{aligned} \quad (27)$$

$$\begin{aligned} \pi_r^{**} - \pi_r^* &= [(\gamma + 1)^{\delta/(\delta+\gamma+1)} - 1](\gamma + 1)\rho_r (\delta\rho_m)^{-\delta/(\delta+\gamma+1)} \\ &\quad \times (\gamma\rho_r)^{-\gamma/(\delta+\gamma+1)} > 0. \end{aligned} \quad (28)$$

That is to say, when $\rho_m/\rho_r < \gamma + 1$, the manufacturer prefers the leader–follower structure rather than the simultaneous move structure. This is consistent with the result under $\rho_m/\rho_r \geq \gamma + 1$. However, the retailer prefers the simultaneous move structure rather than the leader–follower structure. This observation differs from the result under the case

of $\rho_m/\rho_r \geq \gamma + 1$, where the retailer's preference depends on the comparison result of magnitude of $\gamma\rho_r/\rho_m + (\rho_r/\rho_m)^{\delta+1}$ with one (cf. [2, Proposition 2]).

We close this section by summarizing the results for the case of $\rho_m/\rho_r < \gamma + 1$ as the following proposition.

Proposition 2. (1) *Both the manufacturer's and the retailer's advertising investments are always higher at Nash than at Stackelberg.*

(2) *The manufacturer always prefers the leader–follower structure rather than the simultaneous move structure, while the retailer always prefers the simultaneous move structure rather than the leader–follower structure.*

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