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Consumer return policies in presence of a P2P market

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A B S T R A C T

Retailers selling products with valuation uncertainty often offer return policies to consumers to stimulate demand. However, some products that do not meet consumers' expectations cannot be returned to the retailers either because of retailers' strict restrictions on returns or because of short trial period. With the development of e-commerce, consumers who cannot return their products to retailers can resell them directly to others through electronic peer-to-peer (P2P) second-hand goods markets. This paper examines the effect of the presence of a P2P market on a retailer's optimal return policy when the consumers are strategic and uncertain about their valuations. As a benchmark, we first examine the retailer's optimal return policy when there is only a retailer-run resale market. Then, we analyze the retailer's optimal return policy in presence of both the retailer-run resale market and the P2P market. Theoretical and numerical results show that, first, the presence of the P2P market is detrimental to the retailer in most cases. The presence of the P2P market is beneficial to the retailer only when the unit purchasing cost is very high, the consumers' acceptance of products in the P2P market is relatively high and the transaction cost in the P2P market is relatively low; second, the consumer surplus is improved by the presence of P2P market; third, when the retailer-run resale market is the only second-hand products market, returned products are sold out; while in presence of the P2P market, the retailer will hold some inventory when the unit purchasing cost is very low; fourth, the selling price of new products is increased and the selling price of second-hand products in the retailer-run resale market is decreased with the emergence of the P2P market while the refund amount is increased in most cases.

1. Introduction

Consumers often buy products only to learn after using them that the products do not fit their preferences or that the products provide lower value than expected. Such products are called experience goods [3,14], or goods with valuation uncertainty: customers do not fully know their actual valuations for the products until after gaining some experience [3]. These products include fashion apparel, jewelry, video games, textbooks, etc. It is obvious that valuation uncertainty of products discourages consumers from purchasing.

To protect consumers from the risk of dissatisfaction with the purchased products, most retailers offer return policies to consumers, i.e., consumers get a full or partial refund for returned products. Consumer returns are quite common today. According to Appriss Retail [2], the value of returned products is about $351 billion in the United States in 2017, which accounts for about 10% of the total sales. Surprisingly, [11] shows that as high as 95% of consumer returns are not defective. Nevertheless, consumer returns usually cannot be sold as new, which greatly impacts retailers' profits. To earn extra revenue from consumer returns, many retailers resell returned goods as second-hand products. Consumer electronics retailers such as Fry's Electronics, Best Buy, and e-tailers such as J and R, Newegg, resell returned products as open-box items at discounted prices [1]. In this paper, we will consider the retailing of electronic products.

To dissuade customers from returning products, many retailers create a number of hurdles for the customers, such as restocking fees, missing receipts, and conditions imposed by stores for the returned goods to be in like-new condition or even unopened [9]. For example, Best Buy and Circuit City used to charge 15% of the purchase price as a restocking fee on unboxed merchandise [12]. JC Penney requires formal wear to be returned in the "original" condition with the return tag in place [12]. Sears requires that returned products are in their original packaging with all accessories, manuals & parts, and original receipts. Except the strict restrictions on returns, retailers limit the trial period for the consumers.

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For many kinds of products, the trial period may be too short for the customers to evaluate the product fit [9]. For example, for the electronic products, the seven-day free trail period in Tmall.com is very short. Therefore, some products that do not meet consumers' expectations cannot be returned to the retailers either because of retailers' strict restrictions on returns or because of short trial period. This probably explains the vast amount of unwanted goods that are resold in the peer to peer (P2P) markets, such as eBay.com, Amazon.com, Craigslist [9].

The impacts of P2P markets on the retailers are twofold. On the one hand, the presence of P2P markets can alleviate consumers' valuation uncertainty as the consumers know that if their returns are not accepted by the retailers, they can sell them through the P2P markets, which provides them a resale value. Thus, the presence of P2P markets increases consumers' willingness to pay for new products. On the other hand, some consumers may strategically wait to buy second-hand products from P2P markets. Therefore, P2P markets cannibalize the supply and demand of second-hand products of retailers, and also the demand of new products of retailers.

The motivation of this paper is to study the impact of a P2P market on a retailer's optimal return policy. We aim to answer the following two questions. First, how will the presence of a P2P market affect the refund amount and the selling prices of new products and second-hand products? Second, how will the presence of a P2P market affect a retailer's profit and consumers' surplus? To answer these questions, we establish a two-period model where the consumers are strategic and uncertain about their valuations. In the first period, the retailer sells new products. At the beginning of the second period, the actual valuations of consumers who bought new products are realized. They can return the products to the retailer to get a refund. If returns are not accepted by the retailer, they can sell them through the P2P market at an exogenous transaction cost. The retailer resells consumer returns in the retailer-run resale market. Consumers who did not buy new products in the first period can buy second-hand products from the retailer-run resale market or the P2P market if the markets exist. The products in the P2P market are devalued by the consumers as they are not in as good condition as those in the retailer-run resale market.

We contribute to the extant literature by characterizing the impact of the presence of a P2P market on a retailer's optimal return policy, retailer's profit and consumer surplus. We consider the competition between a retailer-run resale market and a P2P market, and differentiate the two markets by considering consumers' devaluations of products in the P2P market and the transaction cost in the P2P market. The coexistence of the two markets is realistic and deserves investigation. As benchmarks, we first examine the retailer's optimal return policy when there is only a retailer-run resale market. Then, we analyze the scenario with both the retailer-run resale market and the P2P market. As there are no analytical expressions of the retailer's optimal decisions in the scenario with only retailer-run resale market and the scenario with both markets, we present numerical examples to study the impact of the P2P market on the retailer's optimal decisions and profit. It is demonstrated that the presence of the P2P market is detrimental to the retailer in most cases. The presence of the P2P market is beneficial to the retailer only when the unit purchasing cost is very high, the consumers' acceptance of products in the P2P market is relatively high and the transaction cost in the P2P market is relatively low. Furthermore, the consumer surplus is improved by the presence of P2P market. When the retailer-run resale market is the only second-hand products market, returned products are sold out. While in presence of the P2P market, the retailer will hold some inventory when the unit purchasing cost is very low. In addition, the selling price of new products is increased while the selling price of second-hand products in the retailer-run resale market is decreased due to the presence of the P2P market. In most cases, the refund amount is increased by the emergence of the P2P market.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model description. Section 4 analyzes the retailer's optimal return policy in two scenarios: scenario with only retailer-run resale market and scenario with both retailer-run resale market and P2P market. Section 5 conducts numerical examples to examine the impact of P2P market. Lastly, Section 6 concludes the paper.

2. Literature review

The most relevant literature to our paper is the research on return policies. We focus on the literature investigating product returns due to consumer uncertainty about their own value derived from the product [3]. is among the first to study the return process between retailers and consumers who are risk-averse. It is proven that a return policy can protect consumers and hence allow the retailer to charge a higher retail price [4]. show that a retailer can profit by offering a full-money-back return policy [8]. is the first paper to study nonrefundable charges to manage opportunistic returns, where a refund is requested even though there is a good match [5]. consider the optimal level of hassle with a return policy when a retailer faces opportunistic returns [23]. indicates that a full return policy is never better than a partial refund return policy [19]. and [21] examine the optimal pricing and restocking fee strategies to manage returns of products with valuation uncertainty in a monopolistic market and a competitive market, respectively [20]. study how the equilibrium return policy offered to consumers is affected by the reverse channel structure in processing product returns [15]. analyze how competing retailers' pricing strategies and physical store assistance levels change as a result of the addition of an online channel, where consumers share a common return probability that can be reduced through an investment in assistance [17]. consider how the extent of wardrobing (how many consumers consider such behavior) and the benefit of wardrobing (how much value can be extracted during the trial period) impact firm pricing decisions and profits where consumers are uncertain about their valuations [1]. consider a retailer adopting a "money-back-guaranteed" (MBG) sales policy which allows consumers to return products to the retailer for a full or partial refund. The retailer can salvage the returned products or resell them as open-box items. It is shown that reselling with MBG increases retail sales and profit. Similar to [1], we assume that the retailer resells consumer returns in the retailer-run resale market.

A paper very relevant to the current one is [12] which investigates a retailer's motivation for running its own resale market in presence of an independent resale market, and derives the retailer's optimal returns and resale policy. There are four main differences between [12] and our paper. First, Lee and Rhee [12] consider two consumer options of product disposition concurrently: returning the product in like-new condition to the retailer, or resell them as used at an independent resale market or at the retailer-run resale market if the retailer offers this business. If the consumer returns are not accepted by the retailer, the consumer can only consume the product until the time for resale and resell it as used at the resale market. Furthermore, the retailer liquidates the consumer returns. Different from [12], our focus is the resale of consumer returns. In our model, if the consumer returns are not accepted by the retailer, the consumers can resell their products in the P2P market. In addition, compared to the products in the retailer resale market, there is a valuation discount for the products in the P2P market. Second, we consider strategic customers who can decide whether to buy new products in the first period or to buy second-hand products in the second period. The consumers have different time discount factor for second-hand products [12]. assume that no prospective customers of a new product delay the purchase and buy a used one later for a low price in the resale market. Third, in [12], consumers use the retailer-run resale market if the resale allowance paid to the consumer is greater than the expected resale value at the independent resale market. That is, there is no trade in the independent resale market. While in our model, the retailer resale market and the P2P market coexist because if the consumer returns are denied, they can be resold in the P2P market. Fourth, the resale prices of used products in the retailer-run resale market and independent resale market are exogenous in [12], while in our model the reselling price in the P2P market is determined by the supply and demand of products in that market and the reselling price in the retailer-run resale market is a decision variable of the retailer.

Another related stream of literature considers the impact of various secondary markets on supply chains [18]. consider retailer's decision to buy back used goods from consumers for profitable resale in a retailer-operated used good market. It is shown that the manufacturer makes higher profits from allowing used-good sales alongside new-good sales than from shutting down the retailer-operated used good market [6]. consider a model with two different suppliers selling two different types of a similar good via a common retailer which could establish a P2P platform for trading of used goods. It is shown that, unlike a monopolistic market, intersupplier competition can be beneficial to suppliers in presence of the P2P platform. Furthermore, the presence of P2P platform may be beneficial to suppliers. In [16], the original technology equipment manufacturer (OEM) sells new products and faces competition from a third-party entrant that purchases the used products, refurbishes them, and resells them in a secondary market. The OEM can directly affect the resale value of the products through a relicensing fee charged to the buyer of the refurbished equipment and can effectively "shut down" the secondary market by charging a high enough fee. The key finding is that it is suboptimal for the OEM to shut down the secondary market when consumers have a high willingness to pay for the refurbished product [24]. examine how the sequential emergence of a retail used goods market and a P2P used goods market affect manufacturer's product upgrade strategy and retailer's pricing strategy. They find that the presence of a P2P used goods market may increase the manufacturer's benefit from product upgrades [7]. study how P2P used goods markets affect manufacturers' incentive to offer a returns policy option to retailers. One of the most important findings is that a P2P market selling used goods highly valued by consumers creates the potential for both margin and volume gains for the manufacturer by offering a returns option. Note that, both [24] and [7] do not consider consumer returns due to valuation uncertainty.

Our paper is relevant to [9] which explores the trade-off between consumer valuation uncertainty and P2P platforms. The consumers could sell the mismatched products on a P2P platform which is a decision maker. It is shown that the profit-maximizing platform does not always extract all surplus from the consumers who sell their products through the platform. Furthermore, due to the emergence of a P2P marketplace, both supply chain partners (i.e., supplier and retailer) will be better off if the product's unit cost is sufficiently high and both will be worse off otherwise. Different from [9], we explore the impact of a P2P market on the optimal return policy of a retailer. In [9], the retailer does not accept consumer returns and the consumers can only sell the unwanted products through the P2P platform, while in the current paper, the consumers sell their products in the P2P market only when their returns are not accepted by the retailer.

3. Model description

We consider a retailer selling electronic products to consumers. The retailer's unit purchasing cost is denoted as c. Each consumer buys at most one unit of the product. Market size is normalized to 1. Consumers face uncertainty in their own valuations. For analytical tractability, similar to [9] and [17], we assume that the customers' valuations V are identically and independently drawn from the uniform distribution on the interval [0, 1], i.e., $V\sim U[0, 1]$. These realizations are not known until the customers purchase the products. To protect consumers from the risk of dissatisfaction with the products, the retailer offers a return policy to the consumers. Under the return policy, the retailer accepts consumer returns and offers a refund amount r to each return.

To prevent the customers from abusing return policies, many retailers imposed conditions on the returned goods to be in like-new condition or even unopened [9]. For example, JC Penney requires formal wear to be returned in the "original" condition with the return tag in place [12]. Sears requires that returned products are in their original packaging with all accessories, manuals & parts, and original receipts. Except the strict restrictions on returns, the retailers limit the trial period for the consumers. For example, Tmall.com offers a seven-day free trial period. For the electronic products, the seven-day trial period in Tmall.com is very short for the consumers to evaluate the values of products. Therefore, some consumers cannot return the products that do not meet their expectations to the retailers either because of retailers' strict restrictions on returns or because of short trial period. We define the fraction of consumers who successfully return the products as the successful return rate denoted by θ [12]. assume that the retailer can strategically control the acceptance rate of product return, while we assume the successful return rate to be exogenous because the return restrictions and the length of trial period sometimes are not determined by the retailers themselves, if the retailers sell their products through department store chains (e.g., Macy's, Nordstrom) and sales platforms (e.g., Tmall.com). For practical and analytical reason, suppose $\theta \geq 1/2$. Otherwise, the retailer's return policy is of little significance to the consumers.

Furthermore, we consider return policies with reselling: the retailer accepts consumer returns and resells them as second-hand products. We assume that $c <= 0.5$ to guarantee that the retailer earns positive profit even if it does not provide return policy. All sales of the returned products are final, i.e., once the returned products are sold, they cannot be returned to the retailer.

Suppose there is a P2P market through which the consumers who purchased new products but are not satisfied with them can trade with those who want to buy second-hand products. A selling price p_e is set in the P2P market to clear all the goods sold there. We assume that there is a fixed transaction cost t (where $t \leq c$) paid to the P2P platform if the consumers use the P2P platform to sell their products [10]. In practice, P2P platform fees are posted online and rarely changed, for example, Amazon, Ebay and Etsy [9]. Furthermore, suppose that the refund amount r is always higher than consumers' net revenue from selling products in the P2P market $p_e - t$, which is reasonable as the retailer only accepts returned products in good condition. Therefore, if a consumer could return the product to the retailer successfully, he will not consider selling it in the P2P market. Only if his return is not accepted, he will consider selling it in the P2P market. This is consistent with most of the practice.

Consumers value second-hand products as inferior to new products, and value second-hand products in the P2P market as inferior to those in the retailer-run resale market, as the products in the P2P market are usually not in as good condition as those in the retailer-run resale market. Specifically, a customer with valuation v for a new product values a second-hand product in the retailer-run resale market as δv and values a second-hand product in the P2P market as $\beta\delta v$, where $\delta(0<\delta<1)$ is the time discount factor and $\beta(0<\beta<1)$ is the valuation discount factor. Suppose the time discount factor δ is uniformly distributed on the interval [0, 1] and each consumer knows his own δ. The valuation discount factor β reflects consumers' acceptance of products in the P2P market compared to those in the retailer-run resale market. Suppose that the consumers share the same value of β. Note that, each consumer differs from others by two factors: V and δ. Suppose factors V and δ are independent. In addition, to guarantee that trade occurs in the P2P market, we suppose $\beta > 2t$. That is, compared to the consumers' acceptance of products in the P2P market, the transaction cost in the P2P market is relatively low.

The selling season consists of two periods. In the first period, the retailer replenishes inventory of new products and the consumers buy new products from the retailer. In the second period, consumers who bought new products in Period 1 realize their actual valuations and those with low valuations return the products to the retailer with successful return rate θ. The other $1-\theta$ fraction of the consumers sell their products through the P2P market. Then the retailer resells consumer returns as second-hand products in the retailer-run resale market at price p_s. Therefore, the retailer's decisions include the selling price of new products in the first period p_n, the selling price of second-hand products in the second period p_s and the refund amount r (where $r \le p_n$). The retailer's goal is to maximize his expected profit.

Consumers are rational decision makers and they make purchase as well as keep/return/resell decisions to maximize their individual expected surplus. Consumers make the two decisions sequentially. Initially, the consumers choose whether to buy new products in the first period, to buy second-hand products from the retailer-run resale market or the P2P market in the second period, or not to buy. If consumers buy new products, they decide whether to keep them or return them to the retailer after privately observing their own valuations at the beginning of the second period. If the returned products are not accepted by the retailer, the consumers decide whether to sell them in the P2P market. Let EU_n, EU_s and EU_e denote the expected consumer surplus from buying a new product, or a second-hand product from the retailer-run resale market or the P2P market, respectively. Suppose that a consumer has decided to purchase a new product. Based on the realized valuation v of the product, the consumer surplus from keeping, reselling, or returning for refund will be $v-p_n$, p_e-p_n and $r-p_n$, respectively.

In summary, the sequence of events is as follows.

At the beginning of Period 1, the retailer decides the selling price of new products p_n, the refund amount r and the selling price of returned products p_s, which are announced to the consumers. The consumers form a belief $\bar{p}_e$ about the selling price in the P2P market in Period 2, and make their purchase decisions based on the belief and the retailer's decisions.

At the beginning of Period 2, the actual valuations of consumers who bought products in Period 1 are realized. They decide whether or not to keep the products. If they decide not to keep the products, they can return them to the retailer and get a refund r with successful return rate θ. If their returns are not accepted by the retailer, they can sell them in the P2P market at selling price p_e with transaction cost t. Consumers who did not buy new products in Period 1 can choose whether to buy second-hand products from the retailer-run resale market at price p_s or from the P2P market at price p_e, or not to buy.

For expositional convenience, denote the demand of new products in Period 1 as q_n, among which q_r^s will be returned back to the retailer and q_p^s will be sold in the P2P market, i.e., q_r^s and q_p^s are supply of second-hand products in the two markets, respectively. Denote q_r^d and q_p^d as demand of second-hand products in the retailer-run resale market and the P2P market, respectively. Table 1 presents the notation used in this paper.

Table 1
Notation.

Notation	Meaning
c	Unit purchasing cost
p_n	Selling price of new products
r	Refund amount
p_s	Selling price of second-hand products in the retailer-run resale market
p_e	Selling price of second-hand products in the P2P market
$\bar{p}_e$	Consumers' belief about selling price in the P2P market
q_n	Demand of new products
q_r^s	Supply of second-hand products in the retailer-run resale market
q_p^s	Supply of second-hand products in the P2P market
q_r^d	Demand of second-hand products in the retailer-run resale market
q_p^d	Demand of second-hand products in the P2P market
V	Consumer's valuation of new products
δ	Consumer's time discount factor over second-hand products
β	Consumers' valuation discount factor over products in the P2P market
θ	Successful return rate
t	Transaction cost in the P2P market
Π	Retailer's profit

To depict consumers' belief in the selling price of the P2P market $\bar{p}_e$, we consider the rational expectations (RE) equilibrium [22]. The RE equilibrium (p_n, r, p_s, p_e, $\bar{p}_e$) is an equilibrium that satisfies the following four conditions: (1) given p_n, r, p_s and $\bar{p}_e$, consumers always make decisions that maximize their surplus; (2) $p_e = \bar{p}_e$; (3) $(p_n, r, p_s) = \arg\max_{p_n,r,p_s} \Pi(p_n, r, p_s)$, where Π is the retailer's profit; (4) given p_n, r, p_s and $\bar{p}_e$, $q_p^s(p_e) = q_p^d(p_e)$. In condition (2), given p_n, r, p_s and $\bar{p}_e$, the actual selling price in the P2P market p_e is exactly equal to consumers' belief $\bar{p}_e$. Condition (3) asserts that, given p_e and $\bar{p}_e$, the retailer determines p_n, r and p_s to maximize his expected profit Π. Condition (4) indicates that, given p_n, r, p_s and $\bar{p}_e$, the selling price in the P2P market p_e is set to clear all the goods sold there.

4. Retailer's optimal return policy

In this section, we analyze the retailer's optimal return policy. Sections 4.1 and 4.2 analyze the scenario with only retailer-run resale market, and the scenario with both markets, respectively. Represent the two scenarios by superscripts "R" and "B", respectively. The superscripts "$*R$" and "$*B$" represent the optimal decisions in the two scenarios.

4.1. Scenario with only retailer-run resale market

In this subsection, we analyze the scenario with only retailer-run resale market. In Scenario R, if the consumers are dissatisfied with the products they purchased in Period 1, they can only return them to the retailer with successful return rate θ.

We first analyze consumers' decisions. In Period 1, if a consumer buys a new product, his expected consumer surplus is $EU_n = E(max\{V, \theta r + (1-\theta)V\} - p_n) = (1+\theta r^2)/2 - p_n$; if he decides to buy a second-hand product from the retailer-run resale market in Period 2, the expected consumer surplus is $EU_s = E(\delta V - p_s) = \delta/2 - p_s$. Therefore, a consumer buys a new product if and only if

$$\frac{1+\theta r^2}{2} - p_n \ge \max\left\{\frac{\delta}{2} - p_s, 0\right\}. \quad (1)$$

From (1), we can derive the range of δ for consumers who buy new products, i.e., $\delta \le \min\{1+\theta r^2 - 2p_n + 2p_s, 1\}$. Since δ is uniformly distributed on the interval [0,1], the demand for new products is:

$$q_n = \begin{cases} 0, & \text{if } \frac{1+\theta r^2}{2} - p_n < 0, \\ \min\{1+\theta r^2 - 2p_n + 2p_s, 1\}, & \text{if } \frac{1+\theta r^2}{2} - p_n \geq 0. \end{cases} \quad (2)$$

According to (2), if $(1+\theta r^2)/2 - p_n < 0$, no consumer will purchase new products. Thus, the retailer's profit will be 0. This is certainly not optimal for the retailer. In the following, we only need to figure out the retailer's optimal decisions under the condition $(1+\theta r^2)/2 - p_n \geq 0$.

In Period 2, for the consumers who have purchased new products in Period 1, if their realized valuation v is less than the refund amount r, they can return them to the retailer with successful return rate θ. That is, only θ fraction of the consumers can successfully return their products, while the other $1-\theta$ fraction can only keep the products. Since $V \sim U[0, 1]$, the proportion of consumers who will successfully return products is $\theta Prob(V < r) = \theta r$. Thus, the quantity of consumer returns is $q_r^s = \theta r q_n$. Given that $(1+\theta r^2)/2 - p_n \geq 0$, the customers who have not purchased new products in Period 1 will purchase second-hand products in Period 2, i.e., $EU_s > 0$ holds. This follows as below. First, the inequality $(1+\theta r^2)/2 - p_n \geq 0$ is equivalent to $EU_n \geq 0$. Second, a consumer does not buy a new product in Period 1 if and only if the inequality $\max\{EU_s, 0\} > EU_n$ holds. As $EU_n \geq 0$ holds, the inequality $EU_s > 0$ holds. Therefore, the demand for second-hand products is $q_r^d = 1 - q_n$. Under the condition $(1+\theta r^2)/2 - p_n \geq 0$, the expressions of q_n, q_r^s and q_r^d are as follows:

$$\begin{cases} q_n = \min\{1+\theta r^2 - 2p_n + 2p_s, 1\}, \\ q_r^s = \theta r q_n, \\ q_r^d = 1 - q_n. \end{cases} \quad (3)$$

Fig. 1 depicts consumers' decisions under different values of parameters V and δ. When the time discount factor δ is large, consumers will buy second-hand products from the retailer-run resale market, while when δ is small, consumers will buy new products. After realizing their true valuations, those who have high valuations will keep the products, while for those with low valuations, θ proportion of them will return the products to the retailer successfully and $1-\theta$ proportion of them will have to keep the products.

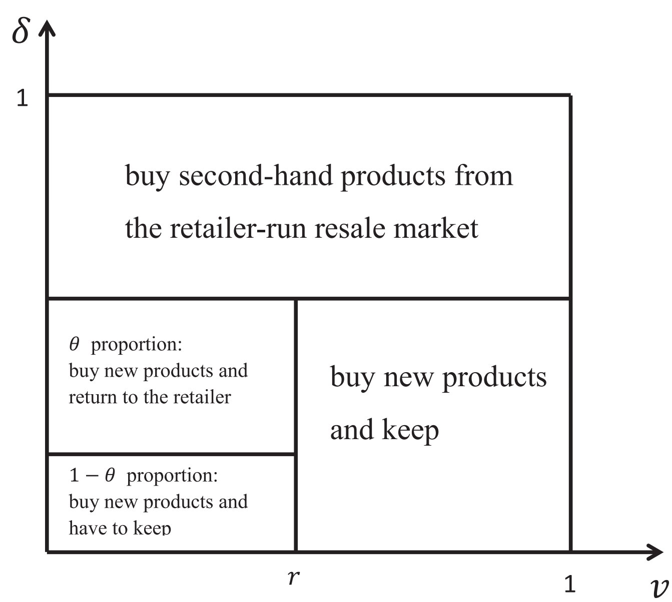

Fig. 1. Consumers' decisions in Scenario *R*.

The retailer's optimization problem is:

$$\max_{p_n, r, p_s} \Pi^R = \underbrace{p_n q_n}_{\text{new product sales}} - \underbrace{r q_r^s}_{\text{returns}} + \underbrace{p_s \min\{q_r^s, q_r^d\}}_{\text{second-hand product sales}} - \underbrace{c q_n}_{\text{purchasing cost}},$$

$$s.t. \quad \frac{1+\theta r^2}{2} - p_n \geq 0. \quad (4)$$

where q_n, q_r^s and q_r^d are as shown in Eq. (3). Proposition 1 establishes the retailer's optimal decisions in Scenario *R*. Proofs of all propositions and corollaries are presented in Appendix.

Proposition 1. *When there is only a retailer-run resale market, the optimal selling price of second-hand products* p_s^{*R} *is the root of the equation* $16\theta p_s^3 - 4[2\theta(1-c) - 1]p_s^2 - 1 = 0$ *on the interval* (0, 1/2) *(which is unique), the optimal refund* $r^{*R} = [1/(2p_s^{*R}) - 1]/\theta$, *and the optimal selling price of new products* $p_n^{*R} = [1 + \theta(r^{*R})^2]/2$.

In scenario *R*, the customers' expected surplus from buying new products is 0, which indicates that the optimal selling price of new products is set high enough to extract customers' expected surplus entirely. The retailer cannot extract customers' expected surplus entirely when customers buy second-hand products as the customers have different value of δ. This result is different from that in [1] which shows that, for both new and second-hand products, the optimal selling prices are set high enough to extract customers' expected surplus entirely. The difference is because [1] do not consider the heterogeneity of customers' time discount factor.

Corollary 1. *In Scenario R,* $p_n^{*R} > r^{*R}$ *and* $p_s^{*R} > r^{*R}$ *hold.*

In Corollary 1, $p_n^{*R} > r^{*R}$ implies that a partial refund is optimal for the retailer and $p_s^{*R} > r^{*R}$ indicates that the retailer is always beneficial from reselling returned products. This is different from the result in [12] which demonstrates that the retailer always give all the revenue from selling a used unit to the owner who demands the resale. The reason for the difference is as follows: in our model, the selling price of second-hand products is determined by the retailer while in [12] the resale price of used products is fixed. Corollary 2 examines the impact of unit purchasing cost and successful return rate on the retailer's decisions and profit.

Corollary 2. *(1) As the unit purchasing cost c increases, the optimal selling price of new products* p_n^{*R} *and the optimal refund* r^{*R} *increase, while the optimal selling price of second-hand products* p_s^{*R} *decreases;*

(2) As the successful return rate θ *increases, the optimal selling price of second-hand products* p_s^{*R} *decreases;*

(3) The retailer's profit Π^{*R} *decreases in c and increases in* θ.

Part (1) of Corollary 2 shows that, as the unit purchasing cost increases, the retailer increases the selling price of new products, which is intuitive. To guarantee that the consumers have incentives to buy new products, the refund amount should be increased correspondingly. As the refund amount increases, more consumers will choose to return their products to the retailer, resulting in an increase in the supply of second-hand products. Thus, the selling price of second-hand products decreases. Part (2) indicates that as the successful return rate increases, the selling price of second-hand products decreases. This is because that as the successful return rate increases, more consumers will return their products successfully to the retailer. The increase in the supply of products in the retailer-run resale market results in the decrease of the selling price of second-hand products. Part (3) demonstrates that the retailer's profit decreases in the unit purchasing cost, which is intuitive. Furthermore, the retailer's profit increases in the successful return rate. The interpretation is as follows. On one hand, as the successful return rate increases, the consumers are more likely to successfully return the products they are dissatisfied with, thus they will have more incentives to buy the products. On the other hand, although higher successful return rate brings more consumer returns, the products returned are sold twice and bring higher marginal profit compared to the products that are not returned. Specifically, the marginal profit for the products that are not returned is $p_n^{*R} - c$, while the marginal profit for the returned products is $p_n^{*R} - c - r^{*R} + p_s^{*R}$, where $p_s^{*R} > r^{*R}$. Therefore, the increase in the successful return rate is beneficial to the retailer.

Corollary 3. *Returned products are sold out, i.e.,* $q_r^s = q_r^d$.

Corollary 3 indicates that there is no left inventory of consumer returns. According to Proposition 1, the selling price of second-hand products p_s^{*R} is higher than the refund amount r^{*R}. Corollary 3 implies that the retailer resells all the returned products to gain additional profit.

4.2. Scenario with both retailer-run resale market and P2P market

In this subsection, we examine the scenario with both retailer-run resale market and P2P market. In Scenario *B*, if the consumers are dissatisfied with the new products they purchased in Period 1, they can return them to the retailer with successful return rate θ. If returns are denied, the consumers can sell them through the P2P market.

We first analyze consumers' purchase decisions. In Period 1, a consumer makes his purchasing decision based on the retailer's decisions and his belief about the selling prices in the P2P market $\bar{p}_e$. The consumer's expected surplus from buying a new product is

$$EU_n = E\max\{V, \theta r + (1-\theta)(\bar{p}_e - t), \theta r + (1-\theta)V\} - p_n = \frac{1 + \theta r^2 + (1-\theta)(\bar{p}_e - t)^2}{2} - p_n. \quad (5)$$

The interpretation of (5) is as follows. Given that a consumer buys a new product, if he keeps the product, the valuation is V; if he returns the product to the retailer successfully with probability θ, he obtains the refund amount r; if the return is denied with probability $1-\theta$, he can keep the product and obtain the valuation V, or resell the product in the P2P market and obtain net revenue $\bar{p}_e - t$.

If the consumer waits until Period 2, he could choose to buy a second-hand product from the retailer-run resale market or from the P2P market, or not to buy. The corresponding expected surpluses are EU_s, $E\bar{U}_e$ and 0, respectively, where

$$EU_s = \delta EV - p_s = \frac{\delta}{2} - p_s, \quad (6)$$

$$E\bar{U}_e = \beta\delta EV - \bar{p}_e = \frac{\beta\delta}{2} - \bar{p}_e. \quad (7)$$

Therefore, a consumer will buy a new product in Period 1 if $EU_n \geq \max\{EU_s, E\bar{U}_e, 0\}$, and will wait until Period 2 if $\max\{EU_s, E\bar{U}_e\} \geq \max\{EU_n, 0\}$. Otherwise, he will leave the market.

In Period 2, the selling prices in the P2P market p_e is observed by the consumers. The consumers who wait until Period 2 make their purchase decisions. A consumer's surpluses from buying a second-hand product in the retailer-run resale market or in the P2P market, or from not buying, are $EU_s = \delta EV - p_s = \delta/2 - p_s$, $EU_e = \beta\delta EV - p_e = \beta\delta/2 - p_e$ and 0, respectively. The consumer will choose the option that maximizes the surplus.

Next, we analyze consumers' keep/resell/return decisions. In Period 2, consumers who bought new products in Period 1 privately observe their own valuations. Based on the realized valuation v of the product, the consumers' surpluses from keeping, reselling, or returning for refund will be $v - p_n$, $p_e - t - p_n$ and $r - p_n$, respectively. If the realized valuation v is lower than r, the consumers will choose to return the products to the retailer. Among the consumers who want to return products, θ proportion will be accepted and the other $1-\theta$ proportion will be denied. If a consumer's return is denied, he will choose to sell his product through the P2P market if the selling price in the P2P market p_e minus the transaction cost t is higher than his realized valuation v.

According to Eq. (5), consumers' expected surplus from buying a new product is $EU_n = [1 + \theta r^2 + (1-\theta)(\bar{p}_e - t)^2]/2 - p_n$. It is obvious that we only need to consider the case where $EU_n \geq 0$ holds. Under the condition $EU_n \geq 0$, according to Eqs. (5)–(7), it can be easily shown that the expressions of q_n, q_r^s, q_r^d, q_p^s and q_p^d are as follows:

$$\begin{cases} q_n = \min\{\delta_1, \delta_2, 1\}, \\ q_r^s = \theta r q_n, \\ q_r^d = \max\left\{1 - \frac{2(p_s - \bar{p}_e)}{1-\beta}, 0\right\}, \\ q_p^s = (1-\theta)(\bar{p}_e - t) q_n, \\ q_p^d = \max\left\{\frac{2(p_s - \bar{p}_e)}{1-\beta} - q_n, 0\right\}, \end{cases} \quad (8)$$

where

$$\delta_1 = 1 + \theta r^2 + (1-\theta)(\bar{p}_e - t)^2 - 2p_n + 2p_s,$$
$$\delta_2 = \frac{1 + \theta r^2 + (1-\theta)(\bar{p}_e - t)^2 - 2p_n + 2\bar{p}_e}{\beta}. \quad (9)$$

Fig. 2 depicts consumers' decisions under different values of parameters V and δ. When the time discount factor δ is relatively large, consumers will buy second-hand products from the retailer-run resale market; when the time discount factor δ is at a medium level, consumers will buy second-hand products from the P2P market; while when δ is small, consumers will buy new products. After realizing their true valuations, those who have high valuations will keep the products, while those with low valuations will want to return the products to the retailer. Among the consumers who want to return the products, θ proportion of them can return the products successfully. For the other $1-\theta$ proportion, if their realized valuation is higher than $p_e - t$, they will keep the product, otherwise, they sell the products through the P2P market.

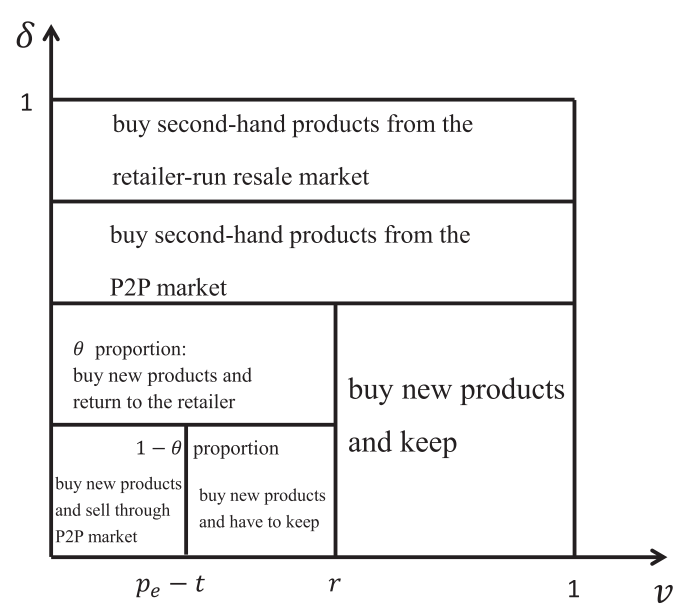

Fig. 2. Consumers' decisions in Scenario *B*.

The retailer's optimization problem is as follows.

$$\max_{p_n, r, p_s} \quad \Pi^B = p_n q_n - r q_r^s + p_s \min\{q_r^d, q_r^s\} - c q_n$$

$$s.t. \quad \begin{cases} \frac{1+\theta r^2 + (1-\theta)(\bar{p}_e - t)^2}{2} - p_n \geq 0, \\ q_n \leq \frac{2(p_s - \bar{p}_e)}{1-\beta} \leq 1, \\ r \quad \geq p_e - t, \\ p_e = \bar{p}_e, \\ q_p^s = q_p^d, \end{cases} \tag{10}$$

where q_n, q_r^s, q_r^d, q_p^s and q_p^d are as shown in Eq. (8).

For ease of expression, denote

$$\begin{cases} p_{e1}^{*B} = & \begin{cases} p_{e1}^*, & \text{if } p_{e1}^* \geq \frac{\sqrt{2\beta+(1-t)^2}-(1-t)}{2}, \\ \frac{\sqrt{2\beta+(1-t)^2}-(1-t)}{2}, & \text{if } p_{e1}^* < \frac{\sqrt{2\beta+(1-t)^2}-(1-t)}{2}, \end{cases} \\ r_1^{*B} = & p_{e1}^{*B} - t, \\ p_{s1}^{*B} = & p_{e1}^{*B}\left[1 + (1-\beta)(1-\theta)(p_{e1}^{*B} - t)\right]/\beta, \\ p_{n1}^{*B} = & \left[1 + \theta\left(r_1^{*B}\right)^2 + (1-\theta)(p_{e1}^{*B} - t)^2\right]/2, \end{cases} \tag{11}$$

and

$$\begin{cases} p_{e2}^{*B} = & \begin{cases} p_{e2}^*, & \text{if } t \leq p_{e2}^* \leq \frac{\sqrt{2\beta+(1-t)^2}-(1-t)}{2}, \\ \frac{\sqrt{2\beta+(1-t)^2}-(1-t)}{2}, & \text{if } p_{e2}^* \geq \frac{\sqrt{2\beta+(1-t)^2}-(1-t)}{2}, \end{cases} \\ r_2^{*B} = & \left[\beta/2p_{e2}^{*B} - 1 - (1-\theta)\left(p_{e2}^{*B} - t\right)\right]/\theta, \\ p_{s2}^{*B} = & p_{e2}^{*B}\left[1 + (1-\beta)(1-\theta)\left(p_{e2}^{*B} - t\right)\right]/\beta, \\ p_{n2}^{*B} = & \left[1 + \theta\left(r_2^{*B}\right)^2 + (1-\theta)\left(p_{e2}^{*B} - t\right)^2\right]/2, \end{cases} \tag{12}$$

where p_{e1}^* is the unique real root of $\partial \Pi_1^B / \partial p_e = 0$, in which

$$\Pi_1^B = \frac{p_e}{\beta^2}\{\beta[1 - (2\theta - 1)(p_e - t)^2 - 2c] + [1 + (1-\beta)(1-\theta)(p_e - t)] [\beta - 2p_e[1 + (1-\theta)(p_e - t)]]\}, \tag{13}$$

and p_{e2}^* is the unique real root of $\partial \Pi_2^B / \partial p_e = 0$, in which

$$\Pi_2^B = \frac{p_e}{\beta^2}\left\{\beta\left[1 - \frac{1}{\theta}\left[\frac{\beta^2}{4p_e^2} - \frac{\beta[1 + (1-\theta)(p_e - t)]}{p_e} + [1 + (1-\theta)(p_e - t)]^2\right] + (1-\theta)(p_e - t)^2 - 2c\right] + [1 + (1-\beta)(1-\theta)(p_e - t)] [\beta - 2p_e[1 + (1-\theta)(p_e - t)]]\right\}. \tag{14}$$

Proposition 2 presents the retailer's optimal decisions in Scenario *B*.

Proposition 2. *When there are both retailer-run resale market and P2P market, the optimal selling price of new products* p_n^{*B}*, the optimal refund* r^{*B}*, the optimal selling price in the retailer-run resale market* p_s^{*B}*, and the selling price in the P2P market in equilibrium* p_e^{*B} *satisfy* $(p_n^{*B}, r^{*B}, p_s^{*B}, p_e^{*B}) = (p_{ni}^{*B}, r_i^{*B}, p_{si}^{*B}, p_{ei}^{*B})$ *if* $\Pi_i^B \geq \Pi_j^B$ $(i, j = 1, 2$ *and* $i \neq j)$*, where* $(p_{ni}^{*B}, r_i^{*B}, p_{si}^{*B}, p_{ei}^{*B})$ *and* Π_i^B $(i = 1, 2)$*are stated in Eqs. (11)–(14).*

Note that, under both $(p_{n1}^{*B}, r_1^{*B}, p_{s1}^{*B}, p_{e1}^{*B})$ and $(p_{n2}^{*B}, r_2^{*B}, p_{s2}^{*B}, p_{e2}^{*B})$, the customers' expected surplus from buying new products in Scenario *B* is $EU_n = [1 + \theta(r_i^{*B})^2 + (1-\theta)(p_{ei}^{*B} - t)^2]/2 - p_{ni}^{*B} = 0$ $(i = 1, 2)$. This indicates that the optimal selling price of new products in Scenario *B* is set high enough to extract customers' expected surplus entirely, which is similar to the result in Scenario *R*.

According to the proof of Proposition 2, we have the following corollary.

Corollary 4. *(1) Under* $(p_{n1}^{*B}, r_1^{*B}, p_{s1}^{*B}, p_{e1}^{*B})$*, there is some left inventory of returned products, i.e.,* $q_r^s > q_r^d$*;*

(2) under $(p_{n2}^{*B}, r_2^{*B}, p_{s2}^{*B}, p_{e2}^{*B})$*, returned products are sold out, i.e.,* $q_r^s = q_r^d$*.*

As there are no analytical expressions of the retailer's optimal decisions, we cannot give the conditions explicitly under which $(p_{ni}^{*B}, r_i^{*B}, p_{si}^{*B}, p_{ei}^{*B})$ $(i = 1, 2)$ is obtained. In Section 5, we will analyze the conditions under which different equilibrium results are obtained.

5. Numerical analysis

According to Propositions 1 and 2, there are no analytical expressions of the retailer's optimal decisions in Scenarios *R* and *B*. In this section, we conduct numerical analysis to study the equilibrium results in Scenario *B* and the impact of the P2P market on the retailer's optimal decisions and profit.

The values of the parameters are presented in Table 2. When combined, these parameters define 4*3*4*7=336 cases.

We first investigate the parameter values under which the conditions $\beta > 2t$ and $t \leq c$ hold. Table 3 presents the parameter values under which $\beta \leq 2t$ or $t > c$ hold, where symbol "-" means all values of that parameter. Table 3 includes 3*4*3+4*3+3*7=69 cases. In the following, we will concentrate on the other 336-69=267 cases.

5.1. Analysis of scenario with both retailer-run resale market and P2P market

In this subsection, we study the equilibrium results in Scenario *B*. Observation 1 analyzes the conditions under which different equilibrium results $(p_{ni}^{*B}, r_i^{*B}, p_{si}^{*B}, p_{ei}^{*B})$ $(i = 1, 2)$ are obtained.

Observation 1. *In Scenario B,* $\Pi_1^B > \Pi_2^B$ *holds when* $c = 0.02$*.*

According to Observation 1, $(p_{n1}^{*B}, r_1^{*B}, p_{s1}^{*B}, p_{e1}^{*B})$ is obtained when $c = 0.02$ while $(p_{n2}^{*B}, r_2^{*B}, p_{s2}^{*B}, p_{e2}^{*B})$ is obtained when $c = [0.05, 0.1, 0.2, 0.3, 0.4, 0.5]$. Referring to Corollary 4, returned products are sold out under $(p_{n2}^{*B}, r_2^{*B}, p_{s2}^{*B}, p_{e2}^{*B})$, while there is some left inventory of returned products under $(p_{n1}^{*B}, r_1^{*B}, p_{s1}^{*B}, p_{e1}^{*B})$. Therefore, Observation 1 implies that, returned products are sold out when $c = [0.05, 0.1, 0.2, 0.3, 0.4, 0.5]$ while there is some left inventory of returned products when $c = 0.02$. That is, when the unit purchasing cost is very low, the retailer will hold some inventory to increase the selling price of second-hand products in the retailer-run resale market. While when the unit purchasing cost is relatively high, the cost burden brought by left inventory is high. Thus, the retailer will sell out consumer returns.

Table 2
Parameter values.

Parameters	Values
β	[0.2,0.4,0.6,0.8]
θ	[0.7,0.8,0.9]
t	[0,0.025,0.05,0.1]
c	[0.02,0.05,0.1,0.2,0.3,0.4,0.5]

Table 3
Parameter values under which $\beta \leq 2t$ or $t > c$ hold.

Parameters	c	t	β	θ
	0.02	[0.025,0.05,0.1]	–	–
	0.05	0.1	–	–
	–	0.1	0.2	–

5.2. The impact of P2P market

In this subsection, we analyze the impact of the P2P market on the retailer's optimal decisions and profit. Observation 2 compares the optimal selling prices of new products and second-hand products in the retailer-run resale market, and also the demand of new products in Scenarios *R* and *B*.

Observation 2. *$p_n^{*B} \geq p_n^{*R}$, $p_s^{*B} \leq p_s^{*R}$ and $q_n^{*B} \leq q_n^{*R}$ always hold.*

First, Observation 2 shows that the selling price of new products in Scenario *B* is always higher than that in Scenario *R*. The reason is that, in Scenario *B*, for the consumers who are not satisfied with the new products they bought, if they cannot return their products to the retailer, they could sell them in the P2P market. That is, the consumers in Scenario *B* have more options to dispose of the products they are not satisfied with. Thus, they have higher willingness to pay for the new products. Therefore, the retailer could charge a higher selling price of new products.

Second, numerical examples show that the selling price in the retailer-run resale market in Scenario *B* (i.e., p_s^{*B}) is always lower than that in Scenario *R* (i.e., p_s^{*R}), which is due to the competition from the P2P market. As the P2P market competes with the retailer-run resale market over customers, the retailer-run resale market will lower the selling price of second-hand products in Scenario *B* than that in Scenario *R*.

Third, the demand of new products in Scenario *B* is lower than that in Scenario *R*. The reason is as follows. Both in Scenarios *R* and *B*, the customers' expected surplus from buying new products is zero. As the selling price in the retailer-run resale market in Scenario *B* is always lower than that in Scenario *R*, more consumers will buy second-hand products in Scenario *B*. Furthermore, consumers could buy second-hand products from the P2P market in Scenario *B*. Therefore, the demand of new products in Scenario *B* is lower than that in Scenario *R*.

Observation 3 compares the optimal refund amount in Scenarios *R* and *B*.

Observation 3. *$r^{*B} \geq r^{*R}$ holds under the parameter values in Table 4.*

Observation 3 indicates that in most cases the optimal refund amount in Scenario *B* is higher than that in Scenario *R*. From Table 4, we observe that, r^{*B} is higher than r^{*R} in the following two cases. First, the unit purchasing cost is relatively low; second, the unit purchasing cost is high, the consumers' acceptance of products in the P2P market is relatively high and the transaction cost in the P2P market is relatively low. In the former case, when the unit purchasing cost is relatively low, the selling price of new products is relatively low. Although the presence of the P2P market stimulates the customers to buy new products, when the selling price of new products is relatively low, this demand stimulation effect is not strong. Recall from Observation 2 that the selling price of new products in Scenario *B* is higher than that in Scenario *R*. To encourage the customers to buy new products, the retailer should raise the refund amount. In the latter case, as the consumers' acceptance of products in the P2P market is relatively high and the transaction cost in the P2P market is relatively low, the consumers could sell their products in the P2P market at a relatively high price. Due to the competition from the P2P market, the retailer offers a higher refund amount than that in Scenario *R*.

Table 4
Parameter values under which $r^{*B} \geq r^{*R}$ holds.

Parameters	c	β	θ	t
	[0.02,0.05,0.1,0.2,0.3]	–	–	–
	0.4	0.4	–	–
	0.4	0.6	[0.7,0.8]	0
	0.4	0.6	0.9	[0,0.025]
	0.4	0.8	[0.7,0.8]	[0,0.025,0.05]
	0.4	0.8	0.9	[0,0.025,0.05,0.1]
	0.5	0.8	0.7	0
	0.5	0.8	[0.8,0.9]	[0,0.025]

Table 5
Parameter values under which $\Pi^{*B} \geq \Pi^{*R}$ holds.

Parameters	c	β	θ	t
	0.5	0.4	[0.7,0.8,0.9]	0
	0.5	0.6	[0.7,0.8,0.9]	[0,0.025,0.05]
	0.5	0.8	[0.7,0.8,0.9]	[0,0.025,0.05,0.1]

Observation 4 compares the retailer's profit in Scenarios *R* and *B*.

Observation 4. *The retailer's profit in Scenario B is higher than that in Scenario R under the parameter values in Table 5.*

Observation 4 indicates that in most cases, the retailer's profit in Scenario *B* is lower than that in Scenario *R*, which implies that the presence of the P2P market is detrimental to the retailer. From Table 4, we observe that, when the unit purchasing cost is very high, the consumers' acceptance of products in the P2P market is relatively high and the transaction cost in the P2P market is relatively low, the presence of the P2P market is beneficial to the retailer. The interpretation of the result is as follows. On one hand, the P2P market competes with the new product market and the retailer-run resale market over customers (competition effect). On the other hand, due to the emergence of the P2P market, the consumers have more options to dispose of the products they are not satisfied with, which stimulates the customers to buy new products (stimulation effect). When the unit purchasing cost is very high, the selling products of new products is very high, if there is no P2P market, the consumers are reluctant to buy new products because with a probability $1-\theta$ the consumers cannot return the products to the retailer. In presence of the P2P market, when the consumers' acceptance of products in the P2P market is relatively high and the transaction cost in the P2P market is relatively low, the consumers could sell their products in the P2P market at a relatively high price. Thus, when the unit purchasing cost is very high, the consumers' acceptance of products in the P2P market is relatively high and the transaction cost in the P2P market is relatively low, the simulation effect of the P2P market is very strong which dominates the competition effect. Therefore, in this case, the presence of the P2P market is beneficial to the retailer.

In practice, different firms have different attitudes towards second-hand goods markets. One opinion is that some consumers who intended to buy new products turn to the second-hand goods markets, thus reducing the sales of new products. Therefore, the presence of second-hand goods markets hurts the profits of firms. For example, Sun Microsystems (Sun), one of the leading firms in the IT server business, was criticized for "deliberately attempting to eliminate the secondary market for its machines worldwide" through their pricing and licensing schemes [13]. The other opinion is that the existence of second-hand goods markets provides a platform for consumers who have bought new products to dispose of their second-hand products, thus promoting consumers to buy new products. Therefore, the existence of second-hand markets contributes to the sales of new products. For instance, IBM and Hewlett Packard create high resale values for their used equipment by facilitating the resale process and secondary use so that the original customers gain a higher net benefit from their new product purchases [16]. Our study sheds light on how the presence of P2P second-hand goods markets impact retailers optimal return policies and profits when the retailers operate their only second-hand goods markets. According to Observation 4, if the unit purchasing cost of the retailers is very high, the consumers' acceptance of products in the P2P market is relatively high and the transaction cost in the P2P market is relatively low, the retailers should embrace the P2P second-hand goods market. Otherwise, they could try to eliminate the P2P second-hand goods market.

Observation 5 compares the consumer surplus in Scenarios *R* and *B*.

Observation 5. *Compared to Scenario R, the consumer surplus is improved in Scenario B.*

Observation 5 states that consumer surplus is improved by the presence of P2P market. Note that, in both scenarios, consumers' expected surpluses from buying new products are both zero. According to Observation 2, the demand of new products in Scenario *B* is lower than that in Scenario *R*. For the consumers who buy second-hand products from the retailer-run resale market, as $p_s^{*B} \leq p_s^{*R}$ holds, consumers' expected surplus in Scenario *B* is higher than that in Scenario *R*. What is more, in Scenario *B*, the consumers could buy second-hand products from the P2P market. For those consumers who buy second-hand products from the P2P market, they obtain higher expected surplus than buying from the retailer-run resale market. Therefore, consumer surplus is improved by the presence of the P2P market.

5.3. The impact of neglecting P2P market

To analyze the importance of capturing the existence of a P2P market, we consider a heuristic as follows: the retailer neglects the existence of the P2P market and adopts the optimal decisions when there is only retailer-run resale market, i.e., p_n^{*R}, r^{*R} and p_s^{*R}. Denote the results in the heuristic by superscript *H*. Therefore, $p_n^{*H} = p_n^{*R}$, $r^{*H} = r^{*R}$ and $p_s^{*H} = p_s^{*R}$ hold in the heuristic. According to the proof of Proposition 2, the retailer's optimization problem in Scenario *B* is as follows:

$$\max_{p_n, r, p_s} \quad \Pi^B = (p_n - \theta r^2 - c)q_n + p_s \min\left\{\theta r q_n, 1 - \frac{2(p_s - p_e)}{1-\beta}\right\}$$

$$s.t. \begin{cases} q_n = \frac{2(p_s - p_e)}{(1-\beta)[1+(1-\theta)(p_e - t)]} = \frac{1+\theta r^2 + (1-\theta)(p_e-t)^2 - 2p_n + 2p_e}{\beta}, \\ \frac{2(p_s-p_e)}{(1-\beta)} \leq 1, \\ \frac{1+\theta r^2 + (1-\theta)(\bar{p}_e - t)^2}{2} - p_n \geq 0, \\ r \geq p_e - t. \end{cases} \tag{15}$$

In the heuristic, the selling price in the P2P market p_e^{*H} are determined as follows: substitute p_n^{*H}, r^{*H} and p_s^{*H} into optimization problem (15) and solve out p_e^{**} from the equations in (15). If p_n^{*H}, r^{*H}, p_s^{*H} and p_e^{**} satisfy the three inequalities in (15), then $p_e^{*H} = p_e^{**}$. We notice that p_n^{*H}, r^{*H}, p_s^{*H} and p_e^{**} always satisfy the first two inequalities in (15), but for most of the numerical examples, they do not satisfy the third inequality. This means that, the refund amount set by the retailer in Scenario *R* (i.e., r^{*R}) is too low, such that the refund amount is lower than consumers' net revenue from selling products in the P2P market. As a result, the consumers will not return their products to the retailer and thus there is no trade in the retailer-run resale market. Therefore, it degenerates into the scenario with only P2P market. According to Appendix E, the retailer's optimization problem in the scenario with only P2P market is (16). Substitute p_n^{*H} into (16) and solve out p_e^{***} from the equations in (16). We notice that p_n^{*H} and p_e^{***} always satisfy the inequalities in (16). Therefore, $p_e^{*H} = p_e^{***}$ holds in the heuristic.

$$\max_{p_n} \quad \Pi^E = (p_n - c)q_n,$$

$$s.t. \begin{cases} \frac{1+(p_e-t)^2}{2} - p_n \geq 0, \\ q_n = \frac{1}{1+p_e-t} = \frac{1+(p_e-t)^2 - 2p_n + 2p_e}{\beta}, \\ p_e - t \geq 0. \end{cases} \tag{16}$$

Denote Π^H as the retailer's profit in the heuristic. Numerical examples demonstrate that the profit loss (defined as $(\Pi^B - \Pi^H)/\Pi^B$) due to neglecting the presence of the P2P market is remarkable, which can be as high as 72%. Furthermore, the profit loss is relatively high when the unit purchasing cost is very high, the consumers' acceptance of products in the P2P market is relatively high and the transaction cost in the P2P market is relatively low. According to the analysis of Observation 4, in this condition, the simulation effect of the P2P market is very strong. Therefore, if the retailer neglects the presence of the P2P market, the profit loss would be significant.

6. Conclusions

In this paper, we consider how the presence of a P2P market affects a retailer's optimal return policy. The consumers are strategic and uncertain about the valuation of products. If consumers' returns are not accepted by the retailer, consumers can sell their products in the P2P market. The retailer resells consumer returns in the retailer-run resale market. Theoretical and numerical results show that, the presence of the P2P market is detrimental to the retailer in most cases. The presence of the P2P market is beneficial to the retailer only when the unit purchasing cost is very high, the consumers' acceptance of products in the P2P market is relatively high and the transaction cost in the P2P market is relatively low. Furthermore, the consumer surplus is improved by the presence of P2P market. Due to the presence of the P2P market, the selling price of new products increases while the selling price of second-hand products in the retailer-run resale market decreases. In most cases, the refund amount increases due to the emergence of the P2P market. Specifically, this holds in the following two cases: first, the unit purchasing cost is relatively low; second, the unit purchasing cost is high, the consumers' acceptance of products in the P2P market is relatively high and the transaction cost in the P2P market is relatively low. In addition, when the retailer-run resale market is the only second-hand products market, returned products are sold out. While in presence of the P2P market, the retailer will hold some inventory when the unit purchasing cost is very low, and the retailer will sell out consumer returns when the unit purchasing cost is relatively high.

There are some directions for future research. First, we assume that the distributions of the consumers' ex ante valuations about the product are identical. In practice, some consumers have more information about the product's valuation. Thus, consumers can be classified into different categories according to the information they have about the product's valuation. Second, we assume that the successful return rate of the consumers is exogenous, while it can be determined endogenously by retailers [12]. Third, the retailer always resells the consumer returns in our model while in practice some retailers salvage the consumer returns to protect their brand image. Fourth, some empirical evidence would be useful to test the validity of the conclusions.

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Appendix A. Proof of Proposition 1

According to (4), the retailer's optimization problem is:

$$\max_{p_n,r,p_s} \quad \Pi^R = (p_n - c)q_n - rq_r^s + p_s q_r^d,$$
$$s.t. \quad \frac{1+\theta r^2}{2} - p_n \geq 0. \tag{A.1}$$

If $1+\theta r^2 - 2p_n + 2p_s > 1$, then $q_n = 1$, $q_r^d = 0$, $q_r^s = \theta r$. Substituting the expressions of q_n, q_r^s and q_r^d into (A.1), we have

$$\max_{p_n,r,p_s} \quad \Pi^R = p_n - c - \theta r^2,$$
$$s.t. \quad \frac{1+\theta r^2}{2} - p_n \geq 0. \tag{A.2}$$

Note that, the constraint functions and the objective function in (A.2) are irrelevant to p_s. We can decrease p_s so that $1+\theta r^2 - 2p_n + 2p_s = 1$. Thus, we only need to solve (A.1) under the condition $1+\theta r^2 - 2p_n + 2p_s \leq 1$, which implies that $q_n = 1+\theta r^2 - 2p_n + 2p_s$. Substituting the expressions of q_n, q_r^s and q_r^d in (3), the retailer's optimization problem can be rewritten as follows:

$$\max_{p_n,r,p_s} \quad \Pi^R = (p_n - c - \theta r^2 + \theta r p_s)q_n,$$
$$s.t. \quad \begin{cases} q_n = 1+\theta r^2 - 2p_n + 2p_s \leq 1, \\ \frac{1+\theta r^2}{2} - p_n \geq 0. \end{cases} \tag{A.3}$$

We divide the feasible domain of (A.3) into two parts: $q_r^d \leq q_r^s$ and $q_r^d > q_r^s$, and look for the optimal solution for each part. When $q_r^d \leq q_r^s$, the optimization problem is:

$$\max_{p_n,r,p_s} \quad \Pi_1^R = (p_n - c)q_n - rq_r^s + p_s q_r^d$$
$$s.t. \quad \begin{cases} \frac{1+\theta r^2}{2} \geq p_n, \\ 1+\theta r^2 - 2p_n + 2p_s \leq 1, \\ q_r^d \leq q_r^s. \end{cases} \tag{A.4}$$

Notice that $\partial \Pi_1^R / \partial p_n + \partial \Pi_1^R / \partial p_s = 1$, thus $\mathbf{d} = (p_n, r, p_s)^T = (1, 0, 1)^T$ is an ascending direction of the objective function in (A.4). So at least one of the constraints $g(p_n, r, p_s) \leq 0$ that satisfy $\nabla g(p_n, r, p_s).\mathbf{d} > 0$ is a strict constraint. It can be easily verified that, among the three constraints in (A.4), only $(1+\theta r^2)/2 \geq p_n$ is a strict constraint, which means that the optimal solution of (A.4) must satisfy $(1+\theta r^2)/2 = p_n$. Substituting $p_n = (1+\theta r^2)/2$ into (A.4), we can obtain the following optimization problem:

$$\max_{r,p_s} \quad \Pi_1^R = (2 - \theta r^2 - 2c - 2p_s)p_s$$
$$s.t. \quad \begin{cases} 2p_s \leq 1, \\ q_r^d \leq q_r^s. \end{cases} \tag{A.5}$$

Since $\partial \Pi_1^R / \partial r < 0$, $\mathbf{d} = (r, p_s)^T = (-1, 0)^T$ is an ascending direction of the objective function in (A.5). At least one of the constraints $g(r, p_s) \leq 0$ that satisfy $\nabla g(r, p_s).\mathbf{d} > 0$ is a strict constraint. It can be easily verified that, among the two constraints in (A.5), only $q_r^d \leq q_r^s$ is a strict constraint. Thus, the optimal solution of (A.5) must satisfy $q_r^d = q_r^s$.

In sum, the optimal solution of optimization problem (A.4) is obtained when $q_r^d = q_r^s$. Therefore, the optimal solution of optimization problem (A.3) must satisfy $q_r^d \geq q_r^s$. We only need to solve optimization problem (A.3) under the condition of $q_r^d \geq q_r^s$. Then, the retailer's optimization problem is:

$$\max_{p_n,r,p_s} \quad \Pi^R = (p_n - c)q_n - rq_r^s + p_s q_r^s$$
$$s.t. \quad \begin{cases} \frac{1+\theta r^2}{2} \geq p_n, \\ 1+\theta r^2 - 2p_n + 2p_s \leq 1, \\ q_r^d \geq q_r^s. \end{cases} \tag{A.6}$$

Since $\partial \Pi^R / \partial p_n + \partial \Pi^R / \partial r = (1+\theta r)q_n > 0$, $\mathbf{d} = (p_n, r, p_s)^T = (1, 0, 1)^T$ is an ascending direction of the objective function of (A.6). Following similar analyzing procedures of optimization problems (A.4) and (A.5), we can prove that the constraint $(1+\theta r^2)/2 \geq p_n$ is strict. Substituting $p_n = (1+\theta r^2)/2$ into (A.6), we obtain the following optimization problem:

$$\max_{r,p_s} \quad \Pi^R = (1 - \theta r^2 - 2c)p_s + 2\theta r p_s^2$$
$$s.t. \quad \begin{cases} 2p_s \leq 1, \\ q_r^d \geq q_r^s. \end{cases} \tag{A.7}$$

Notice that the objective function in (A.7) is a convex function of p_s. For a given r, the two constraints in (A.7) are equivalent to $0 \leq p_s \leq 1/2(1+\theta r)$. Thus, the maximum of the objective function in (A.7) is achieved when p_s is either 0 or $1/2(1+\theta r)$. The value of the objective function is 0 when $p_s = 0$, so $p_s = 0$ can never be the optimal solution. Therefore, the optimal solution of (A.7) should satisfy $p_s = 1/2(1+\theta r)$. Under this condition, we have $q_r^d = q_r^s$. Substituting $q_r^d = q_r^s$ into (A.7), we obtain the following optimization problem:

$$\max_{p_s} \quad \Pi^R = -2p_s^2 + \left[2(1-c) - \frac{1}{\theta}\right]p_s - \frac{1}{4\theta p_s} + \frac{1}{\theta}$$
$$s.t. \quad 2p_s \leq 1. \tag{A.8}$$

It can be easily verified that, there is a unique root p_s^{*R} for the equation $\partial \Pi^R / \partial p_s = 0$ on the interval (0,1/2), which satisfies $\partial \Pi^R / \partial p_s > 0$ when $p_s < p_s^{*R}$ and $\partial \Pi^R / \partial p_s < 0$ when $p_s > p_s^{*R}$. Therefore, the objective function of (A.8) is unimodal and the optimal solution p_s^{*R} of (A.8) is the solution of the equation $\partial \Pi^R / \partial p_s = -16\theta p_s^3 + 8\theta(1-c)p_s^2 - 4p_s^2 + 1 = 0$. Recalling the strict constraints $q_r^d = q_r^s$ and $(1+\theta r^2)/2 = p_n$, we can obtain that $r^{*R} = [1/(2p_s^{*R}) - 1]/\theta$ and $p_n^{*R} = [1+\theta(r^{*R})^2]/2$.

Appendix B. Proof of Corollary 1

(1) Denote

$f(p_s) = \partial \Pi^R / \partial p_s = -16\theta p_s^3 + 4[2\theta(1-c) - 1]p_s^2 + 1.$

We first show that $p_n^{*R} > r^{*R}$. According to Proposition 1,

$$p_n^{*R} - r^{*R} = \frac{1+\theta(r^{*R})^2}{2} - \frac{1}{\theta}\left(\frac{1}{2p_s^{*R}} - 1\right)$$
$$= \frac{1}{2} + \frac{1}{8\theta}\left(\frac{1}{p_s^{*R}} - 2\right)\left(\frac{1}{p_s^{*R}} - 6\right). \tag{B.1}$$

If

$$\left(\frac{1}{p_s^{*R}} - 2\right)\left(\frac{1}{p_s^{*R}} - 6\right) > -2, \tag{B.2}$$

then, $p_n^{*R} - r^{*R} > 1/2 - 1/(4\theta) \geq 0$.

Note that (B.2) holds if and only if $p_s^{*R} > 1/(4-\sqrt{2})$. In the following, we show that $p_s^{*R} > 1/(4-\sqrt{2})$ holds. According to the proof of Proposition 1, p_s^{*R} is the unique root for the equation $f(p_s) = \partial \Pi^R / \partial p_s = 0$ on the interval (0,1/2), which satisfies $\partial \Pi^R / \partial p_s > 0$ when $p_s < p_s^{*R}$ and $\partial \Pi^R / \partial p_s < 0$ when $p_s > p_s^{*R}$. In the following, we calculate $f(1/(4-\sqrt{2}))$,

$$f(1/(4-\sqrt{2}))$$
$$= \theta\left[-\frac{16}{(4-\sqrt{2})^3} + \frac{8(1-c)}{(4-\sqrt{2})^2}\right] + 1 - \frac{4}{(4-\sqrt{2})^2},$$
$$\geq -\frac{4\theta}{(4-\sqrt{2})^2}\left(\frac{4}{4-\sqrt{2}} - 1\right) + 1 - \frac{4}{(4-\sqrt{2})^2},$$
$$\geq -\frac{4}{(4-\sqrt{2})^2}\left(\frac{4}{4-\sqrt{2}} - 1\right) + 1 - \frac{4}{(4-\sqrt{2})^2} > 0. \tag{B.3}$$

The first inequality in (B.3) holds as $c < 1/2$ and the second inequality in (B.3) holds as $\theta \leq 1$. Therefore, $p_s^{*R} > 1/(4-\sqrt{2})$ holds.

(2) Next, we show that $p_s^{*R} > r^{*R}$. According to Proposition 1,

$$p_s^{*R} - r^{*R} = \frac{2\theta (p_s^{*R})^2 + 2p_s^{*R} - 1}{2\theta p_s^{*R}}. \tag{B.4}$$

The larger root of $2\theta (p_s^{*R})^2 + 2p_s^{*R} - 1 = 0$ is $(-1+\sqrt{2\theta+1})/(2\theta)$. To verify that $p_s^{*R} - r^{*R} > 0$, we only need to show that $p_s^{*R} > (-1+\sqrt{2\theta+1})/(2\theta)$. Similar to the proof of Part (1), we calculate $f((-1+\sqrt{2\theta+1})/(2\theta))$,

$$\begin{aligned} & f((-1+\sqrt{2\theta+1})/(2\theta)) \\ = & \frac{\theta^2 + (\sqrt{2\theta+1})^2[2\theta(1-c)+1-2\sqrt{2\theta+1}]}{\theta^2}, \\ \geq & \frac{\theta^2 + (2\sqrt{2\theta+1}-1)^2(1+\theta-2\sqrt{2\theta+1})}{\theta^2}. \end{aligned} \tag{B.5}$$

The inequality in (B.5) holds as $c < 1/2$. It can be easily shown that $\theta^2 + (2\sqrt{2\theta+1}-1)^2[1+\theta-2\sqrt{2\theta+1}]$ increases in θ and thus obtains its minimum value at $\theta = 1/2$. Thus,

$$\theta^2 + (2\sqrt{2\theta+1}-1)^2[1+\theta-2\sqrt{2\theta+1}] \geq \frac{51-36\sqrt{2}}{4} > 0. \tag{B.6}$$

Therefore, $f((-1+\sqrt{2\theta+1})/(2\theta)) \geq 0$ holds, which indicates that $p_s^{*R} \geq (-1+\sqrt{2\theta+1})/(2\theta)$.

Appendix C. Proof of Corollary 2

According to the proof of Proposition 1, p_s^{*R} is the solution of the equation

$$f(p_s) = -16\theta p_s^3 + 4[2\theta(1-c)-1]p_s^2 + 1 = 0. \tag{C.1}$$

(1) Taking derivative of both sides of (C.1) with respect to c, we obtain that

$$\frac{\partial p_s^{*R}}{\partial c} = -\frac{\theta p_s^{*R}}{6\theta p_s^{*R} - 2\theta(1-c)+1}. \tag{C.2}$$

Notice that $6\theta p_s - 2\theta(1-c)+1 = 0$ when $p_s = [2\theta(1-c)-1]/6\theta \triangleq p_s^0$. To compare p_s^0 with p_s^{*R}, substituting p_s^0 into $f(p_s)$, we have

$$f(p_s^0) = \frac{\theta^2[27-8\theta(1-c)^2]}{27\theta^2} > 0. \tag{C.3}$$

Thus, p_s^{*R} is greater than p_s^0, which implies that $6\theta p_s^{*R} - 2\theta(1-c)+1 > 0$. Therefore, $\partial p_s^{*R}/\partial c < 0$, which indicates that p_s^{*R} decreases in c. Since $r^{*R} = [1/(2p_s^{*R})-1]/\theta$ and $p_n^{*R} = [1+\theta(r^{*R})^2]/2$, both of them increase in c.

(2) Taking derivative of both sides of (C.1) with respect to θ, we obtain that

$$\frac{\partial p_s^{*R}}{\partial \theta} = -\frac{p_s^{*R}[2p_s^{*R}-(1-c)]}{6\theta p_s^{*R} - 2\theta(1-c)+1}. \tag{C.4}$$

According to the proof of Part (1), $6\theta p_s^{*R} - 2\theta(1-c)+1 > 0$. To investigate the sign of $2p_s^{*R}-(1-c)$, we only need to compare $p_s^1 = (1-c)/2$ and p_s^{*R}. Substituting p_s^1 into $f(p_s)$, we have

$$f(p_s^1) = 1-(1-c)^2 > 0. \tag{C.5}$$

Thus, p_s^{*R} is greater than p_s^1, which implies that $2p_s^{*R}-(1-c) > 0$. Therefore, $\partial p_s^{*R}/\partial\theta < 0$, which indicates that p_s^{*R} decreases in θ.

(3) Note that the objective function in (4) decreases in c. Thus, the retailer's optimal profit Π^{*R} decreases in c. Furthermore, in the objective function of (A.8), the items relevant to θ are

$$\frac{1}{\theta}\left(1-\frac{1}{4p_s}-p_s\right) = -\frac{(1-2p_s)^2}{4\theta p_s}, \tag{C.6}$$

which increases in θ. Therefore, Π^{*R} increases in θ.

Appendix D. Proof of Proposition 2

Denote

$$\delta_3 = \frac{2(p_s-\bar{p}_e)}{1-\beta},$$

which satisfies $EU_s = E\bar{U}_e$.

It can be easily obtained that

$$\delta_2 - \delta_1 = \frac{(1-\beta)(\delta_1-\delta_3)}{\beta}.$$

Note that, if $\delta_1 \leq \delta_2$, then $\delta_3 \leq \delta_1$. In this case, there will be no trade in the P2P market. If $\delta_1 \geq \delta_2$, then $\delta_3 \geq \delta_1$. In this case, there will be trades in both the P2P market and the retailer-run resale market. We are only interested in the latter case. Therefore, $q_n = \min\{\delta_2, 1\}$.

If $\delta_2 > 1$, then $q_n = 1$ and the retailer's optimization problem is as follow:

$$\begin{aligned} & \max_{p_n} \quad \Pi^B = p_n - \theta r^2 - c, \\ & s.t. \quad \begin{cases} \frac{2(p_s-p_e)}{1-\beta} = 1, \\ \frac{1+\theta r^2+(1-\theta)(p_e-t)^2}{2} \geq p_n, \\ \frac{2(p_s-p_e)}{1-\beta} - 1 = (1-\theta)(p_e-t), \\ r \geq p_e - t. \end{cases} \end{aligned} \tag{D.1}$$

It is obvious that the optimal solution of (D.1) is that $p_n = 1/2$, $r = 0$, $p_e = t$ and $p_s = t + (1-\beta)/2$. Therefore, $\delta_2 = 2t/\beta < 1$ which contradicts with the assumption that $\delta_2 > 1$. Therefore, we only need to solve the retailer's optimization problem (10) under condition $\delta_2 \leq 1$.

The retailer's optimization problem (10) can be rewritten as follows:

$$\begin{aligned} & \max_{p_n, r, p_s} \quad \Pi^B = (p_n - \theta r^2 - c)q_n + p_s \min\left\{\theta r q_n, 1 - \frac{2(p_s-p_e)}{1-\beta}\right\} \\ & s.t. \quad \begin{cases} q_n = \frac{2(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]} = \frac{1+\theta r^2+(1-\theta)(p_e-t)^2-2p_n+2p_e}{\beta} \leq 1, \\ \frac{2(p_s-p_e)}{(1-\beta)} \leq 1, \\ \frac{1+\theta r^2+(1-\theta)(\bar{p}_e-t)^2}{2} - p_n \geq 0, \\ 0 \leq p_e - t \leq r. \end{cases} \end{aligned} \tag{D.2}$$

Note that if the second constraint in (D.2) holds, $q_n \leq 1$ always holds. We divide the feasible domain of (D.2) into two parts: $q_r^d \geq q_r^s$ and $q_r^d < q_r^s$, and look for the optimal solution for each part. When $q_r^d \geq q_r^s$, the optimization problem is:

$$\begin{aligned} & \max_{p_n, r, p_s} \quad \Pi^B = (p_n - \theta r^2 + \theta r p_s - c)q_n \\ & s.t. \quad \begin{cases} q_n = \frac{2(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]} = \frac{1+\theta r^2+(1-\theta)(p_e-t)^2-2p_n+2p_e}{\beta}, \\ \frac{2(p_s-p_e)}{(1-\beta)} \leq 1, \\ \frac{1+\theta r^2+(1-\theta)(\bar{p}_e-t)^2}{2} - p_n \geq 0, \\ r \geq p_e - t, \\ \theta r q_n \leq 1 - \frac{2(p_s-p_e)}{1-\beta}. \end{cases} \end{aligned} \tag{D.3}$$

From the first constrain in (D.3), we obtain that

$$\begin{aligned} p_n = & \frac{1}{2}\Big\{1+\theta r^2 + (1-\theta)(p_e-t)^2 + 2p_e \\ & - \frac{2\beta(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]}\Big\}. \end{aligned} \tag{D.4}$$

Substituting (D.4) into optimization problem (D.3), we have

$$\max_{p_s,p_e,r} \ \Pi^B = \Big[\frac{1}{2}\Big\{1-\theta r^2+(1-\theta)(p_e-t)^2+2p_e - \frac{2\beta(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]}\Big\} + \theta r p_s - c\Big]\frac{2(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]}$$

$$s.t. \begin{cases} \frac{2(p_s-p_e)}{(1-\beta)} \le 1, \\ \frac{\beta(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]} - p_e \ge 0, \\ r \ge p_e - t, \\ \frac{2\theta r(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]} \le 1 - \frac{2(p_s-p_e)}{1-\beta}. \end{cases} \quad (D.5)$$

Note that,

$$\frac{\partial \Pi^B}{\partial r} = \frac{2\theta(p_s-r)(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]} \ge 0. \quad (D.6)$$

Since $\partial\Pi^B/\partial r \ge 0$, $\mathbf{d}=(p_s,p_e,r)^T=(0,0,1)^T$ is an ascending direction of the objective function in (D.5). At least one of the constraints $g(p_s,p_e,r)\le 0$ that satisfy $\nabla g(p_s,p_e,r).\mathbf{d}>0$ is a strict constraint. It can be easily verified that, among the four constraints in (D.5), only $q_r^d \ge q_r^s$ is a strict constraint. Thus, the optimal solution of (D.5) must satisfy $q_r^d = q_r^s$.

In sum, the optimal solution of optimization problem (D.5) is obtained when $q_r^d = q_r^s$. Therefore, the optimal solution of optimization problem (D.2) must satisfy $q_r^d \le q_r^s$. We only need to solve optimization problem (D.2) under the condition of $q_r^d \le q_r^s$. Then, the retailer's optimization problem is:

$$\max_{p_n,r,p_s} \ \Pi^B = (p_n-\theta r^2-c)q_n + p_s\Big[1-\frac{2(p_s-p_e)}{1-\beta}\Big]$$

$$s.t. \begin{cases} q_n = \frac{2(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]} = \frac{1+\theta r^2+(1-\theta)(p_e-t)^2-2p_n+2p_e}{\beta}, \\ \frac{2(p_s-p_e)}{(1-\beta)} \le 1, \\ \frac{1+\theta r^2+(1-\theta)(\bar{p}_e-t)^2}{2} - p_n \ge 0, \\ r \ge p_e - t, \\ \frac{2\theta r(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]} \ge 1 - \frac{2(p_s-p_e)}{1-\beta}. \end{cases} \quad (D.7)$$

From the first constrain in (D.7), we obtain that

$$p_n = \frac{1}{2}\Big\{1+\theta r^2+(1-\theta)(p_e-t)^2+2p_e - \frac{2\beta(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]}\Big\}. \quad (D.8)$$

Substituting (D.8) into optimization problem (D.7), we have

$$\max_{p_n,r} \ \Pi^B = \Big\{\frac{1}{2}\Big[1-\theta r^2+(1-\theta)(p_e-t)^2+2p_e - \frac{2\beta(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]}\Big] - c\Big\} \frac{2(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]} + p_s\Big[1-\frac{2(p_s-p_e)}{1-\beta}\Big]$$

$$s.t. \begin{cases} \frac{2(p_s-p_e)}{(1-\beta)} - 1 \le 0, \\ p_e - \frac{\beta(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]} \le 0, \\ p_e - t - r \le 0, \\ 1 - \frac{2(p_s-p_e)}{1-\beta} - \frac{2\theta r(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]} \le 0. \end{cases} \quad (D.9)$$

Denote the ith constraint in optimization problem (D.9) as g_i ($i=1,\ldots,4$). Taking derivative of Π^B with respect to p_s, p_e and r, we have

$$\frac{\partial\Pi^B}{\partial p_s} + \frac{\partial\Pi^B}{\partial p_e} + \frac{\partial\Pi^B}{\partial r} = 1 + \frac{2(p_s-p_e)}{1-\beta} + \frac{2(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]^2} \Big\{[1+(1-\theta)(p_e-t)]^2 + \frac{2\beta(1-\theta)(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]} + \frac{\theta(1-\theta)r^2}{2} - \frac{(1-\theta)^2(p_e-t)^2}{2} - \frac{1-\theta}{2} + (1-\theta)c - (1-\theta)p_e - \theta r[1+(1-\theta)(p_e-t)]\Big\}.$$

It can be shown that, when $t<c$, $\partial\Pi^B/\partial p_s + \partial\Pi^B/\partial p_e + \partial\Pi^B/\partial r \ge 0$ holds. Taking derivative of g_i with respect to p_s, p_e and r, we have

$$\begin{aligned} &\frac{\partial g_1}{\partial p_s} + \frac{\partial g_1}{\partial p_e} + \frac{\partial g_1}{\partial r} = 0, \\ &\frac{\partial g_2}{\partial p_s} + \frac{\partial g_2}{\partial p_e} + \frac{\partial g_2}{\partial r} = 1 + \frac{\beta(1-\theta)(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]^2} \ge 0, \\ &\frac{\partial g_3}{\partial p_s} + \frac{\partial g_3}{\partial p_e} + \frac{\partial g_3}{\partial r} = 0, \\ &\frac{\partial g_4}{\partial p_s} + \frac{\partial g_4}{\partial p_e} + \frac{\partial g_4}{\partial r} = \frac{2\theta(p_s-p_e)[r(1-\theta)-1-(1-\theta)(p_e-t)]}{(1-\beta)[1+(1-\theta)(p_e-t)]^2} \le 0. \end{aligned} \quad (D.10)$$

Therefore, constraint g_2 is strict. That is, the equation

$$p_e = \frac{\beta(p_s-p_e)}{(1-\beta)[1+(1-\theta)(p_e-t)]}, \quad (D.11)$$

holds. From (D.11), we have

$$p_s = \frac{p_e[1+(1-\beta)(1-\theta)(p_e-t)]}{\beta}, \quad \frac{p_s-p_e}{1+(1-\theta)(p_e-t)} = \frac{(1-\beta)p_e}{\beta}. \quad (D.12)$$

Constraint g_1 can be simplified as $2p_e/\beta \le 1/[1+(1-\theta)(p_e-t)]$ and constraint g_4 can be simplified as $2p_e/\beta \ge 1/[1+(1-\theta)(p_e-t)+\theta r]$.

Substituting (D.12) into (D.9), the retailer's optimization problem is

$$\max_{p_n,r} \ \Pi^B = \frac{p_e}{\beta}\{\beta[1-\theta r^2+(1-\theta)(p_e-t)^2-2c] + [1+(1-\beta)(1-\theta)(p_e-t)][\beta - 2p_e[1+(1-\theta)(p_e-t)]]\}$$

$$s.t. \begin{cases} \frac{1}{1+(1-\theta)(p_e-t)+\theta r} \le \frac{2p_e}{\beta}, \\ p_e - t - r \le 0, \\ \frac{2p_e}{\beta} \le \frac{1}{1+(1-\theta)(p_e-t)}. \end{cases} \quad (D.13)$$

It is obvious that the objective function in (D.13) decreases in r and the first two constraints in (D.13) also decrease in r. Therefore, at least one of the first two constrains is strict. The constraint

$$\frac{1}{1+(1-\theta)(p_e-t)+\theta r} \le \frac{2p_e}{\beta}, \quad (D.14)$$

can be rewritten as

$$r \ge \frac{1}{\theta}\Big\{\frac{\beta}{2p_e} - 1 - (1-\theta)(p_e-t)\Big\} \triangleq g(p_e). \quad (D.15)$$

According to the relationship between $p_e - t$ and $g(p_e)$, we discuss optimization problem (D.13) in the following two cases.

In the first case, assume that $p_e - t \ge g(p_e)$ holds. Thus, in the optimal solution, $r = p_e - t$ should hold. Note that $p_e - t \ge g(p_e)$ holds if and only if $2p_e^2 + 2p_e(1-t) - \beta \ge 0$. The solution of $2p_e^2 +$

$2p_e(1-t)-\beta \geq 0$ is $p_e \geq [\sqrt{(1-t)^2+2\beta}-(1-t)]/2$. Therefore, the retailer's optimization problem is

$$\begin{aligned}\max_{p_e} \quad & \Pi_1^B = \frac{p_e}{\beta}\{\beta[1-(2\theta-1)(p_e-t)^2-2c] \\ & + [1+(1-\beta)(1-\theta)(p_e-t)] \\ & [\beta - 2p_e[1+(1-\theta)(p_e-t)]]\} \\ s.t. \quad & \frac{\sqrt{(1-t)^2+2\beta}-(1-t)}{2} \\ & \leq p_e \leq \frac{\sqrt{[1-t(1-\theta)]^2+2\beta(1-\theta)}-[1-t(1-\theta)]}{2(1-\theta)}. \end{aligned} \tag{D.16}$$

Note that, the feasible region of p_e is nonempty if and only if $\beta \geq 2t$. It can be shown that

$$\begin{aligned}\frac{\partial^2 \Pi_1^B}{\partial p_e^2} = & \frac{2}{\beta^2}\{-2\big[[1-t(1-\theta)]^2+6p_e(1-\theta)[1-t(1-\theta)] \\ & + 6p_e^2(1-\theta)^2\big] \\ & - \beta^2(1-\theta)+\beta[1+12p_e^2(1-\theta)^2+2t^2(1-\theta)^2 \\ & - \theta + t(-4+6\theta)] \\ & + 3p_e[3-4t(1-\theta)^2-4\theta]\big\} \leq 0. \end{aligned}$$

That is, Π_1^B is a concave function in p_e. Thus, the optimal solution of optimization problem (D.16) is

$$\begin{cases} p_{e1}^{*B} = & \begin{cases} p_{e1}^*, & \text{if } p_{e1}^* \geq \frac{\sqrt{2\beta+(1-t)^2}-(1-t)}{2}, \\ \frac{\sqrt{2\beta+(1-t)^2}-(1-t)}{2}, & \text{if } p_{e1}^* < \frac{\sqrt{2\beta+(1-t)^2}-(1-t)}{2}, \end{cases} \\ r_1^{*B} = & p_{e1}^{*B}-t, \\ p_{s1}^{*B} = & p_{e1}^{*B}[1+(1-\beta)(1-\theta)(p_{e1}^{*B}-t)]/\beta, \\ p_{n1}^{*B} = & [1+\theta(r_1^{*B})^2+(1-\theta)(p_{e1}^{*B}-t)^2]/2, \end{cases}$$

where p_{e1}^* is the unique real root of $\partial \Pi_1^B / \partial p_e = 0$.

In the second case, assume that $g(p_e) \geq p_e - t$ holds. Thus, in the optimal solution, $r = [\beta/2p_e - 1 - (1-\theta)(p_e-t)]/\theta$ should hold. Note that $g(p_e) \geq p_e - t$ holds if and only if $2p_e^2 + 2p_e(1-t) - \beta \leq 0$. The solution of $2p_e^2+2p_e(1-t)-\beta \leq 0$ is $p_e \in [t, [\sqrt{(1-t)^2+2\beta}-(1-t)]/2]$. Note that $[\sqrt{(1-t)^2+2\beta}-(1-t)]/2 > t$ holds if and only if $\beta > 2t$. Therefore, the retailer's optimization problem is

$$\begin{aligned}\max_{p_e} \quad & \Pi_2^B = \frac{p_e}{\beta^2}\Big\{\beta\Big[1-\frac{1}{\theta}\Big[\frac{\beta^2}{4p_e^2}-\frac{\beta[1+(1-\theta)(p_e-t)]}{p_e} \\ & + [1+(1-\theta)(p_e-t)]^2\Big] \\ & + (1-\theta)(pe-t)^2-2c\Big]+[1+(1-\beta)(1-\theta)(p_e-t)] \\ & [\beta-2p_e[1+(1-\theta)(p_e-t)]]\} \\ s.t. \quad & t \leq p_e \leq \frac{\sqrt{(1-t)^2+2\beta}-(1-t)}{2}. \end{aligned} \tag{D.17}$$

It can be shown that

$$\begin{aligned}\frac{\partial^2 \Pi_2^B}{\partial p_e^2} = & -\frac{1}{2\theta p_e^3}\{\beta^3+4\beta^2\theta(1-\theta)p_e^3+8\theta p_e^3\big[[1-t(1-\theta)]^2 \\ & + 6p_e^2(1-\theta)^2 \\ & + 6p_e(1-\theta)[1-t(1-\theta)]\big] \\ & + 4\beta(1-\theta)p_e^3\big[2-\theta-12\theta(1-\theta)p_e^2 \\ & - 2t^2\theta(1-\theta)+2t(3\theta-1) \\ & - 3p_e[-1+4\theta(1-t(1-\theta))]\big]\} \leq 0. \end{aligned}$$

The optimal solution of optimization problem (D.17) is

$$\begin{cases} p_{e2}^{*B} = & \begin{cases} p_{e2}^*, & \text{if } t \leq p_{e2}^* \leq \frac{\sqrt{2\beta+(1-t)^2}-(1-t)}{2}, \\ \frac{\sqrt{2\beta+(1-t)^2}-(1-t)}{2}, & \text{if } p_{e2}^* \geq \frac{\sqrt{2\beta+(1-t)^2}-(1-t)}{2}, \end{cases} \\ r_2^{*B} = & \big[\beta/2p_{e2}^{*B}-1-(1-\theta)\big(p_{e2}^{*B}-t\big)\big]/\theta, \\ p_{s2}^{*B} = & p_{e2}^{*B}\big[1+(1-\beta)(1-\theta)\big(p_{e2}^{*B}-t\big)\big]/\beta, \\ p_{n2}^{*B} = & \Big[1+\theta\big(r_2^{*B}\big)^2+(1-\theta)\big(p_{e2}^{*B}-t\big)^2\Big]/2, \end{cases}$$

where p_{e2}^* is the unique real root of $\partial \Pi_2^B / \partial p_e = 0$.

Appendix E. Analysis of scenario with only P2P market

In the scenario with only P2P market, the retailer does not accept consumer returns and there is no retailer-run resale market. Thus, if the consumers are dissatisfied with the products they purchased in Period 1, they can only sell them through the P2P market. Represent the scenario by superscript "E".

We first analyze consumers' decisions. In Period 1, if a consumer buys a new product, the expected consumer surplus is $EU_n = E(max\{V, \bar{p}_e - t\} - p_n) = [1+(\bar{p}_e-t)^2]/2 - p_n$; if he buys a second-hand product from the P2P market in Period 2, the expected consumer surplus is $EU_e = E(\beta\delta V - \bar{p}_e) = \beta\delta/2 - \bar{p}_e$. Therefore, a consumer buys a new product if and only if

$$\frac{1+(\bar{p}_e-t)^2}{2} - p_n \geq \max\left\{\frac{\delta\beta}{2}-\bar{p}_e, 0\right\}. \tag{E.1}$$

From (E.1), we can derive the range of δ for consumers who buy new products, i.e., $\delta \leq \min\{[1+(\bar{p}_e-t)^2-2p_n+2\bar{p}_e]/\beta, 1\}$. Since δ is uniformly distributed on the interval [0,1], the demand for new products is:

$$q_n = \begin{cases} 0, & \text{if } \frac{1+(\bar{p}_e-t)^2}{2}-p_n<0, \\ \min\{[1+(\bar{p}_e-t)^2-2p_n+2\bar{p}_e]/\beta, 1\}, & \text{if } \frac{1+(\bar{p}_e-t)^2}{2}-p_n \geq 0. \end{cases} \tag{E.2}$$

It is obvious that the case where $[1+(\bar{p}_e-t)^2]/2-p_n<0$ is not optimal for the retailer. Thus, we only need to figure out the retailer's optimal decisions under the condition $[1+(\bar{p}_e-t)^2]/2 - p_n \geq 0$.

In Period 2, for the consumers who have purchased new products in Period 1, if their realized valuation v is less than $p_e - t$, they will sell their products through the P2P market. As V is uniformly distributed on the interval [0,1], the supply of second-hand products in the P2P market is $q_p^s = (p_e-t)q_n$. Note that, under the RE equilibrium, $\bar{p}_e = p_e$. Given that $[1+(\bar{p}_e-t)^2]/2-p_n \geq 0$, it can be easily shown that, the customers who have not purchased new products in Period 1 will purchase second-hand products in Period 2, i.e., $EU_e > 0$ holds. Therefore, the demand for second-hand products in the P2P market is $q_p^d = 1 - q_n$. Under the condition $[1+(\bar{p}_e-t)^2]/2-p_n \geq 0$, the expressions of q_n, q_p^s and q_p^d are as follows:

$$\begin{cases} q_n = \min\{1+(\bar{p}_e-t)^2-2p_n+2\bar{p}_e, 1\}, \\ q_p^s = (p_e-t)q_n, \\ q_p^d = 1-q_n. \end{cases} \tag{E.3}$$

As the selling price in the P2P market p_e clear all the products sold there, the equation $q_p^d = q_p^s$ holds. Therefore, the retailer's optimization problem is:

$$\max_{p_n} \quad \Pi^E = (p_n - c)q_n,$$
$$s.t. \begin{cases} \frac{1+(\bar{p}_e - t)^2}{2} - p_n \geq 0, \\ p_e - t \geq 0, \\ q_p^s = q_p^d, \\ \bar{p}_e = p_e, \end{cases} \tag{E.4}$$

where q_n, q_p^s and q_p^d are as shown in Eq. (E.3).

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