

Potentially Self-defeating: Group Buying in a Two-tier Supply Chain

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Abstract: Retailers have an incentive to cooperate in the form of group buying (GB) when a supplier provides quantity discounts, because wholesale price under GB depends on total purchasing quantity rather than individual purchasing (IP) quantity. Most previous studies on GB focus on the benefits that buyers get but ignore the supplier's response to GB. In this paper, we take the supplier's response into consideration, and present a game model with a single supplier and two symmetric competing retailers in two systems: the retailers purchase individually, and the retailers group buy. Under a general quantity discount schedule, each system has a unique sub-game perfect equilibrium. The comparison between IP and GB suggests that GB may sabotage the benefits of all members in the supply chain (i.e., the supplier, the retailers, and the consumer). Retailers may hold contradictory attitudes toward GB before and after the publishing of the discount schedule. These insights are shown to be robust for the case when more than two retailers are involved, as well as the case when the supplier enjoys economies of scale based on the order volume. We suggest that a mixed discount schedule may help prevent the potential damage of GB. In addition, with significant economies of scale, the supplier and retailers may be better off under GB. Then GB can be a favorable purchasing strategy.

Keywords: group buying; quantity discount; economies of scale; price-dependent demand; supply chain management; game theory.

1. Introduction

Retailers have an incentive to cooperate in the form of group buying (GB) when a supplier provides quantity discounts, because wholesale price under GB depends on total purchasing quantity rather than individual purchasing (IP) quantity. GB is commonly observed and widely used in buyer groups, group purchasing organizations, and purchasing consortia. In many European countries, buyer groups account for a significant proportion of sales in food retail distribution [1]. GB is also commonly used by healthcare institutions, schools, and government organizations as well as by small- to medium-sized businesses in other retail industries (e.g., [2]-[6]).

However, there are many controversies related to the influence of GB on both the buyer and seller sides. On the one hand, for the retailers' benefit, a fundamental issue is that GB

may not help to reduce the purchasing cost [7]. For instance, in the United States, group purchasing organizations for healthcare products do not guarantee that hospitals will save money; their “prices were not always lower but often higher than prices paid by hospitals negotiating with vendors directly” [8].

On the other hand, for the suppliers’ benefit, they are under great pressure due to buyer groups’ focus on short-term benefits. For example, many European suppliers concern about their profits when buyer groups are aggregating orders across countries for Europe-wide discounts [1]. Many empirical studies suggest that the existence of buyer groups leads to a reduction in the suppliers’ profit (e.g., [9], [10]). Given these undesirable impacts, GB has potential drawbacks to supply chains.

Previous studies on GB often focus on the benefits received by buyers [11] and do not consider the supplier’s response to GB. In this paper, we think over the supplier’s response and examine how GB affects all members in the supply chain (i.e., the supplier, the retailers, and the consumer). We establish a two-tier supply chain consisting of a single supplier, who dictates a general quantity discount schedule, and two retailers, who compete in the final market. We derive the equilibrium outcomes for IP and GB respectively. Under IP, the retailers purchase individually; and under GB, the retailers group buy.

Based on the comparison between IP and GB, we have the following findings. (1) Without the effect of economies of scale, the supplier is worse off under GB even if he adjusts the discount level to address the aggregation of retailers. (2) For a type of general discount schedules, retailers are likely to suffer losses under GB when they cooperate in a weakly competitive market. GB may become a self-defeating strategy for retailers. (3) The demand quantity is lower under GB for a type of general discount schedules, and thus according to the aggregate consumer utility function, the welfare of consumers is reduced. These insights are shown to be robust for the case with more than two retailers. (4) We suggest that a mixed discount schedule can prevent retailers from choosing to group buy in a weakly competitive market. (5) When the supplier enjoys economies of scale based on the order volume, the supplier probably benefits from GB, while retailers may still be self-defeating. Furthermore, with significant economies of scale (measured by a scale parameter), GB can be a favorable purchase strategy, since the supplier and retailers are better off. The results of this paper are helpful for the managers to understand the potential benefits and drawbacks of GB.

The following is a brief literature review. A large body of theoretical and empirical literature on GB demonstrates that GB can reduce acquisition costs or enhance buying power (e.g., [12]–[17]), but most of these studies do not take the competition among buyers (retailers) into account. Some studies show that GB may influence sellers’ (suppliers’) rivalry or upstream technology choices (e.g., [7], [11], [13], and [18]–[21]). However, these studies usually ignore the primary motive of retailers to form buyer groups, and that is the quantity discount provided by suppliers [1]. In this paper, we regard the supplier as a Stackelberg leader who provides the quantity discount to competing retailers who are able to purchase in group. Keskinocak and Savaşaneril [22] assume the supplier employs an approximately linear quantity discount schedule, and adopt a theoretical approach to deal with group buying among competing buyers. The difference between their model and ours is that we assume the supplier provides a general quantity discount schedule. In addition, our model

incorporates the economies of scale at the supplier's cost and examines his profit, whereas Keskinocak and Savaşaneril [22] think through the supplier's revenue without considering his cost.

In recent years, web-based GB has attracted much theoretical and experimental attention (e.g., [6], [23]-[28]). Anand and Aron [6] develop a theoretical model to show that web-based GB can be a price discovery mechanism in an uncertain market. Chen et al. [25] and their follow-up studies ([26], [27]) analyze the performance of a web-based GB auction model. Chen et al. [28] considers the inventory rationing problem for the seller based on the web-based GB. These studies see models similar to auctions and are based on consumer behavior, which differs from our assumptions. We hold that retailers have no purchasing uncertainty and investigate how GB influences the supply chain.

Our paper is most related to that of Chen and Roma [29]. They consider the competing retailers' choice of group buying under given quantity discount schedules. They find that under GB symmetric competing retailers (i.e., with the same market base and operational cost) always have higher profits, and the supplier also has a chance to be good. In Chen and Roma [29], the supplier offers a quantity discount schedule and keeps the same price for both individual and group purchases. We extend their model and assume the supplier acts as a Stackelberg leader and adjusts the discount level according to retailers' individual or group purchases. Due to the assumption of a different game setting, we draw different conclusions from Chen and Roma's results concerning the effects of GB on the profits of the retailers and the supplier; we suggest that under GB retailers may not always get higher profits and the supplier will always be worse off if there is no economies of scale.

Our study is also bound up with quantity discount. The literature on quantity discount consists of three aspects [29]: price discrimination (e.g., [30]-[33]), channel coordination (e.g., [34]-[38]), and operating efficiency (e.g., [39]-[41]). Some studies focus on the designing of the quantity discount schedule to extract all or some of consumer surplus [22] (e.g., [30]-[33]). Some other studies discuss how quantity discounts address channel coordination under different market conditions (e.g., [34]-[38]) and how to improve the conflict between suppliers and retailers (e.g., [40]-[41]). For more information about the literature on quantity discount, the readers can refer to Dolan [42], Weng [43], and Kanda and Deshmukh [44], where excellent reviews are provided. Similar to Chen and Roma [29], our work is also attached to channel coordination, but we further setup a dynamic game, and examine how GB, which is based on quantity discount, affects all members in the supply chain.

The rest of the paper is organized as follows. Section 2 describes the game model and derives the equilibrium outcomes. Section 3 presents comparisons and discussions. Section 4 provides three extensions: we first of all suggest a mixed discount schedule to prevent retailers from purchasing in group; we then extend the model to a case with more than two retailers; finally considering the benefits of the supplier, we introduce the economies of scale in the model of the supplier's cost, and examine the robustness of the results. Section 5 concludes the paper.

2. Model and analysis

2.1 Model

We consider a two-tier supply chain consisting of a single supplier and two retailers. The supplier sells a single product and provides a quantity discount schedule, as in the power function derived by Schotanus et al. [45]. Specifically, the unit wholesale price $w(q)$ is

$$w(q) = a + \frac{d}{q^e}, \quad de > 0, \quad (1)$$

where q is the purchase quantity, $a \geq 0$ is the base wholesale price, d scales the function, and e is the steepness. Schotanus et al. [45] show that this general discount schedule fits very well with 66 discount schedules found in practice, with e varying from -1.00 to 1.60. In general, the steepness e captures the variation tendency of the wholesale price over the purchase quantity. In particular, with positive steepness (e.g., $e = 1$, two part tariff with $w(q) = a + d/q$), the wholesale price flattens out gradually after a steep fall, while with negative steepness (e.g., $e = -1$, linear quantity discount with $w(q) = a + dq$), the wholesale price decreases persistently with the purchase quantity. In this paper, we assume the steepness e is exogenous.

For tractability, we assume the steepness $e \in [-1, 1]$. Thus, the wholesale price $w(q)$ is convex, and the total cost $w(q)q$ is concave in q . We assume $e \neq 0$ because $e = 0$ is the trivial case of no discount. In addition, under two part tariff (i.e., $e = 1$), the supplier can set the scaling parameter d where retailers have 0 profits, which is a simple case. Thus, we assume $e \neq 1$. Then, the steepness e characterizes the quantity discount schedule into two categories: positive steepness ($0 < e < 1$) and negative steepness ($-1 \leq e < 0$).

In the quantity discount schedule, $de > 0$ is required to ensure that the wholesale price decreases with purchase quantity q ([29], [45]). Then, for positive steepness ($0 < e < 1$), the scaling parameter $d > 0$, and the base wholesale price a represents the theoretical minimum wholesale price (i.e., $q = \infty$). For negative steepness ($-1 \leq e < 0$), the scaling parameter $d < 0$, and a represents a theoretical maximum wholesale price (i.e., $q = 0$). The absolute value of the scaling parameter $|d|$, referred to as the discount level, reflects how quickly the wholesale price decreases with the purchase quantity. The higher the $|d|$, the more effective the demand aggregation by retailers [29]. In practice, suppliers usually adjust the discount level to match variations in the market, such as different times in the product's life cycle, sales promotions, or hot and off-season sales (e.g., [46], [47]). Therefore, given the base wholesale price a (the theoretical minimum or maximum wholesale price), we assume the supplier optimizes the scaling parameter d corresponding to retailers who purchase individually or in group¹. We assume the supplier has a constant operational cost C for each unit product in this section. In the extension, we

¹ The steepness e represents the type of the quantity discount schedule [45]. Theoretically, the supplier can also optimize e and obtain the suitable discount schedule function. However, in practice, the type of the discount schedule may be related to other conditions, such as payment functions. For example, "two part tariff" payment policy involves charging a constant price per unit of purchase and a fixed charge. Compared to the discount level, the schedule type would be much harder to change. Therefore, in this paper, we assume the supplier only optimize the scaling parameter d , while the steepness e is exogenous.

will introduce the economies of scale of the supplier's cost, which is related to the order volume, and examine the robustness of the results.

We assume that retailers are competing with each other and that retailer i 's demand q_i decreases with his own price p_i and increases with his opponent's price p_j . Specifically, we assume

$$q_i = A_i - p_i + \theta \cdot (p_j - p_i), \quad (2)$$

where $i \in \{1, 2\}$, $j = 3 - i$. A_i reflects the retailer i 's market base, which is determined by his location, customer loyalty, brand, or service [29]. $\theta \geq 0$ measures the substitutability between the retailers and embodies the product competition intensity. If θ is zero, there is no competition between retailers. The demand function (2) can be easily derived from the aggregate consumer utility function ([29], [48]), which will be provided in Section 3.3. This demand function is common in economic and marketing literature (e.g., [29], [35], [49]-[50]). We assume that retailer i has operational cost c_i for each unit product. For the reason that homogeneity of the group members is usually an important factor in the formation of a successful purchasing group in practice (e.g., [10], [17], [51]), we assume retailers are symmetric in their market base and operational costs (i.e., $A_1 = A_2 = A$, $c_1 = c_2 = c$).

We model the setting as two Stackelberg games under two systems: IP and GB. First, under each system, the supplier acts as the leader. He optimizes his scaling parameter d and provides the quantity discount schedule. Then, retailers determine their prices according to IP or GB system. To be more specific, we maintain that: (1) Under IP, the retailers purchase individually. Retailer i acts individually in purchasing and determines his retail price p_i . The purchase quantity q_i follows from each demand function. (2) Under GB, the retailers consider to cooperate by combining their orders to obtain a lower wholesale price, and then make their price decisions. Cooperation occurs only if both retailers have higher profits when they purchase together². Otherwise, retailers would still purchase individually. We use backward induction to solve these two games.

2.2 Equilibrium outcomes

In this part, we will solve the equilibrium solutions under both IP and GB. Chen and Roma [29] show that, given the discount schedule, symmetric retailers' profits are always higher when they purchase together. This means that retailers will always choose to cooperate in purchasing under GB, and the supplier will optimize the scaling parameter d based on the total purchase quantity. In contrast, under IP, the supplier will optimize the scaling parameter d based on each retailer's individual purchase quantity.

Let π_i denote retailer i 's profit, and let $Q = q_1 + q_2$ denote the total purchase quantity. Then, the profit functions of retailer i under IP and GB are

$$\text{IP: } \pi_i = (p_i - c - w(q_i))q_i \quad (3)$$

$$\text{GB: } \pi_i = (p_i - c - w(Q))q_i. \quad (4)$$

² We assume GB is initiated from retailers, and there is zero coordination cost under GB. With the opportunity of GB, retailers can always purchase in group if both (all) of them choose to do so. This assumption is consistent with Chen and Roma [29].

Given the general discount schedule, the equilibrium purchase quantities under both systems satisfy the following equations accordingly [29], and closed-form solutions do not exist for general steepness e .

$$\text{IP: } A - a - c = \frac{2+\theta}{1+\theta} q_i + \frac{d}{(q_i)^e} (1 - e), \quad i = 1, 2 \quad (5)$$

$$\text{GB: } A - a - c = \frac{2+\theta}{1+\theta} \frac{Q}{2} + \frac{d}{(Q)^e} \left(1 - \frac{e}{2(1+\theta)}\right) \quad (6)$$

Let Π denote the supplier's profit. His profit function under each system is

$$\text{IP: } \Pi = (w(q_1) - C)q_1 + (w(q_2) - C)q_2 \quad (7)$$

$$\text{GB: } \Pi = (w(Q) - C)Q. \quad (8)$$

Under these two systems, the supplier optimizes the scaling parameter d on conditions that retailers receive positive profits. The equilibrium solutions are listed in Lemma 1. Because the equilibrium outcomes are symmetric, we suppress the subscript “ i ” for each retailer i . We let the subscripts “IP” and “GB” denote the equilibrium outcomes under IP and GB respectively. Proofs of this lemma and all propositions in this paper are provided in the Online Appendix. We impose a number of conditions on the parameters to ensure a reasonable model (e.g., positive prices, retailers' profits, and reasonable scaling parameters). These conditions are explicitly described in Eqns. (A7)–(A11) in the Online Appendix.

Lemma 1. (Equilibrium solutions) There exists a unique optimal scaling parameter d_{IP} (d_{GB}) for the supplier under IP (GB). The equilibrium retail price, purchase quantity, retailer's profit, scaling parameter and supplier's profit under both systems are listed in Table 1.

Table 1. The equilibrium solutions under IP and GB.

Equilibrium Solutions ($\alpha \in \{IP, GB\}$)	
Retail Price	$p_\alpha = A - q_\alpha$
Purchase Quantity	$q_\alpha = \frac{Q_\alpha}{2} = \frac{(1+\theta)}{2(2+\theta)}(X + t_\alpha Y)$
Retailer's Profit	$\pi_\alpha = \frac{(1+\theta)}{4(2+\theta)^2 t_\alpha} (X + t_\alpha Y)((3t_\alpha + \theta t_\alpha - 2 - \theta)X + t_\alpha(2 + \theta - t_\alpha - \theta t_\alpha)Y)$
Scaling Parameter	$d_{IP} = \frac{(q_{IP})^e}{2t_{IP}}(X - t_{IP}Y) \quad d_{GB} = \frac{(Q_{GB})^e}{2t_{GB}}(X - t_{GB}Y)$
Supplier's Profit	$\Pi_\alpha = \frac{(1+\theta)}{2(2+\theta)t_\alpha}(X + t_\alpha Y)^2$

$$t_{IP} = 1 - e, \quad t_{GB} = 1 - \frac{e}{2(1+\theta)}, \quad X = A - a - c, \quad Y = a - C$$

For both retailers, a larger market base A , a lower base wholesale price a or a lower operational cost c is always beneficial [29]. For the supplier, a higher base wholesale price a or a lower supplier operational cost C is always beneficial. Therefore, we assume that the indicator $X = A - a - c$ reflects the operating environment for retailers [29], and the indicator $Y = a - C$ reflects the operational condition for the supplier. Both X and Y remain positive throughout the paper.

3. Comparison

In this section, we compare the equilibrium outcomes between IP and GB. The comparison suggests that GB is detrimental to the supplier's profit. Furthermore, GB may be a self-defeating strategy for retailers. The consumer's welfare can be lower under GB as well because the demand quantity is lower.

3.1 Supplier's profit

The following is a discussion of the discount level $|d|$ and the supplier's profit Π . We have talked about the discount level $|d|$, and it reflects the effect of the purchase quantity on the wholesale price. We assume the incremental discount level of GB to be $\Delta|d| = |d_{GB}| - |d_{IP}|$ and the incremental supplier profit of GB to be $\Delta\Pi = \Pi_{GB} - \Pi_{IP}$. Proposition 1 describes the performance of $\Delta|d|$ and $\Delta\Pi$.

Proposition 1. (Discount level and supplier's profit)

(a) *Discount level:*

- (i) *For negative steepness ($-1 \leq e < 0$), $\Delta|d| < 0$.*
- (ii) *For positive steepness ($0 < e < 1$), when $f(\theta, e) < 0$, there exists $\hat{Y}$ such that for $Y < \hat{Y}$, $\Delta|d| > 0$; when $f(\theta, e) \geq 0$, $\Delta|d| \leq 0$. Here, $f(\theta, e) = 1 - e/[2(1 + \theta)] - 2^e(1 - e)$.*

(b) *Supplier's profit: $\Delta\Pi < 0$.*

Proposition 1(a) suggests that, under GB, the supplier adjusts the discount level according to the sign of the steepness e . Specifically, for negative steepness ($-1 \leq e < 0$), the supplier reduces the discount level, so the wholesale price would be less affected by the purchase quantity. In contrast, for positive steepness ($0 < e < 1$), the supplier may increase the discount level when $f(\theta, e) < 0$. The formula $f(\theta, e)$ reflects the relationship between the aggregation (i.e., the multiplier "2" of $2q_{GB}$) and other parameters (i.e., θ and e). When $f(\theta, e) < 0$, other parameters can counteract the effect of aggregation, which means the effect of the total purchase quantity on the wholesale price under GB would be similar with the effect of the individual purchase quantity under IP. In this circumstance, it is not necessary for the supplier to reduce the discount level to address the aggregation between retailers. Thus, the discount level under GB is higher if q_{GB} is similar to q_{IP} , which occurs when Y is close to 0 (i.e., $Y < \hat{Y}$). However, as Y becomes larger (i.e., $Y > \hat{Y}$), one can find that q_{GB} will be much higher than q_{IP} for positive steepness e . The higher purchase quantity q_{GB} would induce the supplier to reduce the discount level under GB, even without the effect of aggregation.

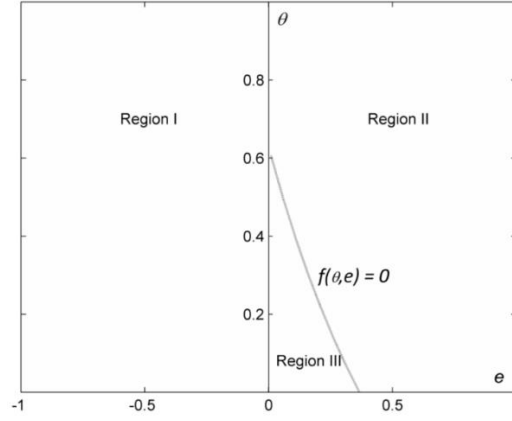


Fig. 1. Region of the direction of the discount level adjustment

We solve $f(\theta, e) = 0$ numerically to characterize the region of the direction for the discount level adjustment. As shown in Fig. 1, the region is divided by the vertical axis and the dashed curve where $f(\theta, e) = 0$ into three regions: Region I and Region II, where the supplier reduces the discount level under GB for negative and positive steepness, and Region III, where the supplier may increase the discount level under GB in a poor operational condition (e.g., low base wholesale price or high operational cost).

Furthermore, Proposition 1(b) suggests that the supplier has less profit under GB. This is because the adjustment of discount level sacrifices either the wholesale price or the purchase quantity, which eventually leads to the profit loss. This conclusion is different from Chen and Roma [29]. Their results hold that given the same linear quantity discount schedule ($e = -1$) under IP and GB, it is quite possible that the purchase quantity is higher under GB, which will overcome the lower wholesale price, and the supplier's profit then will be higher under GB. As shown in Section 3.3, when the supplier optimizes the discount level according to IP and GB, the purchase quantity will always be lower under GB for linear quantity discount schedule. Then the situation mentioned in Chen and Roma's model would not occur in our model.

We now focus on the case if the supplier neglects GB (referred to as the negligent supplier), and consider the supplier's profit loss on the conditions that he fails to provide the optimal discount level when the retailers purchase in group. This will help us understand the necessity of optimizing the discount level promptly according to individual or group purchasing, because the negligent supplier will suffer a great loss in profit. We assume that the negligent supplier still adopts the scaling parameter d_{IP} when retailers employ group buy, and we denote his profit as $\Pi_{GB}(d_{IP})$. In contrast, if the supplier optimizes the discount level under GB, his profit is denoted as $\Pi_{GB}(d_{GB})$, which is listed in Table 1. We examine the profit loss proportion

$$\Delta \Pi_{d_{IP}}^{d_{GB}} = \frac{\Pi_{GB}(d_{GB}) - \Pi_{GB}(d_{IP})}{\Pi_{GB}(d_{GB})}. \quad (9)$$

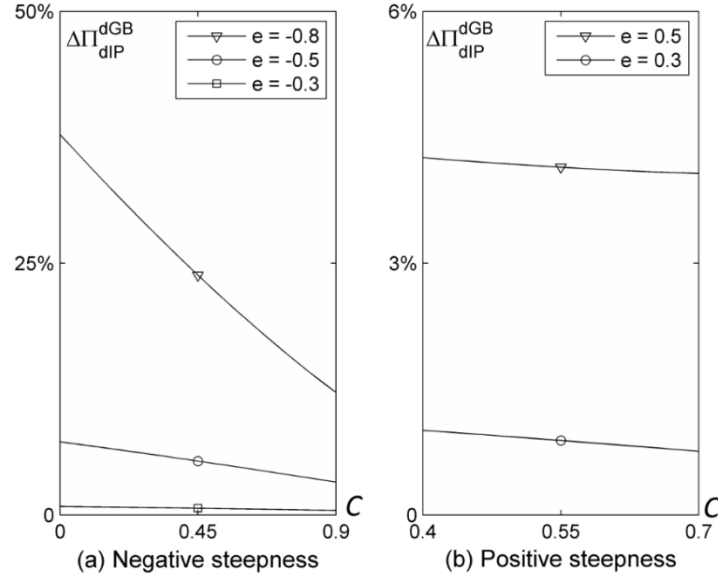


Fig. 2. Negligent supplier's profit comparison

A closed-form solution does not exist for Eqn. (6) if the scaling parameter d is not optimal. Therefore, we calculate the profit loss proportion numerically. Figs. 2(a) and 2(b) illustrate some typical examples of the profit loss associated with the supplier's operational cost C , in which $\theta = 0.5$, $A = 3.1$, $\alpha = 2$, and $c = 0.01$ for negative steepness in Fig. 2(a), and $\theta = 0.2$, $A = 2.5$, $\alpha = 1.5$, and $c = 0.01$ for positive steepness in Fig. 2(b). Note that a lower C indicates a better operational condition for the supplier. Our results show that the negligent supplier would face considerable loss, especially when he provides schedules with steepness that is not close to 0. Furthermore, we obtain that for negative steepness, the gap between d_{IP} and d_{GB} increases with higher Y . Thus, a better operational condition for the negligent supplier (e.g., higher base wholesale price or lower operational cost) may lead to a greater profit loss.

3.2 Retailers' profits

For a given discount level $|d_0|$, symmetric retailers always receive higher profits under GB [29], which is $\pi_{GB}(d_0) \geq \pi_{IP}(d_0)$ (here, we loosely adopt the subscripts "IP" and "GB" to distinguish the retailers' profits when they group buy and individual purchase, and we use the bracket to emphasize the discount level which affects the retailer's profit). However, if d_{IP} is the optimal discount level for the supplier under IP, while d_{GB} is optimal for the supplier under GB, it is possible that $\pi_{GB}(d_{GB}) < \pi_{IP}(d_{IP})$. This means GB hurt the retailers. We summarize the comparison of the retailers' profits between IP and GB in Proposition 2 and denote the incremental retailer profit of GB as $\Delta\pi = \pi_{GB} - \pi_{IP}$; π_{GB} and π_{IP} are listed in Table 1.

Proposition 2. (Retailers' profits)

- (a) For negative steepness ($-1 \leq e < 0$), when $\theta < \hat{\theta}$, then for $X > \hat{k}Y$, $\Delta\pi < 0$; otherwise, $\Delta\pi \geq 0$. Here, $\hat{\theta} = -(8 + 2e - 5e^2 + \sqrt{64 - 96e + 68e^2 - 44e^3 + 17e^4})/[4e \cdot (2 - e)]$, $\hat{k} = (\sqrt{t_{IP}t_{GB}(t_{IP}t_{GB} + (\theta + 2)(\theta + 1)(t_{IP} + t_{GB}) - (\theta + 2)^2)} - t_{IP}t_{GB})/(2 + \theta)$.

(b) For positive steepness ($0 < e < 1$), $\Delta\pi > 0$.

Proposition 2 suggests that for a positive steepness schedule, retailers always have higher profits under GB. For a negative steepness schedule, retailers also benefit from GB if they are in a relatively competitive market, or if they are in a weakly competitive market with poor operating environments (e.g., low market base, high base wholesale price or high operational cost). This result is consistent with the significant development of buyer groups for small retailers in food industries in the European Union. Some small retailers in a fierce market can better participate in business by joining buyer groups than the small independent buyers [1]. In the United States and many other countries, GB also contributes to the survival of retailers in small markets [52].

However, Proposition 2(a) suggests that retailers do not always benefit from their cooperation. For negative steepness, retailers under weak competition may have lower profits under GB when they operate in a relatively good environment (e.g., high market base, low base wholesale price or low operational cost). Intuitively, GB avails a deeper discount, which induces a better profit margin for retailers. However, as operating environment gets better, each retailer's demand increases accordingly. Then the supplier would reduce the discount level to weaken the impact of the increasing demand on the wholesale price. The total quantity under GB would furthermore stimulate a deeper adjustment (Proposition 1(a. i)). This means that the wholesale price will not reduce much under GB. Then the advantage of the retailer's profit margin under GB would dissipate gradually, which eventually leads to lower retailer's profits. This result implies that GB is potentially a self-defeating strategy for retailers, which extends the result in Chen and Roma [29]. According to Chen and Roma [29], given the discount schedule, symmetric retailers are always better off under GB, and they lack the ability to commit to non-group purchase. This inability to commit results in an adjusted discount level and ultimately lower profits for the retailers.

Furthermore, one can obtain $\partial \hat{k} / \partial \theta > 0$ and $\partial t_{GB} / \partial \theta < 0$ for negative steepness, and this means that the weaker the competition intensity among retailers (i.e., smaller θ), the larger the interval of lower profits for retailers under GB. This is because weaker competition intensity induces retailers to purchase more, which leads to a lower discount level, and this impact will be reinforced under GB. Therefore, GB is more likely to be detrimental to retailers in a weakly competitive market (i.e., small θ). To illustrate this, we present some numerical examples about the retailer's relative profit difference, $\Delta\pi_{IP}^{GB} = (\pi_{GB} - \pi_{IP}) / \pi_{GB}$. Given several different competition intensities, Fig. 3 depicts the profit difference associated with the base wholesale price a , in which $e = -0.3$, $A = 3$, $c = 0.1$, and $C = 0.1$. Note that a lower base wholesale price a implies a better operating environment for retailers.

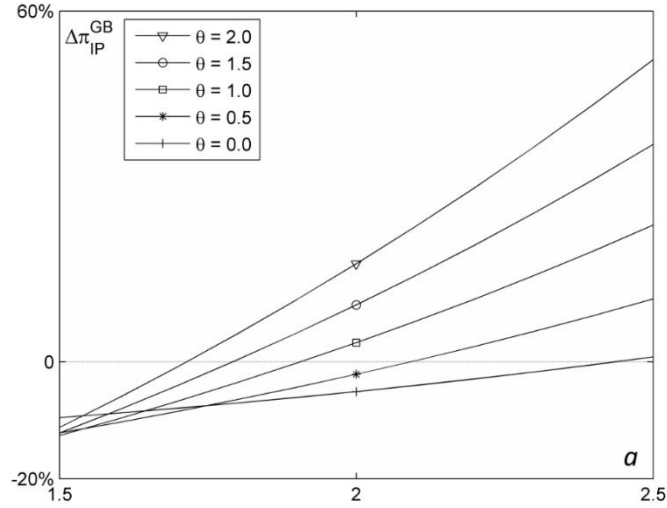


Fig. 3. Retailer's profit comparison

This insight about competition is similar to that of Chen and Roma [29]. They provide two examples of buyer groups who have requirements to limit the competition intensity among members: DPA, a buyer group for cleaning supplies; and Furniture First, a furniture buyer group in the United States. The members in these two groups are geographically exclusive by representation. Our finding, as well as that of Chen and Roma [29] suggests that competition may not be a concern under GB. Retailers in a more competitive market are more likely to have higher profits when they purchase in group.

3.3 Consumers' welfare

In this part, we will discuss the consumers' welfare. We use the value of the aggregate consumer utility function as the welfare of the consumer, denoted as W . The function is defined as follows ([48]):

$$W = A \cdot (q_1 + q_2) - \frac{1}{4}(q_1 + q_2)^2 - \frac{1}{4(1+2\theta)}(q_1 - q_2)^2 - p_1 q_1 - p_2 q_2,$$

where the parameters above have the same meaning as those in the previous section. Note that the demand Eqn. (2) can be derived from this aggregate utility function [48]. For both IP and GB, by substituting the equilibrium demand quantities and rearranging terms, we can obtain the welfare of the consumers in equilibrium:

$$W_\alpha = A \cdot (Q_\alpha) - \frac{1}{4}(Q_\alpha)^2 - \frac{1}{4(1+2\theta)}\left(\frac{Q_\alpha}{2} - \frac{Q_\alpha}{2}\right)^2 - 2\left(A - \frac{Q_\alpha}{2}\right)\frac{Q_\alpha}{2} = \left(\frac{Q_\alpha}{2}\right)^2 \quad \alpha \in \{IP, GB\}. \quad (10)$$

Then, if the total demand increases, the consumer's welfare increases accordingly. We assume the incremental consumer's welfare of GB as $\Delta W = W_{GB} - W_{IP}$ and the incremental total demand quantity of GB as $\Delta Q = Q_{GB} - Q_{IP}$. It is easy to obtain from Table 1 that $\Delta Q = (1 + \theta)(t_{GB} - t_{IP})Y/(2 + \theta)$. Then, since $e \cdot (t_{GB} - t_{IP}) = e^2 \cdot (1 - 1/[2(1 + \theta)]) > 0$, we have $e \cdot \Delta Q > 0$. This means that for positive (or negative) steepness, the total demand quantity is higher (or lower) under GB. Thus, the welfare of consumers increases (or decreases) under GB for positive (or negative) steepness (i.e., $e \cdot \Delta W > 0$). Furthermore, the competition intensity affects the welfare of consumers. A less competitive market induces retailers to set higher prices, so the total demand is lower

and consumers' welfare decreases accordingly. In the next section, we will provide numerical examples to illustrate the impact of GB on consumers' welfare, as well as on the supplier's profit and retailers' profits.

3.4 Discussion

For the supplier and the retailers under GB, two effects influence their benefits: the Group Buying Effect (GB Effect) through retailers' cooperation and the Supplier Decision Effect (SD Effect) through the adjustment of the discount level. The GB Effect is beneficial for retailers and detrimental to the supplier. The SD effect helps the supplier limit damage and cut retailers' profits. The GB Effect stems from retailers' inability to commit not to group purchase, and SD Effect can be considered as a "protection of stress".

These two effects counteract each other and lead to profit differences between IP and GB. In particular, for the negative steepness quantity discount, if retailers operate in a good environments and a weakly competitive market (i.e., $X > \hat{k}Y$ and $\theta < \hat{\theta}$ in Proposition 2), the SD Effect will be strong enough to outweigh the GB Effect, and retailers will be worse off under GB. In this situation, retailers may hold contradictory attitudes toward GB before and after the providing of the discount schedule. To be more specific, before the discount schedule is provided, retailers may prefer none GB. However, when the discount schedule is provided, retailers will always prefer GB. Under this contradiction, the retailers thought they have benefited from GB, but the fact is they have not, and even get less profits.

In terms of all members' benefits (i.e., the supplier's profit, the retailers' profits, and the consumer's welfare), the impact of GB changes with the sign of the steepness e . For positive steepness, the retailers' profits and the consumer's welfare are higher under GB, whereas the supplier's profit is lower. A GB opportunity increases downstream welfare (i.e., retailers and consumers). For negative steepness, the supplier and consumers are worse off under GB, whereas retailers' profits may be higher or lower due to their operational environments (indicated by market base, operational cost and base wholesale price). This is not an intuitive result for GB induced by quantity discounts, suggesting that a GB opportunity potentially sabotages all members' benefits.

We provide representative examples to illustrate the potential detriments of GB. Fig. 4 depicts the relative benefit differences under GB and IP of the supplier, retailers, consumers, and the gross welfare of the supply chain (i.e., $(\Pi_{GB} - \Pi_{IP})/\Pi_{GB}$, $(\pi_{GB} - \pi_{IP})/\pi_{GB}$, $(W_{GB} - W_{IP})/W_{GB}$, and $[(\Pi_{GB} + 2\pi_{GB} + W_{GB}) - (\Pi_{IP} + 2\pi_{IP} + W_{IP})]/(\Pi_{GB} + 2\pi_{GB} + W_{GB})$, respectively). Figs. 4(a) and 4(b) show the differences associated with the competition intensity θ and the market base A respectively, in which $e = -0.6$, $a = 0.7$, $c = 0.1$, and $C = 0.1$ for both examples, $A = 1.2$ for Fig. 4(a) and $\theta = 0.3$ for Fig. 4(b). Note that the larger A is, the better is the operating environments for retailers. Both vertical axes measure the benefit differences.

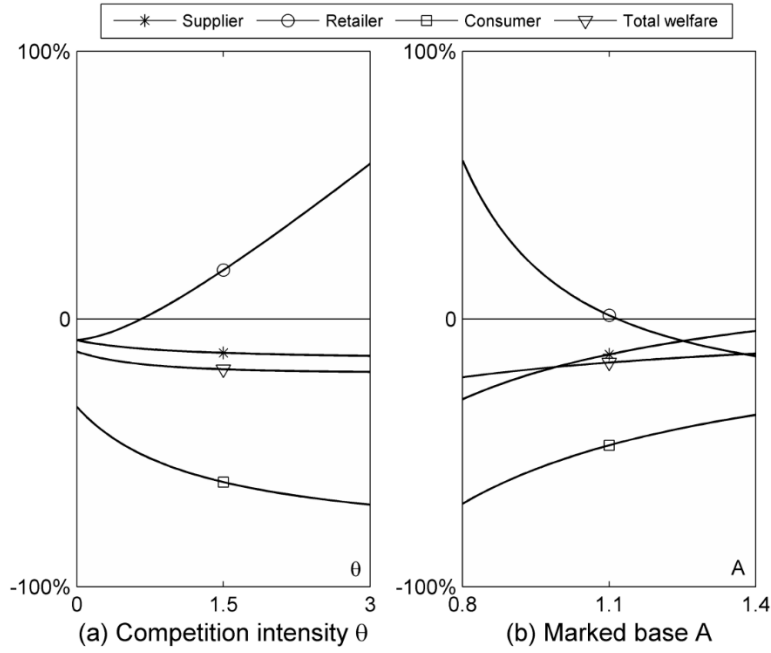


Fig. 4. Potential detriments of GB

An intuitive implication from these examples is that a GB opportunity may not add value to the entire supply chain even when retailers benefit from GB. For buyer managers who see GB as an option, it is worth assessing whether GB is a suitable strategy for both their own profits and the total welfare of the supply chain. GB could be advantageous from the perspective of buyer managers but detrimental on a large scale.

In contrast, for seller managers, they are not only able to adjust the discount level promptly according to GB, but also improve their operating conditions (e.g., to reduce operational cost) to limit the damage from GB. In the extension, we suggest that even under GB, a mixed discount schedule can make retailers choose individual purchase, and this helps protect the supplier from damage. In addition, we show that seller managers can benefit from GB by exploring the effect of economies of scale based on the order volume.

4. Extension

4.1 Mixed discount schedule

In this section, we suggest a mixed discount schedule could prevent retailers from choosing to purchase in group.

In Eqn. (1), the unit wholesale price $w(q)$ continuously decreases with the purchasing quantity for an infinite interval. However, in practice, a supplier usually provides the schedule with a maximum quantity limit beyond which the wholesale price would not decrease anymore (e.g., [45]). Therefore, we employ a mixed discount schedule consisting of the quantity discount schedule and the fixed wholesale price. Specifically, the wholesale price is denoted as $\tilde{w}(q)$ with the following form:

$$\tilde{w}(q) = \begin{cases} a + \frac{d_{IP}}{q^e} & \text{if } q \leq q_{IP} \\ a + \delta & \text{if } q > q_{IP} \end{cases} \quad (11)$$

where the base wholesale price a and the purchase quantity q are the same as the original discount schedule in Eqn. (1). d_{IP} and q_{IP} are shown in Table 1. δ is part of the fixed wholesale price and is assumed to be chosen by the supplier. It is easy to determine that when the order quantity is smaller than q_{IP} , the wholesale price is the same as that in Eqn. (1). When the order quantity exceeds q_{IP} , the wholesale price remains at the fixed wholesale price $a + \delta$. To evaluate the validity of this schedule, we need to check the existence of δ such that the retailers' profits are higher under IP.

We denote the retailer's profit under GB as $\tilde{\pi}_{GB}$ in this extension and the incremental profit of GB for each retailer as $\Delta\tilde{\pi} = \tilde{\pi}_{GB} - \pi_{IP}$. By solving the optimal problems for retailers and comparing the profits, we obtain the validation of this schedule as shown in proposition 3. The proposition suggests that in a weakly competitive market (i.e., small θ), the supplier could adopt this mixed discount schedule to prevent retailers from choosing group buying. The conditions of the parameters that ensure non-negativity of variables in this section are described in Eqns. (A8)–(A11) and (A20)–(A22) in the Online Appendix.

Proposition 3. (Mixed discount schedule)

If $\max\{-1, \underline{e}\} \leq e < \bar{e}$, $e \neq \underline{e}$, $e \neq 0$, and $\theta < \tilde{\theta}$, there exists $\delta \in (\underline{\delta}, \bar{\delta})$, such that

$$\Delta\tilde{\pi} < 0, \quad \text{where} \quad \underline{e} = \frac{-\sqrt{121X^2+16XY+64Y^2}-(11X-2Y)}{10Y}, \quad \bar{e} = \frac{\sqrt{121X^2+16XY+64Y^2}-(11X-2Y)}{10Y},$$

$$\tilde{\theta} = \frac{(3-11e)X+(1-e)(3+5e)Y}{[4e \cdot (X-(1-e)Y)]}, \quad \underline{\delta} = X - \sqrt{\frac{(2+\theta)^2 \pi_{IP}}{(1+\theta)}}, \quad \bar{\delta} = \frac{3X-(1-e)Y}{4}; \quad \pi_{IP} \text{ is listed in Table 1.}$$

We provide two typical examples of this mixed quantity discount schedule in Fig. 5: one schedule with negative steepness where $e = -0.2$, $a = 0.6$, and one schedule with positive steepness where $e = 0.2$, $a = 0.5$. For both examples, $A = 1$, $c = 0.1$, $C = 0.1$, and $\theta = 0.5$. We set the parameter δ equal to the lower limit $\underline{\delta}$ as shown in Proposition 3. These examples reveal that the fixed price $a + \delta$ is not the minimum wholesale price in the discount schedule. There would be a break point in this mixed discount schedule. This result implies that in order to prevent retailers from choosing group buying, the supplier could set the fixed price a bit higher than the equilibrium wholesale price under IP. This mixed discount schedule helps prevent the potential damage of GB.

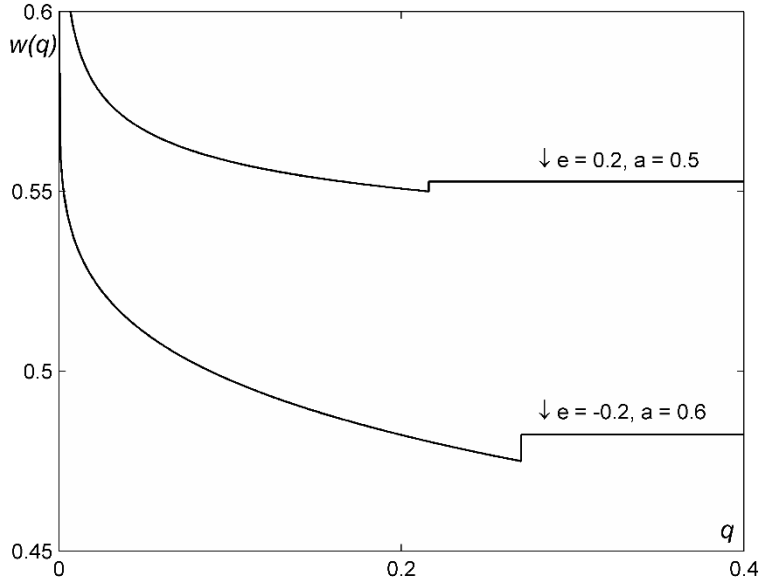


Fig. 5. Two mixed discount schedules

4.2 More than two retailers

We now extend the two-retailer model (2-retailer case) to the case with n retailers (n -retailer case, $n \geq 2$) and examine how the results in Section 3 are influenced by the number of retailers. Similar to the practice in Section 2, we derive the retailer i 's demand function from Shapley and Shubik [48] as

$$q_i = A_i - p_i + \theta \cdot \left(\frac{1}{n-1} \sum_{j=1, j \neq i}^n p_j - p_i \right), i = 1, 2, \dots, n. \quad (12)$$

This setting is similar with the extension of Chen and Roma [29], who adopt numerical investigation for a special case of three retailers, including two symmetric retailers with small market bases and one retailer with a big market base. Different from their model, we theoretically employ a Stackelberg game and focus on the influence of the number of the retailers. We still assume that the retailers are symmetric in the market base and the operational cost. With the same notation as in the previous section, we further denote the subscript “ n ” for the n -retailer case (the first subscript in the double-subscript notation in equilibrium solutions if necessary). Given the discount schedule, we then reach the equilibrium of the individual purchase quantity under IP satisfies the same condition as that in Eqn. (4), and the total equilibrium quantity Q_n under GB satisfies the following in Eqn. (13):

$$A - a - c = \frac{2+\theta}{1+\theta} \frac{Q_n}{n} + \frac{d}{Q_n^e} \left(1 - \frac{e}{n \cdot (1+\theta)} \right), \quad Q_n = \sum_{i=1}^n q_i. \quad (13)$$

Then, similar to Lemma 1, we can determine that under IP, the equilibrium solutions of the n -retailer case are the same as the solutions in Table 1, except that the supplier receives n pieces of profit instead of two pieces. Under GB, the equilibrium solutions in the n -retailer case are similar to Table 1, but the term $t_{GB} = 1 - e/[2(1+\theta)]$ is replaced with $t_{n,GB} = 1 - e/[n \cdot (1+\theta)]$. We omit the equilibrium outcomes here. Proposition 4 includes the comparison of the equilibrium outcomes, and the influence of the number of retailers.

The conditions of the parameters that ensure non-negativity of variables in this section are described in Eqns. (A8)–(A11) and (A23) in the Online Appendix.

Proposition 4. *(Comparison of the equilibrium solutions in the n-retailer case)*

(a) *Supplier's profit:*

(i) $\Delta \Pi_n < 0$.

(ii) $\frac{\partial(\Pi_{n,GB})}{\partial n} > 0$, $\frac{\partial(\Pi_{n,GB}/n)}{\partial n} < 0$.

(b) *Retailers' profits:*

(i) When $-1 \leq e < 0$, if $\theta < \hat{\theta}_n$, then for $X > \hat{k}_n Y$, $\Delta \pi_n < 0$; otherwise, $\Delta \pi_n \geq 0$.

Here, $\hat{\theta}_n = -\left(4n + ne - (2n + 1)e^2 + \sqrt{16n^2 - 24n^2e + (9n^2 + 16n)e^2 - 22ne^3 + (8n + 1)e^4}\right) / [2ne \cdot (2 - e)]$,

$\hat{k}_n = \left(\sqrt{t_{IP}t_{n,GB}(t_{IP}t_{n,GB} + (\theta + 2)(\theta + 1)(t_{IP} + t_{n,GB}) - (\theta + 2)^2) - t_{IP}t_{n,GB}} \right) / (2 + \theta)$.

(ii) If $-1 \leq e < 0$, $\frac{\partial(\hat{\theta}_n)}{\partial n} > 0$, $\frac{\partial(\hat{k}_n)}{\partial n} < 0$.

(c) *Demand quantity:*

(i) $e \cdot \Delta Q_n > 0$.

(ii) $\frac{\partial(Q_{n,GB})}{\partial n} > 0$, $e \cdot \frac{\partial(Q_{n,GB}/n)}{\partial n} > 0$.

As the results shown in part (a. i), (b. i), and (c. i), the main results in the 2-retailer case remain valid as the number of competing retailers increases. In particular, for negative steepness, retailers may still be self-defeating, and all members of the supply chain may be worse off (i.e., the supplier, retailers, and consumers). In addition, part (a. ii) of Proposition 4 suggests that, as the number of retailers increases, the supplier's total profit under GB is increasing, whereas his average profit from each retailer is decreasing accordingly. Moreover, part (b. ii) implies that the region for lower retailer's profit under GB may expand along with the increasing number of retailers (i.e., increased upper bound of the competition intensity θ). Therefore, buyer groups with a large number of members should pay more attention to the risks of the potential damage of GB.

4.3 Economies of scale based on the order volume

The effect of economies of scale refers to the reduction in unit cost as production scale expands. Besides the market reasons, the effect of economies of scale is one of the most important reasons for suppliers to provide quantity discounts in practice [53]. In this part, we extend economies of scale in the model of the supplier's cost and test the robustness of the results of the previous sections.

Recalling the result in Proposition 1(b), without the effect of economies of scale, the supplier's profit is always lower under GB. Then, will the supplier benefit from GB if there are economies of scale? According to the empirical research carried out by Dobson etc. [1], suppliers enjoy significant economies of scale associated with logistic or handling costs when retailers combine their orders. This means that, instead of small and fragmented orders under IP, the aggregating orders under GB will bring cost advantage for suppliers. Therefore, taking this observation into consideration, we assume the supplier's unit cost is related to

the order volume, denoted as $C(q)$. To be more specific, we assume that under IP retailers submit their orders q_1 and q_2 separately, and the supplier's unit costs are $C(q_1)$ and $C(q_2)$ respectively; while under GB, retailers submit one single order $Q = q_1 + q_2$, and the supplier's unit costs are $C(Q)$. In other words, if the total purchase quantity Q under GB is larger than the individual purchase quantity q_1 (or q_2) under IP, the supplier's unit cost under GB would be lower compared to the unit cost under IP.

Leaving all other assumptions unchanged, we introduce a generalized exponential function to capture the economies of scale. We assume that the supplier's unit cost $C(q)$ satisfies the following function,

$$C(q) = C_0 q^{-\eta} \quad (14)$$

where, C_0 is a constant, representing the basic cost for the supplier; q is the volume of the order; $\eta > 0$, is the scale factor and captures the significance of economies of scale. It is easy to see that, when $\eta = 0$, there are no economies of scale for the supplier. The above unit cost function or the form of total cost function is common in empirical and theoretical literature (e.g., [54]-[56]). Then, under IP, the supplier's unit cost is $C_0 q_i^{-\eta}$, $i = 1, 2$; while under GB, the supplier's unit cost is $C_0 Q^{-\eta}$, where Q is the total purchase quantity.

Adopting the same procedure to derive the equilibrium solutions as in the previous section, we get it through that the equilibrium of purchasing quantities under GB and IP satisfy following functions respectively,

$$\text{IP: } X + t_{IP} \cdot a = 2 \cdot \frac{2+\theta}{1+\theta} \cdot \widetilde{q}_{IP} + C_0(1-\eta)t_{IP} \cdot (\widetilde{q}_{IP})^{-\eta} \quad (15)$$

$$\text{GB: } X + t_{GB} \cdot a = \frac{2+\theta}{1+\theta} \cdot \widetilde{Q}_{GB} + C_0(1-\eta)t_{GB} \cdot (\widetilde{Q}_{GB})^{-\eta} \quad (16)$$

where, $\widetilde{q}_{IP}$ is the equilibrium individual purchasing quantity under IP (we suppress the subscript "i" for each retailer i since the equilibrium outcomes are symmetric); $\widetilde{Q}_{GB}$ is the equilibrium total purchasing quantity under GB; X , t_{IP} , and t_{GB} means the same as in the previous section.

Since closed-form solutions do not exist for general scale factor η , we analyze the equilibriums numerically. Firstly, given the scale factor η , as well as the parameters A , a , c , θ , e , and C_0 , we can compute the numerical results of $\widetilde{q}_{IP}$ and $\widetilde{Q}_{GB}$ by Eqns. (15) and (16). Secondly, by substituting $\widetilde{q}_{IP}$ and $\widetilde{Q}_{GB}$ into the first-order conditions for the retailers under both IP and GB, we can get the equilibrium discount levels and corresponding wholesale prices. We then calculate the supplier's and the retailers' profits accordingly by Eqns. (3), (4), (7), and (8). The numerical study is carried out with a wide range of parameters by normalizing A to 1 and c to 0.1, and varying a between 0.2 and 0.8, e between -1 and 1. The parameters are restricted within the range that ensures positive price, wholesale price, quantity, profit, and reasonable discount level. We focus on the retailers' and supplier's profit differences, $\Delta\pi = \pi_{GB} - \pi_{IP}$ and $\Delta\Pi = \Pi_{GB} - \Pi_{IP}$.

The following figure illustrates a typical numerical example of the profit differences, in which $\theta = 0.5$, $e = -0.5$, $C_0 = 0.1$. The vertical axis represents the scale factor η and the horizontal axis represents the base wholesale price a , noting that a smaller a suggests a better operating environment for retailers. There are altogether four regions in this figure: Region I, the retailers are worse off and the supplier is better off under GB; Region II, both the retailers and the supplier are better off; Region III, both the retailers and the supplier are

worse off; Region IV, the retailers are better off and the supplier is worse off. This figure suggests that, GB is still a potentially self-defeating strategy for retailers when they operate in a relatively good environment (small α , Region I and Region III). This finding is consistent with that in the previous section. In addition, attentions need to be paid to Region I and Region II, because in these two regions the supplier benefits from GB with significant economies of scale (η is not close to 0).

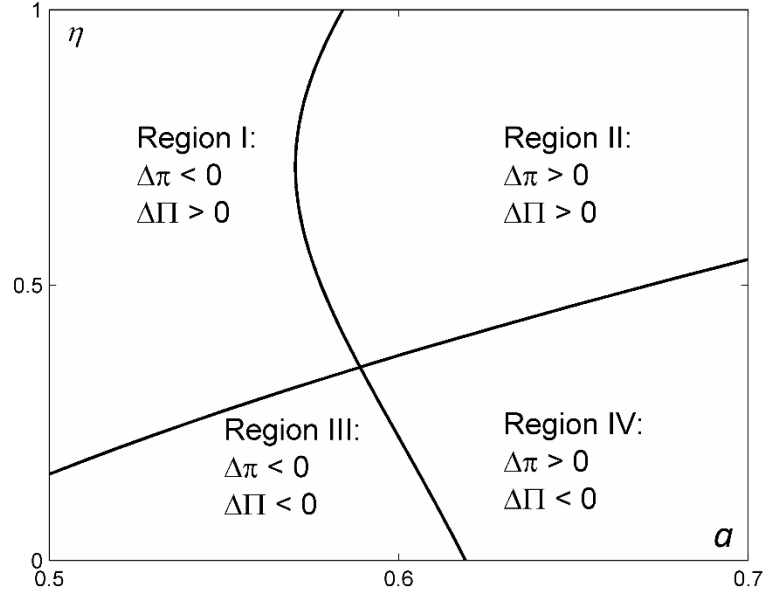


Fig. 6. Profit differences when there are economies of scale for the supplier

For a better understanding, we take a special case into consideration and in this case the scale factor is $\eta = 1$. When $\eta = 1$, the supplier's unit cost under IP is $C_0/q_i, i = 1, 2$, while the unit cost under GB is C_0/Q , where Q is the total purchase quantity. After rearranging the term C_0 , one can find that $\Pi_{IP} = \sum_{i=1}^2 w(q_i) \cdot q_i - 2C_0$ and $\Pi_{GB} = w(Q) \cdot Q - C_0$ (here, we loosely adopt the subscripts "IP" and "GB" to distinguish these two systems). Then C_0 does not influence the supplier's optimal decision under either IP or GB. Since under IP the supplier's total cost is $2C_0$, while under GB the supplier's total cost is C_0 , and the supplier will be better off under GB if C_0 is large enough.

In this special case, the situation amounts to that the supplier needs to pay a fixed cost for each order. This fixed cost, for instance, can be the logistic cost of supplement which is arranged by the supplier. Instead of delivering products to fulfill each retailer's order under IP, the supplier only needs to deliver once under GB. In fact, according to the empirical research from Dobson etc. [1], when retailers choose group buying, delivery in bulk is the key factor for economies of scale at suppliers. Then, the supplier can benefit from GB if the logistic cost is large enough.

In addition, our analysis suggests that, GB may lead to a Pareto improvement for both the supplier and retailers (i.e., Region II). Therefore, in a market with some small independent retailers and a supplier who enjoys significant logistic or handling economies, the introduction of group buying is a good policy, for it benefits the supply chain.

5. Conclusion

Group buying (GB) is widely used in many industries. Despite the fact that many previous studies about GB often focused on the influence of GB on one side of the supply chain, this paper focuses on a dynamic game and shows that, depending on different operational conditions, GB can be either detrimental or beneficial to the supply chain.

On the one hand, GB can hurt the supply chain. Given the quantity discount schedule, symmetric retailers who purchase together always earn more profits than those who do not. Thus, they always prefer purchasing in group [29]. A rational supplier would recognize the drawback of this cooperation and adjust the discount level accordingly. We show that, without the effect of economies of scale, the supplier's profit remains lower under GB. In addition, under negative steepness discount schedules, it is quite possible for the retailers to be worse off when they cooperate in a weakly competitive market, and the consumers' welfare is hurt by GB due to less supply in the market. Thus, under negative steepness discount schedules, GB potentially sabotages all members' benefits (i.e., the supplier, the retailers, and the consumer). Schotanus et al. [45] hold that 60% of the discount schedules collected in practice fit well with negative steepness; therefore, the potential harm of GB should be taken seriously. On the other hand, GB can be a favorable purchasing strategy. Our result implies that, when retailers are in a market with poor operating environments (e.g., low market base, high base wholesale price or high operational cost), and when the supplier enjoys significant logistic or handling economies, GB increases the retailers' and supplier's profits.

An interesting implication is that retailers probably hold contradictory attitudes toward GB before and after the supplier's publishing of the quantity discount schedule. This contradiction stems from the retailer's inability to commit to non-group buy and the supplier's adjustment for discount level. This implication is robust with multiple retailers and can be enhanced with an increase in the number of retailers. Therefore, we suggest the buyer managers think over the consequences of their cooperation, for the reason that GB may benefit neither themselves nor the supply chain. For seller managers, in addition to adjusting the discount level promptly according to GB, we suggest them adopt a mixed discount schedule because it helps prevent the potential damage of GB. Furthermore, seller managers can benefit from GB by exploring the effect of economies of scale based on the order volume.

This model certainly has some limitations. For example, our assumption of symmetric retailers seems not to be comprehensive. If retailers are asymmetric, the analysis is less tractable. Based on the results of Chen and Roma [29], we conjecture that if the asymmetric level (i.e., the market base or operational cost) between retailers is not large, the insights from our model will not change. Another limitation is the assumption of the cost structure. Instead of a simple operational cost for retailers, future studies could consider the inventory cost, transportation cost, and costs associated with the maintenance of purchasing groups. Different cost structures for retailers characterize GB in different industries. Furthermore, contract administration fees charged by buyer groups to suppliers are common in practice and can be incorporated into future studies.

References

- [1] P. W. Dobson, R. Clarke, S. Davies, M. Waterson. Buyer power and its impact on competition in the food retail distribution sector of the European Union, *Journal of Industry, Competition and Trade*, 1(3) (2001) 247–281.
- [2] M. Essig. Purchasing consortia as symbiotic relationships: Developing the concept of “consortium sourcing”, *European Journal of Purchasing & Supply Management*, 6(1) (2000) 13–22.
- [3] M. Kofman. Group purchasing arrangements: Issues for states, *State Coverage Initiatives Issue Brief*, 4(3) (2003) 1–6.
- [4] J. Nollet, M. Beaulieu. The development of group purchasing: An empirical study in the healthcare sector, *Journal of Purchasing and Supply Management*, 9(1) (2003) 3–10.
- [5] H. Walker, L. Knight, C. F. Harland, Z. Greenwell. The developing role of purchasing confederations in complex supply networks, *British Academy of Management*, September (2003) 15-17.
- [6] K. S. Anand, R. Aron. Group buying on the web: A comparison of price-discovery mechanisms, *Management Science*, 49(11) (2003) 1546–1562.
- [7] Q. Hu, L. B. Schwarz. Controversial role of GPOs in healthcare-product supply chains, *Production and Operations Management*, 20(1) (2011) 1-15.
- [8] W. J. Scanlon. Group purchasing organizations: Pilot study suggests large buying groups do not always offer hospitals lower prices, *GPO report* (2002), GAO-02-690T.
- [9] E. Elhauge. The exclusion of competition for hospital sales through group purchasing organizations, (2002), available at <http://www.law.harvard.edu/faculty/elhauge>.
- [10] J. Nollet, M. Beaulieu. Should an organization join a purchasing group? *Supply Chain Management*, 10(1) (2005) 11–17.
- [11] J. D. Dana. Buyer groups as strategic commitments, *Games and Economic Behavior*, 74(2) (2012) 470–485.
- [12] T. Chipty. Horizontal integration for bargaining power: Evidence from the cable television industry, *Journal of Economics and Management Strategy*, 4(2) (1995) 375–397.
- [13] R. Inderst, C. Wey. Bargaining, mergers, and technology choice in bilaterally oligopolistic industries, *RAND Journal of Economics*, 34(1) (2003) 1–19.
- [14] S. Chae, P. Heidhues. Buyers' alliances for bargaining power, *Journal of Economics and Management Strategy*, 13(4) (2004) 731-754.
- [15] H. P. Marvel, H. Yang. Group purchasing, nonlinear tariffs and oligopoly, *International Journal of Industrial Organization*, 26(5) (2008) 1090–1105.
- [16] Q. J. Hu, L. B. Schwarz, N. A. Uhan. The impact of group purchasing organizations on healthcare-product supply chains, *Manufacturing & Service Operations Management*, 14(1) (2012) 7–23.
- [17] W. Yu. Cooperative purchasing in small and medium-sized enterprises, *Supply Chain Strategies, Issues and Models*, in: U. Ramanathan and R. Ramanathan(Eds.), Springer – Verlag, London, 2014, 193–208.
- [18] H. Horn, A. Wolinsky. Bilateral monopolies and incentives for merger, *RAND Journal of Economics*, 19(3) (1988) 408–419.

- [19] C. M. Snyder. A dynamic theory of countervailing power, *RAND Journal of Economics*, 27 (1996) 747–769.
- [20] C. M. Snyder. Why do larger buyers pay lower prices? Intense supplier competition, *Economics Letters*, 58 (1998) 205–209.
- [21] Y. Chen, X. Li. Group buying commitment and sellers' competitive advantages, *Journal of Economics & Management Strategy*, 22(1) (2013) 164–183.
- [22] P. Keskinocak, S. Savaşaneril. Collaborative procurement among competing buyers, *Naval Research Logistics*, 55(6) (2008) 516–540.
- [23] R. J. Kauffman, B. Wang. Bid together, buy together: On the efficiency of group-buying business models in internet-based selling, in: P. B. Lowry, J. O. Cherrington, R. R. Watson(Eds.), *The E-Business Handbook*, CRC Press, Florida, 2002, 99–137.
- [24] R. J. Kauffman, H. Lai, C. Ho. Incentive mechanisms, fairness and participation in online group-buying auctions, *Electronic Commerce Research and Applications*, 9 (2010) 249–262.
- [25] J. Chen, X. Chen, X. Song. Bidder's strategy under group buying auction on the internet, *IEEE Transactions on Systems, Man and Cybernetics, Part A*, 32(6) (2002) 680–690.
- [26] J. Chen, X. Chen, X. Song. Comparison of the group-buying auction and the fixed-pricing mechanism, *Decision Support Systems*, 43(2) (2007) 445–459.
- [27] J. Chen, X. Chen, R. J. Kauffman, X. Song. Should we collude? Analyzing the benefits of bidder cooperation in online group-buying auctions, *Electronic Commerce Research and Applications*, 8 (2009) 191–202.
- [28] Z. Chen, X. Liang, H. Wang, et al. Inventory rationing with multiple demand classes: The case of group buying, *Operations Research Letters*, 40(5) (2012) 404–408.
- [29] R. Chen, P. Roma. Group buying of competing retailers, *Production and Operations Management*, 20(2) (2011) 181–197.
- [30] J. M. Buchanan. The theory of monopolistic quantity discounts, *Review of Economic Studies*, 20(3) (1953) 199–208.
- [31] W. Y. Oi. A Disneyland dilemma: Two-part tariffs for a Mickey Mouse monopoly, *The Quarterly Journal of Economics*, 85 (1971) 77–96.
- [32] K. S. Moorthy. Managing channel profits: Comment, *Marketing Science*, 6(4) (1987) 375–379.
- [33] A. Gabor. A note on block tariffs, *Review of Economic Studies*, 23 (1955) 32–41.
- [34] A. Jeuland, S. Shugan. Managing channel profits, *Marketing Science*, 2(3) (1983) 239–272.
- [35] C. A. Ingene, M. E. Parry. Channel coordination when retailers compete, *Marketing Science*, 14(4) (1995) 360–377.
- [36] C. A. Ingene, M. E. Parry. Supplier-optimal wholesale pricing when retailers compete, *Marketing Letters*, 9(1) (1998) 65–77.
- [37] X. Qi, J. F. Bard, G. Yu. Supply chain coordination with demand disruptions, *Omega*, 32(4) (2004) 301–312.
- [38] J. Raju, Z. J. Zhang. Channel coordination in presence of a dominant retailer, *Marketing Science*, 24(2) (2005) 254–262.
- [39] J. Crowther. Rationale for quantity discounts, *Harvard Business Review*, 42(2) (1964) 121–127.

- [40] F. Erhun, P. Keskinoca, S. Tayur. Dynamic procurement, quantity discounts, and supply chain efficiency, *Production and Operations Management*, 17(5) (2008) 543–550.
- [41] R. Mansini, M. W. P. Savelsbergh, B. Tocchella. The supplier selection problem with quantity discounts and truckload shipping, *Omega*, 40(4) (2012) 445–455.
- [42] R. Dolan. Quantity discounts: Managerial issues and research opportunities, *Marketing Science*, 6(1) (1987) 1–22.
- [43] Z. K. Weng. Channel coordination and quantity discounts, *Management Science*, 41(9) (1995) 1509–1522.
- [44] A. Kanda, S. G. Deshmukh. Supply chain coordination: Perspectives, empirical studies and research directions, *International Journal of Production Economics* 115(2) (2008) 316–335.
- [45] F. Schotanus, J. Telgen, L. de Boer. Unraveling quantity discounts, *Omega*, 37(3) (2009) 510–521.
- [46] R. R. Perdue. Perishability, yield management, and cross-product elasticity: A case study of deep discount season passes in the Colorado ski industry, *Journal of Travel Research*, 41 (2002) 15–22.
- [47] J. M. Ruteri, Q. Xu. Supply chain management and challenges facing the food industry sector in Tanzania, *International Journal of Business and Management*, 4(12) (2009) 70–80.
- [48] L. Shapley, M. Shubik. Price strategy oligopoly with product variation, Cowles Foundation paper 291, reprinted from *Kyklos*, 22(1) (1969) 30–44.
- [49] T. W. McGuire, R. Staelin. An industry equilibrium analysis of downstream vertical integration, *Marketing Science*, 2(2) (1983) 161–191.
- [50] A. A. Tsay, N. Agrawal. Channel dynamics under price and service competition, *Manufacturing and Service Operations Management*, 2(4) (2000) 372–391.
- [51] Y. Polychronakis, A. Syntetos. “Soft” supplier management related issues: An empirical investigation, *International Journal of Production Economics*, 106(2) (2007) 431–449.
- [52] S. Eaton. Joint purchasing: Assisting the survival of small retailers in small markets, *European Competition Law Review*, 30(6) (2009) 301–306.
- [53] C. L. Munson, M. J. Rosenblatt. Theories and realities of quantity discounts: An exploratory study, *Production and Operations Management*, 7(4) (1998) 352–369.
- [54] A. Silberston. Economies of scale in theory and practice, *The Economic Journal*, 82(325) (1972) 369–391.
- [55] P. Flynn. Biomass energy: Cost and scale issues, (2010) Available at http://www.aeri.ab.ca/sec/new_res/docs/presentations/Biomass_Economics_060927.pdf (accessed on 1st December, 2013).
- [56] A. Atasu, L. B. Toktay, L. N. Van Wassenhove. How collection cost structure drives a manufacturer’s reverse channel choice, *Production and Operations Management*, (2013). DOI: 10.1111/j.1937-5956.2012.01426.x.

Appendix

This appendix provides the proofs for the Lemma 1 and Propositions 1-4.

Proof of Lemma 1:

- a) (IP situation)** First, we consider the equilibrium in IP situation for retailers. By Eqn. (5), the equilibrium purchase quantity q_{IP} satisfies the following function

$$A - a - c = \frac{2+\theta}{1+\theta} q_{IP} + \frac{d}{(q_{IP})^e} (1 - e).$$

By taking the derivative with respect to d and rearranging terms yield

$$\left(\frac{2+\theta}{1+\theta} - de \cdot (1 - e)(q_{IP})^{-1-e}\right) \frac{\partial q_{IP}}{\partial d} = (e - 1)(q_{IP})^{-e}. \quad (A1)$$

By Eqn. (A1), we can obtain the second-order derivative

$$\frac{\partial^2 q_{IP}}{\partial d^2} = e \cdot (q_{IP})^{-1} \left(\frac{\partial q_{IP}}{\partial d}\right)^2 \left(d \cdot (1 + e)(q_{IP})^{-1} \frac{\partial q_{IP}}{\partial d} - 2\right). \quad (A2)$$

Then, we consider the decision for the supplier. By Eqn. (7), taking the first-order derivative with respect to d and setting it to 0 yield

$$\frac{\partial \Pi}{\partial d} = 2 \left((a - c) \frac{\partial q_{IP}}{\partial d} + (q_{IP})^{1-e} + d \cdot (1 - e)(q_{IP})^{-e} \frac{\partial q_{IP}}{\partial d} \right) = 0. \quad (A3)$$

By solving Eqn. (A3), we can solve the optimal scaling parameter d_{IP} .

Considering the second-order derivative, by Eqns. (A1)-(A3), we have

$$\begin{aligned} \frac{\partial^2 \Pi_{IP}}{\partial d^2} \Big|_{d_{IP}} &= 2 \left((a - c + d \cdot (1 - e)(q_{IP})^{-e}) \frac{\partial^2 q_{IP}}{\partial d^2} + (1 - e)(q_{IP})^{-e} \frac{\partial q_{IP}}{\partial d} \left(2 - de \cdot (q_{IP})^{-1} \frac{\partial q_{IP}}{\partial d} \right) \right) \\ &= 2 \left(\frac{-(q_{IP})^{1-e}}{\frac{\partial q_{IP}}{\partial d}} e \cdot (q_{IP})^{-1} \left(\frac{\partial q_{IP}}{\partial d}\right)^2 \left(d \cdot (1 + e)(q_{IP})^{-1} \frac{\partial q_{IP}}{\partial d} - 2\right) + (1 - e)(q_{IP})^{-e} \frac{\partial q_{IP}}{\partial d} \left(2 - de \cdot (q_{IP})^{-1} \frac{\partial q_{IP}}{\partial d} \right) \right) \\ &= 4(q_{IP})^{-e} \frac{\partial q_{IP}}{\partial d} \left(1 - de \cdot (q_{IP})^{-1} \frac{\partial q_{IP}}{\partial d} \right) \\ &= -\frac{2(2+\theta)}{(1+\theta)(1-e)} \left(\frac{\partial q_{IP}}{\partial d}\right)^2 \\ &\leq 0. \end{aligned}$$

That is, the supplier's profit is always concave at d_{IP} under IP situation. This implies that there exists a unique optimal scaling parameter under IP situation.

Then by Eqns. (A1) and (A3), we can solve the closed-form scaling parameter d_{IP} and equilibrium purchase quantity q_{IP} as listed in Table 1. The rest equilibrium solutions can be easily obtained by substituting d_{IP} and q_{IP} .

- b) (GB situation)** Similar as IP situation, we first consider the equilibrium for retailers.

By Eqn. (6), the aggregated quantity Q_{GB} satisfies the following function

$$A - a - c = \frac{2+\theta}{1+\theta} \cdot \frac{Q_{GB}}{2} + \frac{d}{(Q_{GB})^e} \left(1 - \frac{e}{2(1+\theta)} \right).$$

Taking the derivative with respect to d and rearranging terms yield

$$\left(\frac{2+\theta}{1+\theta} - de \cdot \left(2 - \frac{e}{1+\theta} \right) (Q_{GB})^{-1-e}\right) \frac{\partial Q_{GB}}{\partial d} = -(2 - \frac{e}{1+\theta})(Q_{GB})^{-e}. \quad (A4)$$

By Eqn. (A4), we can obtain the second-order derivative

$$\frac{\partial^2 Q_{GB}}{\partial d^2} = e \cdot (Q_{GB})^{-1} \left(\frac{\partial Q_{GB}}{\partial d} \right)^2 \left(d \cdot (1+e)(Q_{GB})^{-1} \frac{\partial Q_{GB}}{\partial d} - 2 \right). \quad (A5)$$

Consider the decision for the supplier. By Eqn. (8), taking the first-order derivative with respect to d and setting it to 0 yield

$$\frac{\partial \Pi}{\partial d} = (a-c) \frac{\partial Q_{GB}}{\partial d} + (Q_{GB})^{1-e} + d \cdot (1-e)(Q_{GB})^{-e} \frac{\partial Q_{GB}}{\partial d} = 0. \quad (A6)$$

By solving Eqn. (A6), we can solve the optimal scaling parameter d_{GB} .

$$(a-c + d \cdot (1-e)(Q_{GB})^{-e}) \frac{\partial^2 Q_{GB}}{\partial d^2} + (1-e)(Q_{GB})^{-e} \frac{\partial Q_{GB}}{\partial d} \cdot \left(2 - de \cdot (Q_{GB})^{-1} \frac{\partial Q_{GB}}{\partial d} \right)$$

By Eqns. (A4)-(A6), the second-order derivative

$$\begin{aligned} \left. \frac{\partial^2 \Pi_{GB}}{\partial d^2} \right|_{d_{GB}} &= (a-c + d \cdot (1-e)(Q_{GB})^{-e}) \frac{\partial^2 Q_{GB}}{\partial d^2} \\ &\quad + (1-e)(Q_{GB})^{-e} \frac{\partial Q_{GB}}{\partial d} \left(2 - de \cdot (Q_{GB})^{-1} \frac{\partial Q_{GB}}{\partial d} \right) \\ &= \frac{-(Q_{GB})^{1-e}}{\frac{\partial Q_{GB}}{\partial d}} e \cdot (Q_{GB})^{-1} \left(\frac{\partial Q_{GB}}{\partial d} \right)^2 \left(d \cdot (1+e)(Q_{GB})^{-1} \frac{\partial Q_{GB}}{\partial d} - 2 \right) \\ &\quad + (1-e)(Q_{GB})^{-e} \frac{\partial Q_{GB}}{\partial d} \left(2 - de \cdot (Q_{GB})^{-1} \frac{\partial Q_{GB}}{\partial d} \right) \\ &= 2(Q_{GB})^{-e} \frac{\partial Q_{GB}}{\partial d} \left(1 - \left(\frac{2+\theta}{(1+\theta)(2-\frac{e}{1+\theta})} (Q_{GB})^e \frac{\partial Q_{GB}}{\partial d} + 1 \right) \right) \\ &= -\frac{2(2+\theta)}{(1+\theta)(2-\frac{e}{1+\theta})} \left(\frac{\partial Q_{GB}}{\partial d} \right)^2 \leq 0. \end{aligned}$$

That is, the supplier's profit is always concave at d_{GB} under GB situation. This implies that there exists a unique optimal scaling parameter under GB situation.

Then by Eqns. (A4) and (A6), we can solve the closed-form scaling parameter d_{GB} and equilibrium purchase quantity q_{GB} as listed in Table 1. The rest equilibrium solutions can be easily obtained by substituting d_{GB} and q_{GB} .

To ensure that the various price, quantity and profit expressions are reasonable (i.e., nonnegative) and well behaved, we impose the following conditions.

$$e \cdot (X - t_{GB}Y) > 0 \quad (A7)$$

$$A - a - c > 0 \quad (A8)$$

$$a - C > 0 \quad (A9)$$

$$(3t_{IP} + \theta t_{IP} - 2 - \theta)X + t_{IP}(2 + \theta - t_{IP} - \theta t_{IP})Y \geq 0 \Leftrightarrow \theta \leq \frac{(3t_{IP}-2)X + t_{IP}(2-t_{IP})Y}{e \cdot (X - t_{IP}Y)} \quad (A10)$$

$$(3t_{IP} - 2)X + t_{IP}(2 - t_{IP})Y \geq 0 \Leftrightarrow e \leq \frac{\sqrt{9X^2 + 4XY + 4Y^2} - 3X}{2Y} \quad (A11)$$

$$(3t_{GB} + \theta t_{GB} - 2 - \theta)X + t_{GB}(2 + \theta - t_{GB} - \theta t_{GB})Y \geq 0 \Leftrightarrow \theta \geq -\frac{(4-6e)X + (4-e^2)Y}{(4-2e)X + (4+2e)Y} \quad (A12)$$

$$(4 - 6e)X + (4 - e^2)Y \geq 0 \Leftrightarrow e \leq 2 \frac{\sqrt{9X^2 + 4XY + 4Y^2} - 3X}{2Y}. \quad (A13)$$

Eqn. (A7) ensures a reasonable discount scaling parameter (i.e., $de > 0$). Eqns. (A8) and (A9) ensure positive purchase quantities, wholesale prices, and finite retail prices under both situations. $\pi_{IP} \geq 0$ requires Eqn. (A10). By Eqns. (A7), (A10), and $\theta \geq 0$, we need $(3t_{IP} - 2)X + t_{IP}(2 - t_{IP})Y \geq 0$ for the problem to be feasible, which is Eqn. (A11).

Eqn. (A12) assures $\pi_{GB} \geq 0$, which is always satisfied when Eqn. (A13) holds. Eqn. (A11) guarantees Eqn. (A13).

Then, Eqns. (A7)-(A11) are needed in the analysis. This completes the proof. $\square$

Note that $t_{GB} - t_{IP} = e \cdot (1 - 1/[2(1 + \theta)])$, we have

$$e \cdot (t_{GB} - t_{IP}) > 0. \quad (\text{A14})$$

Proof of Proposition 1:

a) (Discount level) To examine $\Delta|d| = |d_{GB}| - |d_{IP}|$, it suffices to examine $|d_{IP}|/|d_{GB}|$ when $|d_{GB}| \neq 0$, which is always satisfied when Eqns. (A7)-(A9) hold. By Table 1, after some algebra we have

$$\frac{|d_{IP}|}{|d_{GB}|} = \frac{d_{IP}}{d_{GB}} = \frac{1}{2^e} \cdot \frac{t_{GB}}{t_{IP}} \left(\frac{X+t_{IP}Y}{X+t_{GB}Y} \right)^e \frac{X-t_{IP}Y}{X-t_{GB}Y}. \quad (\text{A15})$$

(i) Negative steepness ($-1 \leq e < 0$)

We show $|d_{IP}|/|d_{GB}| > 1$. First, we need to show that

$$X + t_{IP}Y \leq 2(X + t_{GB}Y). \quad (\text{A16})$$

It suffices to show

$$(t_{IP} - 2t_{GB})Y < X. \quad (\text{A17})$$

The inequality (A17) always holds since $1 + e > 0$, $e/(1 + \theta) < 0$, and

$$t_{IP} - 2t_{GB} = (1 - e) - 2 \left(1 - \frac{e}{2(1+\theta)} \right) = -(1 + e) + \frac{e}{1+\theta} < 0.$$

By (A15) and (A16), we have

$$\begin{aligned} \frac{|d_{IP}|}{|d_{GB}|} &= \left(\frac{X + t_{IP}Y}{2(X + t_{GB}Y)} \right)^e \frac{t_{IP}t_{GB}Y - t_{GB}X}{t_{IP}t_{GB}Y - t_{IP}X} \\ &> \left(\frac{X + t_{IP}Y}{2(X + t_{GB}Y)} \right)^0 \frac{t_{IP}t_{GB}Y - t_{GB}X}{t_{IP}t_{GB}Y - t_{IP}X} \\ &> 1. \end{aligned}$$

The last inequality is obtained by Eqns. (A7) and (A14).

(ii) Positive steepness ($0 < e < 1$)

We have

$$f(\theta, e) = 1 - \frac{e}{2(1+\theta)} - 2^e(1 - e) = t_{GB} - 2^e t_{IP}.$$

When $f(\theta, e) < 0$, if $Y = 0$, then

$$\frac{|d_{IP}|}{|d_{GB}|} = \frac{1}{2^e} \cdot \frac{t_{GB}}{t_{IP}} < 1.$$

Since $|d_{IP}|/|d_{GB}|$ is continuous with Y at 0, then there exists $\hat{Y}$, where $0 < (1 - e/[2(1 + \theta)])\hat{Y} \leq X$, such that as long as $Y \in (0, \hat{Y})$, $|d_{IP}|/|d_{GB}| < 1$.

When $f(\theta, e) \geq 0$, then by Eqns. (A7), (A14), and (A15), we have

$$\begin{aligned} \frac{|d_{IP}|}{|d_{GB}|} &\geq \left(\frac{X + t_{IP}Y}{X + t_{GB}Y} \right)^e \frac{X - t_{IP}Y}{X - t_{GB}Y} \\ &= \left(\frac{X^2 - (t_{IP}Y)^2}{X^2 - (t_{GB}Y)^2} \right)^e \left(\frac{X - t_{IP}Y}{X - t_{GB}Y} \right)^{1-e} \\ &> 1. \end{aligned}$$

b) Supplier's profit: To show $\Delta\pi < 0$, by Table 1, it suffices to show

$$\frac{(X + t_{GB}Y)}{\sqrt{t_{GB}}} - \frac{(X + t_{IP}Y)}{\sqrt{t_{IP}}} = (\sqrt{t_{GB}} - \sqrt{t_{IP}}) \left(\frac{X}{\sqrt{t_{IP}t_{GB}}} - Y \right) < 0$$

which can be directly obtained by Eqns. (A7) and (A14). This completes the proof. $\square$

Proof of Proposition 2:

By Table 1 and Eqn. (A9), we have

$$\Delta\pi = \frac{(1 + \theta)Y^2}{4(2 + \theta)^2} \cdot \frac{t_{GB} - t_{IP}}{t_{IP}t_{GB}} G(Z)$$

where $Z = X/Y$, $G(Z) = \beta_1 Z^2 + \beta_2 Z + \beta_3$, and

$$\beta_1 = 2 + \theta$$

$$\beta_2 = 2t_{IP}t_{GB}$$

$$\beta_3 = t_{IP}t_{GB} \cdot [(1 + \theta)(1 - t_{IP} - t_{GB}) + 1].$$

By Eqns. (A8) and (A9), we have $Z > 0$ and

$$G'(Z) = 2\beta_1 Z + \beta_2 > 0. \quad (\text{A18})$$

a) Positive steepness ($0 < e < 1$)

Now we show $\Delta\pi > 0$. It suffices to show $G(Z) > 0$ by Eqn. (A14).

By Eqn. (A7), we have $Z > t_{GB}$. Then,

$$G(t_{GB}) = \frac{t_{GB}}{2(1+\theta)} \cdot h(\theta) \quad (\text{A19})$$

where $h(\theta) = \gamma_1 \theta^2 + \gamma_2 \theta + \gamma_3$, in which $\gamma_1 = 2e \cdot (2 - e)$, $\gamma_2 = 8 + 2e - 5e^2$, and $\gamma_3 = 8 - 5e - e^2$. It is easy to obtain that $\gamma_1 > 0$, $\gamma_2 > 0$, and $\gamma_3 > 0$.

We have $h'(\theta) = 2\gamma_1 \theta + \gamma_2 > 0$ and $h(0) = \gamma_3 > 0$. This suggests that $h(\theta) > 0$.

Then, we have $G(t_{GB}) > 0$ by Eqn. (A19).

By Eqn. (A18), we obtain $G(Z) > 0$.

b) Negative steepness ($-1 \leq e < 0$)

To show that $\Delta\pi < 0$, it suffices to show $g(Z) > 0$ by Eqn. (A14).

By Eqns. (A7)-(A9), we have $0 < Z < t_{GB}$.

Examine $G(0)$ and $G(t_{GB})$,

$$G(0) = \beta_3 = t_{IP}t_{GB} \cdot \left(-\theta + e \cdot \left(\theta + \frac{3}{2} \right) \right) < 0.$$

By Eqn. (A18), if $G(t_{GB}) > 0$, there exists $\hat{k} \in (0, t_{GB})$, such that for $\hat{k} < Z < t_{GB}$, $G(Z) > 0$.

To show $G(t_{GB}) > 0$, it suffices to show $h(\theta) > 0$ by Eqn. (A19). Now we consider $h(\theta)$.

It is easy to obtain that $\gamma_1 < 0$ and $h(0) = \gamma_3 > 0$. Then, there exists $\hat{\theta} \in (0, +\infty)$, such that for $\theta < \hat{\theta}$, $h(\theta) > 0$. Solving $h(\theta) = 0$, we have

$$\hat{\theta} = -\frac{(8+2e-5e^2+\sqrt{64-96e+68e^2-44e^3+17e^4})}{4e \cdot (2-e)}.$$

Note that the other root is discarded, because it is negative.

Now, considering $G(Z)$, we solve $G(Z) = 0$, then

$$\hat{k} = \frac{(\sqrt{t_{IP}t_{GB}(t_{IP}t_{GB}+(\theta+2)(\theta+1)(t_{IP}+t_{GB})-(\theta+2)^2})-t_{IP}t_{GB})}{2+\theta}$$

here the other root is discarded, because it is negative and occurs outside the domain of consideration.

Then we have that, when $\theta < \hat{\theta}$, for $\hat{k} < Z < t_{GB}$, $G(Z) > 0$. This completes the proof. $\square$

Proof of Proposition 3:

Chen and Roma [25] have shown that given the discount schedule as Eqn. (1), the equilibrium aggregated purchase quantity under GB is always higher than the equilibrium individual purchase quantity under IP.

Then, under GB situation, the equilibrium wholesale price would be the fixed price $a + \delta$ under discount schedule Eqn. (11). The retailer i 's equilibrium purchase quantity satisfies:

$$\frac{\partial \pi_i}{\partial p_i} = (1 + w'(Q))q_i - (p_i - c - w(Q))(1 + \theta) = 0.$$

After some algebra, suppressing the subscript “ i ”, we can obtain the equilibrium purchase quantity $\tilde{q}_{GB}$ and the retailer's profit $\tilde{\pi}_{GB}$ as

$$\tilde{q}_{GB} = \frac{(1+\theta)}{(2+\theta)}(X - \delta)$$

$$\tilde{\pi}_{GB} = \frac{(1+\theta)}{(2+\theta)^2}(X - \delta)^2.$$

To assure the retailer's profit is lower under the mixed discount schedule under GB situation, δ needs to satisfy following inequalities

$$\begin{cases} 2\tilde{q}_{GB} \geq q_{IP} \\ \tilde{\pi}_{GB} < \pi_{IP} \end{cases} \Leftrightarrow \begin{cases} \delta \leq X - \frac{(2+\theta)}{2(1+\theta)}q_{IP} \\ \delta > X - \sqrt{\frac{(2+\theta)^2}{(1+\theta)}}\pi_{IP} \end{cases}.$$

Then, δ exists if and only if

$$X - \sqrt{\frac{(2+\theta)^2}{(1+\theta)}}\pi_{IP} < X - \frac{(2+\theta)}{2(1+\theta)}q_{IP}$$

which can be simplified to

$$\theta < \frac{l(e)}{4e \cdot (X - (1-e)Y)}$$

in which $l(e) = \rho_1 e^2 + \rho_2 e + \rho_3$, $\rho_1 = -5Y$, $\rho_2 = -11X + 2Y$, $\rho_3 = 3X + 3Y$. Then we have $\rho_2^2 - 4\rho_1\rho_3 = 121X^2 + 16XY + 64Y^2 > 0$. Solve $l(e) = 0$, we can obtain

$$\begin{aligned} \bar{e} &= \frac{\sqrt{121X^2 + 16XY + 64Y^2} - (11X - 2Y)}{10Y} \\ \underline{e} &= \frac{-\sqrt{121X^2 + 16XY + 64Y^2} - (11X - 2Y)}{10Y} \end{aligned}$$

such that $l(\bar{e}) = l(\underline{e}) = 0$. It is easy to verify that $\underline{e} < 0 < \bar{e} < 1$, since $l(0) = 3X + 3Y > 0$, $l(1) = -8X < 0$, and $\alpha_h < 0$. Then, for $\underline{e} < e < \bar{e}$, $l(e) > 0$.

Note that

$$\tilde{\theta} = \frac{h(e)}{4e \cdot (X - (1-e)Y)}.$$

For $\max\{-1, \underline{e}\} \leq e < \bar{e}$, $e \neq \underline{e}$, and $e \neq 0$, we have $\tilde{\theta} > 0$ by Eqn. (A22), which is the condition that guarantees a reasonable discount scaling parameter. Then, if $\theta < \tilde{\theta}$, there exists δ such that the retailer's profit is lower under the mixed discount schedule under GB situation.

We add the following Eqns. (A20)-(A22) that are used for the mixed discount schedule.

$$a + \delta \geq 0 \Leftrightarrow \delta \geq -a \quad (\text{A20})$$

$$X - \delta \geq 0 \Leftrightarrow \delta \leq A - a - c \quad (\text{A21})$$

$$e \cdot (X - t_{IP}Y) > 0 \quad (\text{A22})$$

Eqn. (A20) ensures positive fixed wholesale price. Eqn. (A21) ensures positive purchase quantity and retail price under GB. A reasonable discount scaling parameter under IP requires Eqn. (A22). We also need Eqns. (A8)-(A11) to ensure that the purchase quantity, the retail price, and the retailer's profit are reasonable under IP situation. This completes the proof. $\square$

Proof of Proposition 4:

Similar as Lemma 1, we can obtain the equilibrium solutions in the n -retailer case. We list the equilibrium solutions under GB in Table 2. We suppress the superscript “ n ” in this proof.

Table 2. The equilibrium solutions under GB in the n -retailer case

Equilibrium Solutions	
Retail Price	$p_{GB} = A - q_{GB}$
Purchase Quantity	$q_{GB} = \frac{Q_{GB}}{n} = \frac{(1+\theta)}{4(2+\theta)}(X + t_{GB}Y)$
Retailer Profit	$\pi_{GB} = \frac{(1+\theta)(X + t_{GB}Y)}{4(2+\theta)^2 t_{GB}} ((3t_{GB} + \theta t_{GB} - 2 - \theta)X + t_{GB}(2 + \theta - t_{GB} - \theta t_{GB})Y)$
Scaling Parameter	$d_{GB} = \frac{(Q_{GB})^e}{t_{GB}}(X - t_{GB}Y)$
Supplier Profit	$\Pi_{GB} = \frac{n \cdot (1+\theta)}{4(2+\theta)t_{GB}}(X + t_{GB}Y)^2$

$$t_{GB} = 1 - \frac{e}{n \cdot (1+\theta)}$$

The comparison of the equilibrium results are similar with Proposition 1 and 2, thus omitted here. We mainly focus on influence of the amount of retailers.

a) (Supplier profit) By table 2, we have

$$\frac{\partial(\Pi_{GB}/n)}{\partial n} = -\frac{e \cdot (X - t_{GB}Y)}{4(2+\theta)(t_{GB})^2 n^2} (X + t_{GB}Y).$$

Then, $\partial(\Pi_{GB}/n)/\partial n < 0$ by Eqn. (A23), which is the condition that guarantees a reasonable discount scaling parameter in the n -retailer case.

Then, we have

$$\begin{aligned} \frac{\partial \Pi_{GB}}{\partial n} &= \frac{\Pi_{GB}}{n} + \frac{\partial(\Pi_{GB}/n)}{\partial n} = \frac{(1+\theta)(X+t_{GB}Y)}{4(2+\theta)t_{GB}} \left(X + t_{GB}Y - \frac{e \cdot (X - t_{GB}Y)}{t_{GB}n^2} \right) \\ &\geq \frac{(1+\theta)(X+t_{GB}Y)}{4(2+\theta)t_{GB}} \left(X + t_{GB}Y - \frac{e \cdot (X - t_{GB}Y)}{1} \right) \\ &= \frac{(1+\theta)(X+t_{GB}Y)}{4(2+\theta)t_{GB}} ((1-e)X + (1+e)t_{GB}Y) \\ &> 0 \end{aligned}$$

b) (Retailer profit) For $\hat{\theta}$

$$\hat{\theta} = \frac{-1}{2ne \cdot (2-e)} (4n + ne - (2n+1)e^2 + \sqrt{16n^2 - 24n^2e + (9n^2 + 16n)e^2 - 22ne^3 + (8n+1)e^4}).$$

We have

$$\frac{\partial \hat{\theta}}{\partial n} = \frac{e}{2n^2(e-2)} \cdot \frac{8n-11ne+(4n+1)e^2+\sqrt{16n^2-24n^2e+(9n^2+16n)e^2-22ne^3+(8n+1)e^4}}{\sqrt{16n^2-24n^2e+(9n^2+16n)e^2-22ne^3+(8n+1)e^4}}.$$

It is easy to obtain that, for $-1 \leq e < 0$, $\partial \hat{\theta} / \partial n > 0$.

For $\hat{k}$, we have

$$\frac{\partial \hat{k}}{\partial n} = \frac{\partial \hat{k}}{\partial t_{GB}} \cdot \frac{\partial t_{GB}}{\partial n} = \frac{e}{n^2(1+\theta)} \cdot \frac{\partial \hat{k}}{\partial t_{GB}}$$

where

$$\frac{\partial \hat{k}}{\partial t_{GB}} = \frac{t_{IP}}{(2+\theta)} \left(\frac{(\sqrt{t_{IP}t_{GB}+(\theta+2)(\theta+1)(t_{IP}+t_{GB})-(\theta+2)^2}-\sqrt{t_{IP}t_{GB}})^2 + t_{GB}(\theta+2)(\theta+1)}{2\sqrt{t_{IP}t_{GB}(t_{IP}t_{GB}+(\theta+2)(\theta+1)(t_{IP}+t_{GB})-(\theta+2)^2)}} \right) > 0.$$

Then $\partial \hat{k} / \partial n < 0$, by $-1 \leq e < 0$.

c) (Supply) By Table 2, we have

$$\Delta Q = \frac{(1+\theta)(t_{GB}-t_{IP})Y}{(2+\theta)} = e \cdot \left(1 - \frac{1}{n \cdot (1+\theta)}\right) \frac{(1+\theta)Y}{(2+\theta)}.$$

Then

$$e \cdot \Delta Q = e^2 \cdot \left(1 - \frac{1}{n \cdot (1+\theta)}\right) \frac{(1+\theta)Y}{(2+\theta)} > 0.$$

For Q_{GB} , we have

$$\frac{\partial Q_{GB}}{\partial n} = \frac{(1+\theta)}{2(2+\theta)} (X+Y) > 0$$

and

$$e \cdot \frac{\partial (Q_{GB}/n)}{\partial n} = \frac{e^2 Y}{2n^2 \cdot (2+\theta)} > 0.$$

Now, we impose the condition used in the analysis. Here, we denote the superscript “ n ” for the n -retailer case.

$$e \cdot (X - t_{n,GB}Y) > 0 \tag{A23}$$

$$(3t_{n,GB} + \theta t_{n,GB} - 2 - \theta)X + t_{n,GB}(2 + \theta - t_{n,GB} - \theta t_{n,GB})Y \geq 0 \Leftrightarrow \theta \geq -\frac{n \cdot (n-3e)X + (n^2 - e^2)Y}{n \cdot (n-e)X + n \cdot (n+e)Y} \tag{A24}$$

$$n \cdot (n - 3e)X + (n^2 - e^2)Y \geq 0 \Leftrightarrow e \leq n \cdot \frac{\sqrt{9X^2 + 4XY + 4Y^2} - 3X}{2Y} \tag{A25}$$

We use Eqn. (A23) in the n -retailer case instead of Eqn. (A7) in the 2-retailer case to guarantee a reasonable discount scaling parameter. Eqns. (A8)-(A11) are also needed to ensure that, the parameters are reasonable in the n -retailer case.

Eqn. (A24) assures $\pi_{n,GB} \geq 0$, which is always satisfied when Eqn. (A25) holds. Eqn. (A11) guarantees Eqn. (A25).

Therefore, we need Eqns. (A8)-(A11) and (A23) in the analysis. This completes the proof.

□