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LOT-SIZING RULE AND FREEZING THE MASTER PRODUCTION SCHEDULE UNDER CAPACITY CONSTRAINT AND DETERMINISTIC DEMAND*

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This paper investigates the performance impact of lot-sizing rule (LSR) selection and freezing of the master production schedule (MPS) in multi-item single-level systems with a single resource constraint under deterministic demand. The results of the study show that the selection of LSRs and the parameters for freezing the MPS have a significant impact on total cost, schedule instability, and the service level of the system. However, the selection of LSRs does not significantly influence the selection of the MPS freezing parameters. The basic conclusions concerning the performance of the freezing parameters under a capacity constraint agreed with previous research findings without consideration of capacity constraints.
(MASTER PRODUCTION SCHEDULING; CAPACITATED LOT-SIZING; SCHEDULE INSTABILITY; CAPACITY CONSTRAINT, COMPUTER SIMULATION)

1. Introduction

Master production scheduling is a very important activity in manufacturing planning and control. The quality of master production schedules (MPS) significantly influences the total cost, schedule instability, and service level of a production inventory system. The MPS drives the material requirements planning (MRP) system and provides an important link between the forecasting, order entry, and production planning activities on the one hand and the detailed planning and scheduling of components and raw materials on the other. Frequent adjustments to the MPS can induce major changes in detailed MRP schedules. These changes can lead to increases in production and inventory costs and deterioration in customer service levels. This phenomenon is called "schedule instability" or "MRP nervousness." Maintaining a stable MPS in view of changing customer requirements, adjustments in sales forecasts, and unforeseen suppliers or production problems is a difficult proposition for many firms (Sridharan, Berry, and Udayabhanu 1988).

Several methods have been suggested to reduce schedule instability in MRP systems (Blackburn, Kropp, and Millen 1986, 1987). One frequently used method involves the

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freezing of the MPS. Several studies have proposed alternative ways of freezing the MPS (Sridharan, Berry, and Udayabhanu 1987, 1988) and have compared the effectiveness of freezing the MPS against other methods (Sridharan and LaForge 1990; Kadipasaoglu and Sridharan 1995). Although these studies address an important managerial issue in manufacturing planning and control and provide guidelines to help managers in their selection of MPS freezing parameters, they do not consider capacity constraints, and whether the conclusions and guidelines under uncapacitated situations will be valid under capacitated situations is unknown. Considering the fact that most manufacturing companies face capacity constraints and that freezing the MPS can influence a firm's ability to fully utilize its capacity, capacity constraints may significantly influence the selection of the MPS freezing parameters. Furthermore, previous studies on MPS freezing have assumed that there is only one end item in the MPS. In reality, many companies use the same capacity to manufacture several products. Developing and maintaining MPS for multiple items is far more challenging when capacity constraints exist, than for a single item with no capacity constraints.

This study extends earlier work on MPS freezing in two major ways. (1) It will investigate the impact of MPS freezing and lot-size ruling (LSR) selection on system performance under a single resource constraint, and thus extends earlier studies on MPS freezing from an environment of unlimited capacity to one of limited capacity. (2) It will examine master production scheduling and capacitated lot-sizing for multi-item systems, thus extending the focus of the earlier MPS freezing studies from single products to multiple products. Specifically we will do the following:

1. Evaluate the performance of some lot-sizing heuristics for multi-item single-level systems with a single resource constraint under deterministic demand in a rolling horizon environment;
2. Investigate the impact of freezing the MPS on the performance of multi-item single-level systems with a single resource constraint; and
3. Study the impact of LSR selection on the selection of MPS freezing parameters under deterministic demand with a single resource constraint.

2. Related Research

Because of the importance of maintaining a stable MPS and the difficulty of balancing the cost, schedule instability, and customer service levels in making MPS freezing decisions, a number of researchers have investigated a variety of ways of freezing the MPS. Blackburn, Kropp, and Millen (1986, 1987) investigated five different strategies for reducing MRP nervousness and found freezing the MPS to be among the most effective. Sridharan, Berry, and Udayabhanu (1987, 1988) developed a method to measure schedule instability and studied the impact of MPS freezing upon inventory costs and schedule instability in single-level MPS systems under deterministic demand. Sridharan and Berry (1990b) presented a framework for designing MPS freezing methods under deterministic demand. They also compared the relative importance of the MPS freezing parameters in influencing the total cost and schedule instability of the system. Sridharan and Berry (1990a), Sridharan and LaForge (1990, 1994), and Lin and Krajewski (1992) extended the studies by Sridharan, Berry, and Udayabhanu (1987, 1988) from a case of deterministic demand to a case of demand uncertainty by introducing forecasting errors into the system.

Zhao and Lee (1996) and Zhao and Lam (1997) extended the studies by Sridharan, Berry, and Udayabhanu (1987, 1988) from single-level systems to multi-level MRP systems and found that some findings in single-level systems cannot be generalised to multi-level systems. Zhao and Lee (1993) investigated the impact of forecasting errors on the performance of the MPS freezing parameters by simulating the forecasting process, as well as the master production scheduling and MRP processes. Zhao, Lee, and Goodale (1995) and Zhao and Xie (1998) also investigated the impact of LSR selection on the selection of MPS freezing parameters under demand uncertainty. Their studies indicated that LSRs had a significant impact on system performance, and that the selection of LSRs significantly influenced the selection of some MPS freezing parameters.

Ho and Carter (1996) evaluated the effectiveness of three rescheduling procedures for dampening the nervousness in a multi-stage MRP system using a factor-2 simulator under uncertainty. They found that the performance of a dampening procedure depended on the operating environment of the firm. They also found that the reduction of system nervousness, as measured by the frequency of schedule disruptions, does not lead to a better system performance. Ho and Ireland (1998) examined the impact of forecasting errors on the scheduling instability in a MRP operating environment. They found that a higher degree of forecasting errors may not cause a higher degree of schedule instability and that the selection of an appropriate LSR can mitigate schedule instability.

King and Benton (1987) compared alternative procedures for determining the master production schedule utilizing the super bill and covering set techniques in an assemble-to-order environment. They used the available-to-promise (ATP) lead-time as the performance criterion and the result showed that the super bill outperformed the covering set technique.

Recently, Yeung, Wong, and Ma (1998) provided an intensive review of the literature that examines the parameters affecting the effectiveness of MRP systems. They pointed out that one of the major limitations of previous research is that capacity constraints are not included in most of the studies. It is uncertain whether the results found under uncapacitated systems can be applied to capacitated systems. Zhao and Lee (1993, 1996) and Zhao, Lee, and Goodale (1995) expressed the same view. Our own literature review shows that only Ho and Carter (1996) considered capacity constraints in the job shop when studying the effectiveness of three procedures for dampening MRP nervousness. However, they did not consider capacity constraints in developing the MPS. This study attempts to fill this gap in the literature by investigating the performance of MPS freezing parameters in a multi-item system under a single resource constraint.

To investigate the impact of MPS freezing on system performance under a single resource constraint, we must first select some appropriate LSRs under a capacity constraint. Many researchers have studied lot-sizing problems for multi-item single-level production and inventory systems with a single resource constraint. Maes and Van Wassenhove (1988) provided a general review and experimental comparison of the performance for most of the multi-item single-level capacitated dynamic lot-sizing procedures that can be found in the literature of that time. Almost all optimal algorithms suggested in the literature use the branch and bound technique (or implicit enumeration) to obtain a final optimal solution for a mixed integer programming formulation, while some sub-optimal algorithms consist of approximation approaches based on either mixed integer programming formulation or linear programming formulation. In comparison to the optimal or sub-optimal algorithms based on mathematical programming, heuristics based on common sense are relatively simple and tractable. These heuristics are usually based on modifications of uncapacitated LSRs. Dixon and Silver (1981) suggested a simple heuristic based on a marginal cost analysis of set-up and holding costs using a priority index derived from the Silver-Meal algorithm (1973). This heuristic ensured feasibility through the unidirectional forward procedure. Dogramaci, Panayiotopoulos, and Adam (1981) proposed two heuristics: one similar to that suggested by Dixon and Silver (1981), the other being the so-called four-step algorithm. The four-step algorithm starts with the lot-for-lot (LFL) schedule for each item, followed by steps to check for feasibility and steps for improvement. Karni and Roll (1982) proposed another heuristic of this type, which starts with the optimal Wagner-Whitin solution (Wagner and Whitin 1958) for each item. In another study, Gunther (1987) presented a heuristic that was similar to that of Dixon and Silver (1981), but used a different priority index derived from Groff's algorithm (Groff 1979).

Based on previous research findings (Maes and Van Wassenhove 1986; Gunther 1988), we selected four relatively simple heuristic procedures. The four heuristics are DPA [four-step algorithm developed by Dogramaci, Panayiotopoulos, and Adam (1981)], DS (heuristic proposed by Dixon and Silver 1981), KR (heuristic proposed by Karni and Roll 1982), and GU (heuristic proposed by Gunther 1987). Because previous studies did not consider the freezing of the MPS in a rolling-time horizon, we have to first evaluate the performance of the four LSRS under our own experimental settings.

3. Research Design

The methodology used in this study is computer simulation. This section describes the design and implementation of the simulation model and summarizes the independent and dependent variables of the experimental design.

3.1. *Simulation Procedures*

The simulated manufacturing company is assumed to operate in a make-to-order environment, and production scheduling is based on known demand and available capacity under rolling-time horizons. The company is assumed to produce five different finished products all requiring a single aggregate resource. It is also assumed that no dependency or absorption relationships exist among these products. The lead-times for all the items are assumed to be zero. Demands, order releases, and order receipts all occur at the end of the periods, and all orders must be satisfied whenever possible. If there is not sufficient capacity to produce all the products demanded, we produce the maximum quantity possible, and demand not satisfied will become loss of sales. This reflects a major difference between a manufacturing system with a capacity constraint as in this study, and a system without capacity constraints. We will discuss the features of capacitated systems in more detail later in this section.

The simulation model was modified from the one used by Zhao and Lam (1997) and Zhao and Lee (1996). The simulation model consists of two phases, which are discussed below.

PHASE I: DEMAND AND CAPACITY GENERATION. The first phase of the simulation generates demand for all the products and the available aggregate capacity representing a single resource. The overall market and the detailed product mix are two crucial concerns of demand management (Vollmann, Berry, and Whybark 1992). Therefore, two parameters, demand variation (DV) and product-mix variation (MV), are varied in the demand generation function to generate demand for each of the five items. The following demand generation function is used to generate demand for each of the five items during 300 periods:

$$A_t = A \times (1 + \text{DV} \times R), \tag{1}$$

$$A_{it} = A_t \times p_i \times (1 + \text{MV} \times R), \tag{2}$$

where

i = item index ($1 \leq i \leq 5$),
t = time period index ($1 \leq t \leq 300$),
A = mean total demand per period for all items,
A_t = total demand for all items in period t,
A_{it} = demand for item i in period t (the sum of A_{it} for all i equal to A_t),
p_i = mean demand proportion for item i (the sum of p_i for all i equal to 1),
DV = magnitude of the noise component for total demand,
MV = magnitude of the noise component for product-mix,
R = a standard normal random variant.

In this study, A is assumed to be a constant of 5,000, and the values of p_i are assumed to be 10, 20, 25, 15, and 30% for items 1 to 5, respectively. DV and MV can be varied to generate different demand patterns. The maximum values for MV and DV used in this study are 40%. In order to make demand A_t and demand A_{it} non-negative, we set a lower and an upper bound on R at -2.5 and $+2.5$, respectively. The values of the demand generation parameters are shown in Table 2 and will be discussed later.

In this study, we assume that the capacity absorption for each unit of all items is equal to one. That is, one unit of resource is required to produce exactly one unit of finished product. Relaxing this assumption will not influence the conclusion because the demand for each product can always be measured by the units of the resource needed to produce the product. Available capacity is generated by varying a capacity tightness (CT) parameter. CT is defined as the ratio between the total capacity available and the total demand needed, which is the inverse of capacity utilisation. Once the demand for all items is generated for 300 periods, the total capacity available can be calculated by multiplying the total demand by the CT factor. We assume that the capacity available in each period is a constant for the entire simulation run and is equal to the total capacity available divided by the total number of periods.

PHASE II: MPS DEVELOPMENT PHASE. The second phase of the simulation model develops the MPS in a rolling-time horizon environment using the set of parameters generated in phase I. The procedure for developing the MPS without considering capacity constraints is well described by Zhao and Lam (1997) and Zhao and Lee (1996). A complication that arose during this study was an unfeasibility problem related to the capacity constraint. In the following paragraphs, we describe how we dealt with this problem.

A MPS is developed for the entire planning horizon in each replanning cycle based on known demand. Only the schedules within the freezing interval are implemented as originally planned. Beyond the frozen interval, the MPS is subject to revision. After each replanning cycle, the production schedule is rolled a certain number of periods (replanning periodicity) ahead, and demand for more distant future periods (not previously scheduled) are appended to the schedule. Net requirements for an item for the non-frozen periods within the new planning horizon are calculated using the following equation:

$$\text{Nrequire}(t) = \text{Grequire}(t) - \text{Endinv}(t-1), \tag{3}$$

where

Nrequire(t) is the net requirement for period t,

Grequire(t) is the demand for period t generated in Phase I,

Endinv($t-1$) is the ending inventory for period $t-1$.

When Nrequire(t) is less than zero, Endinv($t-1$) is set to minus Nrequire(t), and Nrequire(t) is set to zero. After the net requirements are determined for a number of periods into the future (planning horizon), the MPS can be developed for these periods utilizing the selected LSR. Under the capacity constraint, however, a feasible MPS for these periods may not exist, even though there is a feasible schedule for all 300 periods. Obviously, the necessary and sufficient condition for feasibility is that in each period the cumulative capacity needed to produce all demanded items before this period does not exceed the cumulative capacity available before this period. In order to ensure that a feasible MPS can be obtained that will meet the net demand, the cumulative capacity available is checked against the cumulative demand period by period in a forward manner before the lot-sizing heuristics are used to determine the MPS. If capacity is insufficient, the demand for one or more items in this period will be reduced by a value to ensure feasibility. Demand for the item with the lowest unit shortage cost will be reduced first, followed by the item with the second lowest unit shortage cost if capacity is still insufficient. This procedure is repeated until capacity is sufficient to ensure a feasible MPS. The reduced demand quantity for an item will become loss of sales. A cumulative loss of sales is recorded as a system performance measure, and the shortage cost is also incorporated into the total cost calculation.

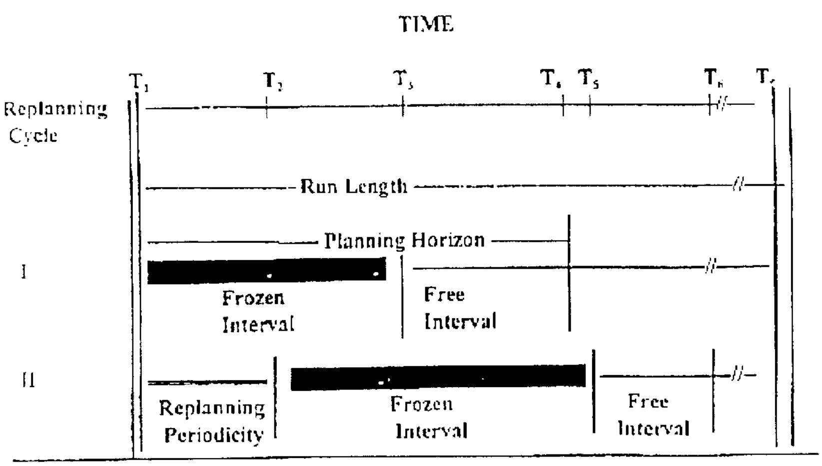
After the feasibility check is completed and demand for some items revised, the net requirements are calculated. Based on the net requirements and the capacity available, the MPS is developed for these periods using the capacitated LSRS. As time goes by, more demand information will become available, and the next planning cycle begins. This process is repeated until a MPS is developed for all 300 periods. To avoid excess changes, management often chooses to implement a portion of a MPS according to the original plan. The portion of the MPS that is not changed is referred to as "frozen." The number of periods for which MPS schedules are frozen depends on the planning horizon (PH), the freezing proportion (FP), the replanning periodicity (RP), and the freezing method (ZM). Figure 1 illustrates the major parameters for freezing the MPS under a rolling-time horizon.

The PH is defined as the number of periods for which the production schedules are developed in each replanning cycle. The frozen interval is the number of scheduled periods within the PH for which the schedules are implemented according to the original plan. The free interval is the number of scheduled periods beyond the frozen interval. This portion is subject to change based on new demand information when the time horizon is rolled forward. The FP refers to the ratio of the frozen interval relative to the PH. The higher the FP, the more stable the production schedule, and the lower the schedule instability of the MPS system. However, a higher FP may also increase the loss of sales and the total cost.

The RP is the number of periods between successive replannings. When the RP is equal to four periods, demand for future periods is entered into the PH every four periods and the MPS is revised. The greater the RP, the less frequently replanning will occur and computational requirements will be reduced. However, a higher RP may also increase both the probability of capacity-related unfeasibility (more loss of sales) and total cost.

In addition to the parameters shown in Figure 1, Sridharan, Berry, and Udayabhanu (1987, 1988) suggested two methods of freezing the MPS: a period-based method and an order-based method. Because multiple end-items are involved in this study and each end-item has its own ordering cycle, it is difficult to implement the order-based ZM. Therefore we only used the period-based ZM in this study.

This process is repeated until a MPS is developed for all 300 periods. After a MPS has been developed and implemented for all items, performance measures are calculated to evaluate the performance of the MRP system. The simulation procedure is summarized in Figure 2.



Adopted and modified from Zhao and Lee (1993)

FIGURE 1. Demonstration of MPS Freezing Parameters

Step A:	Select a demand variation (DV), a product mix variation (MV) and a capacity tightness (CT); Generate the demands and calculate the capacity, then go to step B;
Step B:	Select a maximum natural ordering cycle (T), a unit shortage cost (SC), a planning horizon (PH), a freezing proportion (FP), a replanning periodicity (RP), and a capacitated LSR (LSR) to decide the lot sizes, then go to step C.
Step C:	Calculate the starting period (LS) and the finishing period (LF) for lot-sizing decisions; Check the feasibility condition and reduce the demands if necessary; Develop MPS using the capacitated LSR for periods between LS and LF; Implement MPS within the frozen interval, calculate the ending inventories, and update performance measures, then go to step D.
Step D:	If the end of the simulation has not been reached, roll the schedule RP periods ahead and go to Step C. Otherwise, record the performance measures and go to Step E.
Step E:	If all the combinations of different T, SC, PH, FP, RP and LSR have been exhausted, then go to step F; Otherwise, go to step B and select at least one different value of T, SC, PH, FP, RP or LSR.
Step F:	If all the combinations of different DV, MV and CT have been exhausted, then stop; Otherwise, go to step A and select at least one different value of DV, MV and CT.

FIGURE 2. Simulation Procedure

3.2. *Independent Variables*

There are three major groups of independent variables in this simulation experiment. The first group of independent variables are the "environmental factors" or "operating conditions" of the systems, which include DV, MV, CT, maximum natural ordering cycle (T) and unit shortage cost (SC). The second group of independent variables are the parameters for freezing the MPS, which include the PH, the FP, and the RP. The third is the capacitated LSR. The number of levels of these parameters and their values are shown in Table 1 and are discussed below.

TABLE 1
Independent Variables of the Experimental Design

Variable Number	Variable Name	Label	Number of Levels	Values
	Operating Conditions (Environmental Factors)			
1	Demand variations	DV	3	Low, Medium, High
2	Product mix variations	MV	3	Low, Medium, High
3	Capacity tightness	CT	3	Low, Medium, High
4	Maximum natural ordering cycle	T	2	4 and 8 periods, respectively
5	Unit shortage cost	SC	3	Low, Medium, High
	Parameters for Freezing the Master Production Schedule			
6	Planning horizon	PH	2	4 and 8 maximum natural ordering cycles, respectively
7	Freezing proportion	FP	5	0.00, 0.25, 0.50, 0.75, 1.00
8	Replanning periodicity	RP	4	0.25, 0.50, 0.75, 1.00 times of frozen intervals, respectively
	Capacitated Lot-Sizing Procedures			
9	Lot-sizing rules	LSR	4	DPA, DS, GU, KR

TABLE 2
Demand Parameters

Average Total Demand (A)			5,000		
Item (i)	1	2	3	4	5
Average Demand Proportion (p_i)	20%	10%	25%	15%	30%
Average Demand	1,000	500	1,250	750	1,500

DV (Demand Variations) Parameters

Level	Low	Medium	High
Value	10%	20%	40%

MV (Product Mix Variations) Parameters

Level	Low	Medium	High
Value	10%	20%	40%

3.2.1. ENVIRONMENTAL FACTORS.

Demand variation (DV). As shown in Table 2, the DV factor is set at 10, 20, and 40% of the average total demand, respectively, which represents the low, medium, and high levels of variations in the normal random noise component of the total demand for the five products.

Product-mix variation (MV). MV is set at 10, 20, and 40% of the average proportion of individual item demand, respectively, to represent the low, medium, and high levels of variations in the normal random noise component of the product-mix proportion for the five products.

Capacity tightness (CT). It is set at 1.25, 1.11, and 1.01, respectively, to represent low, medium, and high levels of CT. The three levels of CT correspond to 80, 90, and 99% of resource utilization, respectively.

Maximum natural ordering cycle (T). Inventory carrying costs and production set-up costs/ordering costs are two major cost parameters in MPS settings. In a single-level uncapacitated system, Sridharan, Berry, and Udayabhanu (1987, 1988) fixed the holding cost at $1 per unit per period, and changed the production set-up costs to have different natural ordering cycles. A similar method is also used in this study. The unit holding cost per period for each item is randomly generated from the set [0.1, 0.5, 1.0, 2.0] and then is fixed for all the testing problems. In multi-item single-level systems, although complete enumeration of all possible combinations of natural ordering cycles for different items may be of interest, it is not very practical because of the combination problem. Without loss of generality, the natural ordering cycle for item 1 is assumed to be the maximum cycle among all items. The set-up cost for item 1 is varied so that the natural ordering cycle for this item (T) is 4 and 8 periods, respectively. The set-up costs for the other items (items 2, 3, 4, and 5) are designed so that their natural ordering cycles are randomly selected from the set [1, 2, . . . , T] and then are fixed for all testing problems. Table 3 shows the cost parameters generated using this procedure.

Unit shortage cost (SC). In this simulation experiment, shortages can occur as a result of insufficient capacity in certain periods during any replanning cycle. Whenever capacity is insufficient to meet demand for all items, demand that cannot be met will become loss of sales. A SC parameter is used to reflect loss of profit and the negative effect on future sales. The SC for an item is assumed to be a certain proportion of the unit value of the item. Assuming a period is a day and the unit holding cost per year (365 days) of an item is 25% of the unit value, the unit value for an item is 365/25% (1,460) times of the unit holding cost per period for the item. For example, the unit holding cost per period for item 1 is $1.00, so the unit value of item 1 is $1,460. In this study, the SC is set at 10, 20, and 40% of the item

TABLE 3
Cost Parameters

	Item 1	Item 2	Item 3	Item 4	Item 5
	Unit Holding Cost ($/unit/period) (Generated randomly from [0.50, 1.00, 1.50, 2.00] and then fixed)				
All Sets	1.00	0.50	1.00	2.00	1.00
	Production Set-up Cost ($/set-up)				
$T = 4$	8,000 (4)	500 (2)	625 (1)	3,000 (2)	3,000 (2)
$T = 8$	32,000 (8)	2,000 (4)	2,500 (2)	12,000 (4)	12,000 (4)
	Unit Shortage Cost ($/unit)				
SC = Low	146	73	146	292	146
SC = Medium	292	146	292	584	292
SC = High	584	292	584	1,168	584

Note: The values in the parentheses indicate the corresponding natural ordering cycles calculated using the average demand of that item.

value for the three levels, low, medium and high respectively. The values of the SC are also shown in Table 3.

3.2.2. PARAMETERS FOR FREEZING THE MPS.

Planning horizon (*PH*). Previous research found that the performance of the MRP system is improved when the PH is a multiple of the natural ordering cycle (Blackburn and Millen 1980; Carlson, Beckman and Kropp 1981). Zhao and Lam (1997) and Zhao and Lee (1996) found that the PH had a significant impact on total cost and instability within multi-level MRP systems. In this study, to reduce the number of combinations of independent variables, the PH is set at four and eight times of T, respectively.

Freezing proportion (*FP*). FP has been found to significantly influence the performance of multi-level MRP systems. In this study the FP is set at 0.00, 0.25, 0.50, 0.75, and 1.00, respectively. A FP of 0.00 means a frozen interval equal to one period. This case may be used as a benchmark for evaluating FP performance.

Replanning periodicity (*RP*). RP refers to the time periods between replanning cycles. Zhao and Lam (1997) and Zhao and Lee (1996) found that a RP equal to the frozen interval resulted in the best system performance. However, we do not know whether this is also true in single-level capacitated systems. Therefore, the RP is set at 0.25, 0.50, 0.75, and 1.00 times of the frozen interval, respectively, in this study.

3.2.3. CAPACITATED LOT-SIZING PROCEDURES.

Capacitated lot-sizing rule (*LSR*). The LSR used in this study includes DPA, DS, KR, and GU. All four lot-sizing heuristics are approaches based on common sense, but they fall into two categories (Maes and Van Wassenhove 1988): the DS and GU rules are unidirectional period-by-period heuristics, while the DPA and KR rules are improvement heuristics. The unidirectional period-by-period heuristics work their way through the problem from the first period to the last period in what is essentially a myopic, single-pass constructive algorithm. To indicate the viability of producing demand for a future period in the period under consideration, they used a number of priority indices derived from uncapacitated single-level LSRS. Priority indices are calculated for all items and for all future periods to evaluate whether the production lot in the current period should include demand in a future period for an item. The demand for a future period will be included in the current production lot if the index is positive and the capacity is available. In order to ensure feasibility in future periods in the PH, demand with a negative index is also included in the current production lot, otherwise unfeasibility will occur in future periods. Demand splitting is also permitted if necessary.

The improvement heuristics start with a solution for the entire PH without considering the capacity constraint. Most of the time, this initial solution will be unfeasible. In subsequent steps, feasibility is enforced, and improvements on the initial solution are made by shifting production lots left or right. Demand splitting is also permitted if necessary. The feasibility-enforcing steps try to eliminate unfeasibility at the minimum additional cost. The improvement steps try to further reduce cost without introducing new unfeasibilities. Shifting procedures are used to determine how to shift the quantities around based on trade-offs between set-up and inventory carrying costs.

3.3. *Dependent Variables*

The following three criteria will be used as the dependent variables of the experimental design.

Total cost (TC) is the sum of the production set-up costs, inventory carrying costs, and the shortage cost for all items within the length of the simulation run.

Schedule instability or nervousness (SI) is measured by the following equation:

$$I = \frac{\sum_{i=1}^{n} \sum_{k>1} \sum_{t=M_k}^{M_k+N-1} |Q_{ti}^k - Q_{ti}^{k-1}|}{S} \tag{4}$$

where

i = item index

n = total number of items

t = time period index

k = planning cycle index

Q_{ti}^k = scheduled order quantity for item i in period t during planning cycle k

M_k = beginning period of planning cycle k

N = length of PH

S = total number of orders in all planning cycles.

Sridharan, Berry, and Udayabhanu (1988) used a similar formula to measure MPS instability in single-level uncapacitated systems.

Service level (SL) is the ratio of the cumulative production quantity (i.e., the original cumulative demand minus cumulative shortage due to capacity limitation) to the original cumulative total demand for all items.

The values of the dependent variables are computed for each combination of independent variables. For each combination of independent variables, five runs are made to reduce random effects. The data will be analyzed using the analysis of variance (ANOVA) procedure to test the hypotheses presented in the next section.

4. Research Hypotheses

Four general hypotheses are tested in this study:

HYPOTHESIS 1. *The LSR will significantly influence the total cost, SI and service level of multi-item single-level systems with a single resource constraint.*

HYPOTHESIS 2. *The parameters for freezing the MPS will significantly influence the total cost, SI and service level of multi-item single-level systems with a single resource constraint.*

HYPOTHESIS 3. *Environmental factors (CT, DV, MV, T, SC) will significantly influence the performance of the LSRS.*

HYPOTHESIS 4. *The selection of LSRs will significantly influence the selection of the parameters for freezing the MPS.*

Hypothesis 1 is concerned with the impact of LSR selection on the performance of multi-item single-level systems with a single resource constraint. Although many LSRS have been developed to solve lot-sizing problems under capacity constraints, no comprehensive evaluation of these rules has been performed to examine their performance under a rolling horizon in combination with MPS freezing. Testing Hypothesis 1 will allow us to investigate their performance in developing MPS under a rolling horizon with a portion of the MPS frozen.

Hypothesis 2 is concerned with the impact of the MPS freezing parameters on system performance under a capacity constraint. The intention for testing this hypothesis is to see whether the conclusions made under the assumption of unlimited constraint can be generalized to the environment with capacity constraints. Under capacity constraints, freezing the MPS will also influence capacity utilization because if schedules with certain periods are frozen, unused capacity in these periods can no longer be used even though there may be an additional need for capacity in future periods. Therefore the capacity constraints should influence the impact of the MPS freezing parameters on the performance of the MPS system.

Hypothesis 3 is concerned with the impact of environmental factors on the performance of the LSRS. Because of the different logic used by each lot-sizing approach, the performance of the LSRS will be different under different operating conditions. Testing this hypothesis will allow us to investigate the performance of LSRS under different conditions.

Hypothesis 4 is concerned with the impact of the selection of LSRS on the selection of the MPS freezing parameters. Testing this hypothesis will allow us to examine whether and how the LSRS influence the performance and the selection of the MPS freezing parameters.

5. Results

In order to test the above hypotheses, the output from the simulation program was analysed using the ANOVA procedure. To meet the assumptions of ANOVA, the inverse square root transformation of TC, the logarithm transformation of SI, and square transformations of SL were made based on residual analysis and suggestions by the SAS package. Table 4 shows the main and two-way interaction effects of the independent variables for each of the dependent variables.

Examination of the results in Table 4 shows that most of the main and two-way interaction effects are significant in influencing TC, SI, and SL at a 5% significance level. To examine the impact of the independent variables on the dependent variables, Duncan's multiple-range test was performed to rank the performance of the LSRS and MPS freezing parameters. The results are presented around the hypotheses shown in Section 4.

5.1. *Impact of LSR Selection on System Performance*

From the results in Table 4, we can see that the LSR significantly influences the TC, SI, and SL at a 5% significance level. Table 5 shows the relative total cost (RTC), the relative schedule instability (RSI), and the service level (SL) for the four LSRS. The RTC and the RSI are calculated by dividing the lowest TC and the lowest SI (setting to 100 as benchmark) into the TC and SI for a specific LSR. Table 5 shows the following:

1. The DPA rule produces the lowest TC, and the DS and GU rules produce the lowest SI.

2. The performance differences of the DS and GU rules are not statistically significant according to any of the three performance measures.

3. The KR rule performs worst according to both TC and SI.

4. The KR rule results in a significantly higher SL than the DPA rule, and the DPA rule also results in a significantly higher SL than both the DS and GU rules. However, the service levels are very high (above 99.2%) for all four LSRS, and the differences between them are within 0.15%.

TABLE 4

ANOVA Results for Total Cost, Schedule Instability, and Service Level

Dependent Variables		TC*		SI†		SL‡	
Source	DF	*F* Value	Pr > F	*F* Value	Pr > F	*F* Value	Pr > F
DV	2	99,999.99	0.0001	24,249.30	0.0001	99,999.99	0.0001
MV	2	1,276.12	0.0001	1,226.96	0.0001	152.03	0.0001
DV*MV	4	476.85	0.0001	121.15	0.0001	401.27	0.0001
CT	2	99,999.99	0.0001	3,293.45	0.0001	99,999.99	0.0001
DV*CT	4	3,304.38	0.0001	10,924.39	0.0001	40,810.58	0.0001
MV*CT	4	33.14	0.0001	12.96	0.0001	148.05	0.0001
SC	2	20,941.40	0.0001	0.00	1.0000	0.00	0.9998
DV*SC	4	2,711.31	0.0001	0.00	1.0000	0.00	1.0000
MV*SC	4	2.46	0.0435	0.00	1.0000	0.00	1.0000
CT*SC	4	4,514.37	0.0001	0.00	1.0000	0.00	1.0000
T	1	99,999.99	0.0001	99,999.99	0.0001	24,758.65	0.0001
DV*T	2	38,340.34	0.0001	2,098.37	0.0001	6,546.48	0.0001
MV*T	2	206.93	0.0001	10.57	0.0001	3.96	0.0190
CT*T	2	55,824.76	0.0001	2,784.88	0.0001	9,009.77	0.0001
SC*T	2	6,689.72	0.0001	0.00	1.0000	0.00	1.0000
LSR	3	316.70	0.0001	99,999.99	0.0001	1,028.99	0.0001
DV*LSR	6	19.59	0.0001	2,843.55	0.0001	107.74	0.0001
MV*LSR	6	2.69	0.0129	202.47	0.0001	0.48	0.8263
CT*LSR	6	182.84	0.0001	15,824.33	0.0001	712.58	0.0001
SC*LSR	6	64.82	0.0001	0.00	1.0000	0.00	1.0000
T*LSR	3	372.33	0.0001	16,285.76	0.0001	15.81	0.0001
PH	1	8,739.64	0.0001	4,300.64	0.0001	19,145.80	0.0001
DV*PH	2	918.21	0.0001	210.61	0.0001	5,169.88	0.0001
MV*PH	2	5.99	0.0025	19.22	0.0001	2.70	0.0674
CT*PH	2	1,094.86	0.0001	10,652.25	0.0001	6,327.70	0.0001
SC*PH	2	177.54	0.0001	0.00	1.0000	0.00	1.0000
T*PH	1	3,459.55	0.0001	553.45	0.0001	892.16	0.0001
LSR*PH	3	249.13	0.0001	2,340.67	0.0001	55.88	0.0001
FP	4	29,974.74	0.0001	99,999.99	0.0001	45,658.20	0.0001
DV*FP	8	2,516.48	0.0001	112.33	0.0001	12,702.38	0.0001
MV*FP	8	37.66	0.0001	87.28	0.0001	45.54	0.0001
CT*FP	8	605.48	0.0001	996.46	0.0001	8,278.97	0.0001
SC*FP	8	445.97	0.0001	0.00	1.0000	0.00	1.0000
T*FP	4	5,250.48	0.0001	5,437.37	0.0001	1,468.81	0.0001
LSR*FP	12	160.13	0.0001	4,954.95	0.0001	95.03	0.0001
PH*FP	4	384.13	0.0001	5,253.94	0.0001	1,752.58	0.0001
RP	3	1,191.27	0.0001	46,360.51	0.0001	3,542.68	0.0001
DV*RP	6	67.59	0.0001	7.61	0.0001	988.80	0.0001
MV*RP	6	3.32	0.0029	2.11	0.0483	6.45	0.0001
CT*RP	6	9.55	0.0001	400.14	0.0001	617.14	0.0001
SC*RP	6	11.49	0.0001	0.00	1.0000	0.00	1.0000
T*RP	3	343.54	0.0001	36.09	0.0001	133.04	0.0001
LSR*RP	9	7.56	0.0001	30.71	0.0001	0.15	0.9982
PH*RP	3	50.85	0.0001	39.39	0.0001	122.87	0.0001
FP*RP	12	679.51	0.0001	5,213.52	0.0001	2,029.73	0.0001

Notes: * Based on residual analysis and suggestions by SAS, inverse square root transformation of TC [i.e., 1/sqrt(TC)] was made to satisfy the assumptions of ANOVA. † The observations for FP = 1.0 were not included in the analysis because SI is always equal to zero when FP = 1.0. Based on residual analysis and suggestions by SAS, log transformation of SI [i.e., log 10(SI)] was made to satisfy the assumptions of ANOVA. ‡ Based on residual analysis and suggestions by SAS, square transformation of SL (i.e., SL**2) was made to satisfy the assumptions of ANOVA.

TABLE 5
Overall Performance of Lot-Sizing Rules

	Total Cost		Schedule Instability		Service Level	
LSR	RTC*	RANK§	RSI†	RANK§	SL‡	RANK§
DPA	100.0	1	818	3	99.281	2
DS	102.2	2~3	100	1~2	99.227	3~4
GU	101.7	2~3	106	1~2	99.228	3~4
KR	104.5	4	3,016	4	99.370	1

Notes: * RTC represents relative total cost. For each independent variable, the lowest total cost among all its values is set at 100. The relative total costs of the other values of this independent variable are obtained by dividing the lowest total cost into the total costs of these values. † RSI represents relative schedule instability. For each independent variable, the lowest non-zero schedule instability among all its values is set at 100. The relative schedule instabilities of the other values of this independent variable are obtained by dividing the lowest non-zero schedule instability into the schedule instabilities of these values. ‡ SL represents service level. It is the % of the demand satisfied. § RANK is obtained using Duncan's multiple range test at 5% significance level.

These observations indicate that the LSR significantly influences the TC, and SI, and the SL. While DPA and KR rules result in the best performance in terms of total cost and service level, respectively, the DS and the GU rules are probably the best choice of LSR when all three performance criteria are considered. These two unidirectional period-by-period heuristics seem to represent the best trade-offs between TC, SI, and SL. Overall, these results support Hypothesis 1.

Because previous studies used different experimental settings and included different LSRS, the results of this study cannot be directly compared with the results of previous studies. However, Maes and Van Wassenhove (1986) also found that the DPA rule performed best according to TC under some operating conditions and a rolling horizon.

It is also important to note that the most complicated rule proposed by Karni and Roll (1982) was outperformed by simpler rules according to all three performance criteria. Since the KR rule starts with the optimal Wagner-Whitin solution for each item without considering a capacity constraint, the order quantities are normally much higher than the capacity available. When the capacity constraint is considered, there is a very high probability that initial quantities cannot be produced within the planned period because of insufficient capacity. A shifting procedure is used to move the quantity that cannot be produced in the period to either earlier or later periods, thus new set-up costs will have to be incurred. However, the shifting procedure does not make an optimal allocation of the production quantities in the new and the old production periods to achieve the minimum inventory carrying costs. This leads to higher inventory carrying costs and a poor TC performance of the KR rule.

The DPA rule starts with the LFL solution and produces just the necessary quantity needed in any one period. Therefore demands from different periods are not lumped together in the initial solution. The probability of having insufficient capacity to produce the initial quantity is low, thus the need to split orders due to the capacity constraint is low. However, there is a need to combine orders to realize higher cost savings during set-up. The shifting procedure helps decide what orders should be lumped together under capacity constraints. This procedure does not result in excess inventory carrying costs, but often does result in higher set-up and shortage costs than does the KR procedure.

The DS and GU procedures modified the uncapacitated LSRS as proposed by Silver and Meal (1973) and Groff (1979), respectively, to take into consideration the capacity constraint. An order quantity will not be increased if there is no capacity remaining in the current period even though there can be incremental cost savings by including more demand in the order in future periods. If there is capacity left in the current period, but further lumping cannot save

TABLE 6
Summary of Findings on the Impact of MPS Freezing Parameters

Authors (year)	Settings	Planning Horizon (PH)	Freezing Proportion (FP)	Replanning Periodicity (RP)
Current study	Deterministic demand Single level, multiple items Single resource constraint	Higher PH results in lower TC, higher SI, and higher SL	Higher FP results in higher TC, lower SI, and lower SL	Higher RP results in lower TC, lower SI, and higher SL
Sridharan, Berry, and Udayabhanu (1987)	Deterministic demand Single level, single item No resource constraint	Higher PH results in lower cost of errors	Higher FP results in higher cost of error	N/A*
Sridharan, Berry, and Udayabhanu (1988)	Deterministic demand Single level, single item No resource constraint	Higher PH results in higher SI and then SI levels off	Higher FP results in lower SI	N/A*
Sridharan and Berry (1990b)	Deterministic demand Single level, single item No resource constraint	Higher PH results in lower cost of errors, and higher SI when WW⁺ is used, a lower SI when SM⁺ is used	Higher FP results in higher cost of error and lower SI	Higher RP results in lower TC and lower SI
Sridharan, Berry, and Udayabhanu (1990a)	Stochastic demand Single level, single item No resource constraint	Higher PH results in higher cost of errors and higher SI	Higher FP results in higher cost of error and lower SI	Higher RP results in lower TC and lower SI
Sridharan and LaForge (1990)	Stochastic demand Single level, single item No resource constraint	N/A*	Higher FP results in lower SI and lower SL	N/A*
Sridharan and LaForge (1994)	Stochastic demand Single level, single item No resource constraint	N/A*	Higher FP results in higher inventory but no major loss in SL	N/A*
Zhao and Lee (1996)	Deterministic demand Multi-level, single item No resource constraint	Higher PH results in lower TC and lower SI	Higher FP results in lower TC and lower SI	Higher RP results in lower TC and lower SI

Note: * "N/A" means that the impact of the MPS parameter was not presented in the study. ⁺ "WW" means that the Wagner-and-Whitin rule is used and "SM" means that the Silver-Meal rule is used.

costs, capacity will not be used unless demand for future periods in the PH cannot be met in this replanning cycle. The solution was arrived at in one unidirectional period-by-period step, and a shifting procedure was unnecessary. As a result, the SIS of the DS and GU rules are significantly lower than those found using the KR and DPA rules. Both the KR and DPA rules use the shifting procedure to achieve feasibility and cost savings in the entire PH. Since orders are often adjusted to meet the capacity constraint and to reduce costs, the SIS for these two rules are significantly higher than the SIS for the DS and GU rules.

5.2. *Impact of MPS Freezing Parameters on System Performance*

The ANOVA results in Table 4 show that the main effects of all three MPS freezing parameters (PH, FP, and RP) significantly influence TC, SI, and SL at a 5% significance level. To investigate the differences in the performance impacts of freezing parameters with and without capacity constraints, we summarized the major findings of this study and the previous studies in Table 6.

The results in Table 6 indicate that a longer PH produces a lower TC, a higher SL, but also a higher SI. Therefore a trade-off must be made between TC, SI, and SL in order to select the proper PH under a capacity constraint. This finding is in general agreement with the result reported by Sridharan, Berry, and Udayabhanu (1987, 1988) in single-level systems without considering a capacity constraint. However, the finding concerning the impact of the PH on SI contradicts with the finding reported by Zhao and Lee (1996) in multi-level systems with no consideration of a capacity constraint. Zhao and Lee (1996) found that a longer PH resulted in both lower TC and lower SI in multi-level systems where there were no capacity constraints. The differences in the findings are due to the difference between single-level and multi-level systems. In a multi-level system, a minor change in the MPS can cause major changes in the production schedules for the dependent items. When the PH is short, the orders toward the end of that PH may not be of optimal quantity. In the next replanning cycle, the sizes of these orders will be revised as more demand information is added to the new PH. This will cause a series of changes in the planned schedules for the dependent components. When the PH is long, nervousness is reduced, resulting in a lower SI. In a single-level system, however, changes in the MPS do not result in so many nervous changes in the dependent schedule; thus a longer PH does not result in a lower SI. Under a capacity constraint, a longer PH will allow for more opportunities to make MPS adjustments to better utilize the capacity. These opportunities reduce unfeasibility caused by limited capacity and thus lead to higher service levels but a higher SI.

Table 6 also shows that a higher FP results in higher TC, lower SI, and a lower SL. This result agrees with the findings of Sridharan, Berry, and Udayabhanu (1987, 1988), who considered single-level systems under deterministic demand with no capacity constraints. However, these findings differ from the findings made by Zhao and Lee (1996) in multi-level systems. Zhao and Lee (1996) found that a higher FP resulted in both lower TC and lower SI. This is because freezing the MPS can result in more benefits in multi-level systems than in single-level systems. Again, in order to select a proper FP under a capacity constraint, a trade-off must be made between TC, SL, and SI.

Results in Table 6 also indicate that a higher RP results in lower TC, a higher SL, and lower SI. Therefore the system performance will be at its best when replanning is carried out after the entire frozen interval has passed (RP = 1.0). This finding agrees with the finding by Zhao and Lee (1996) in multi-level systems under deterministic demand without considering capacity constraints. Sridharan and Berry (1990a) also found a similar relationship between TC, SI, and RP in single-level systems under demand uncertainty. However, they did not investigate this relationship in their deterministic studies (Sridharan, Berry, and Udayabhanu 1987, 1988; Sridharan and Berry 1990b).

Our overall results in Table 6 show that most of the conclusions from a single-level system without capacity constraints are also true under capacity constraints. However, there are significant differences between single-level systems and multi-level systems. The results support Hypothesis 2.

The impact of the environmental factors (CT, DV, MV, T, and SC) on the performance of the MPS freezing parameters was also examined. However, the results show that none of them significantly influence the relative performance ranking of the MPS freezing parameters. Thus these results are not presented here.

5.3. *Impact of Environmental Factors on Performance of Lot-Sizing Rules*

From the results in Table 4, we can see that all two-way interactions between the LSRS and the environmental factors (CT, DV, MV, T, and SC) significantly influence TC at a 5% significance level. All interactions between the LSRS and the environmental factors except LSR*SC also significantly influence SI. For SL, the interactions between the LSR, DV, CT, and T are significant, but between LSR, MV, and SC they are not. To focus on the most significant result using the space available, we only examine the interaction effects that are significant at a 5%

TABLE 7

Performance of LSRs Under Different Capacity Tightness

Dependent Variables / Independent Variables		Total Cost		Schedule Instability		Service Level	
CT	LSR	RTC*	RANK§	RSI†	RANK§	SL‡	RANK§
Low	DPA	100.4	2~3	1,462	3	99.894	1~4
	DS	101.5	3	106	1~2	99.893	1~4
	GU	100.0	1~2	121	1~2	99.893	1~4
	KR	106.5	4	5,037	4	99.897	1~4
Medium	DPA	132.2	1	1,183	3	99.633	2~4
	DS	136.3	2~3	100	1~2	99.616	2~4
	GU	135.3	2~3	106	1~2	99.616	2~4
	KR	142.2	4	4,593	4	99.660	1
High	DPA	246.1	1	591	3	98.314	2
	DS	251.5	2~4	190	1~2	98.174	3~4
	GU	251.6	2~4	192	1~2	98.176	3~4
	KR	251.6	2~4	2,303	4	98.554	1

Notes: * RTC represents relative total cost. The lowest TC is set at 100. The RTC of the CT and LSRs are obtained by dividing the lowest TC into the TC of the specific conditions. † RSI represents relative schedule instability. The lowest SI among all its values is set at 100. The RSIs of the CT and LSRs are obtained by dividing the lowest SI into the SIs of the specific conditions. ‡ SL represents service level. It is the percentage of demand satisfied. § RANK represents relative rank of the four LSRs obtained using Duncan's multiple range test with a significance level of 5%.

significance level according to all three performance criteria. The impacts of CT, DV, and T on the performance of the LSRS are presented in Tables 7–9, respectively, and are discussed below.

IMPACT OF CAPACITY TIGHTNESS (CT). Table 7 shows the performance ranking of the LSRS according to all three criteria under different CT conditions. From Table 7, we can make the following important observations.

1. In terms of TC, the DPA rule performs best and is significantly better than the other three lot-sizing rules when CT is either medium or high. When CT is low, its performance worsens, but not significantly over the performance of the GU rule. The performance of the DPA rule improves when capacity is tight because it starts with lot-for-lot logic, then makes adjustments to take advantage of the set-up economy and to make sure that the capacity constraint is not violated. Since the lot-for-lot rule provides a relatively even distribution of the workload in the initial solution, it has less need for adjustment than other rules in order to achieve capacity feasibility under a tight capacity constraint. When CT is low, the relative advantage of DPA over the GU and DS rules decreases, making its performance close to that of the GU and DS rule in terms of TC. The GU rule produces the best performance according to TC when CT is low. When CT is medium or high, the performance of the GU rule is not significantly different from that of the DS rule, but is significantly better than that of the KR rule. When CT is high, the performance of the GU rule is not significantly different from the performance of the DS and KR rules. The KR rule produces the highest TC under all levels of CT. In general, as capacity becomes tighter, the differences in performance for TC between different lot-sizing rules become less significant.

2. According to SI, both the DS and GU rules perform the best under all three levels of CT, and their performances are not significantly different from each other, but are significantly better than the performance of the two myopic period-by-period heuristic rules. Therefore these two rules should be used if the objective is to minimise the SI. The KR rule produces the highest SI under all three levels of CT.

3. In terms of SL, the performances of the four lot-sizing rules are not significantly different from each other when CT is low. As capacity tightens, the differences in SL performance become more significant. The SL of the KR rule is significantly higher than those of the other rules when CT is medium and high. The SL of the DPA rule is also significantly higher than under the DS and GU rules when CT is high. However, the DS and GU rules do not show a significant difference in performance in SL under all levels of CT. Although the differences in SL performance are statistically significant in some cases, the service levels are above 98% under all conditions. The differences between different LSRS may not be significant from a practitioner's perspective.

Overall, the results in Table 7 support research Hypothesis 3 as it relates to CT. The result in Table 7 has several important managerial implications. First, as the capacity becomes tighter, the selection of LSRS has a lower impact on TC and SI and thus makes the LSR selection less important. Second, if the company wants to mainly reduce TC and capacity is tight, the DPA rule would be the best choice. If the company also wants to reduce SI, or when the capacity is not tight, the GU rule seems to be the best choice because it represents the best trade-off between TC and SI. The selection of LSR does not seem to produce any practical difference in terms of SL.

IMPACT OF DEMAND PATTERNS (DV). To examine the impact of demand patterns on the performance of LSR, LSRS are ranked according to the three performance criteria under different DV. The results are shown in Table 8. Examination of the results shows the following.

1. The DPA rule performs best according to TC under all levels of DV. DPA is ranked as third best performer under all the levels according to SI. In terms of SL, different LSRS do not seem to produce any practical differences.

2. The KR rule produces the highest TC and SI under all levels of DV. However, it also results in the highest SL among the four LSRS even though the difference in performance is not significant from a practical perspective. The DS and GU rules do not produce significantly different SI and SL under all levels of DV. Both LSRS produce the lowest level of SI under all demand patterns, thus they are the best for minimising SI. According to TC, however, they are outperformed by the DPA rule. The difference in TC between DS and GU is not significant when DV is low or medium. It becomes significant when DV is high.

Overall, the results in Table 8 support research Hypothesis 3 as it relates to DV. However, the relative ranking of the LSR is not much changed by the DV. If all three criteria are considered, the GU and DS rules seem to perform well under most conditions while DPA performs best according to TC under all conditions. The result concerning the impact of MV on LSR performance is not presented here because MV did not have practical impact on the performance of the LSR. This may be because all items are assumed to consume a unit capacity in the production process. If capacity consumption rates are different for different items, the results might be different.

IMPACT OF COST PARAMETERS (T). Table 9 shows the performance ranking of the LSRS according to all three criteria under different T. Examination of the results in Table 9 shows the following.

1. T does significantly influence the relative ranking of the LSRS according to TC but not according to SI and SL. When T is eight periods, the DPA rule performs best according to TC, followed by the DS, GU, and KR rules. When T is equal to four periods, however, the KR rule becomes the best performing rule according to TC, while the remaining three rules show no significant difference from each other. The reason that the KR rule performs worst when T is longer is because its initial solution without considering a capacity constraint will consist of large order sizes. When the size of the order is large, it is more difficult to achieve feasibility when the shifting procedure is used. Therefore the relative performance of the improvement heuristics will decrease as T increases.

TABLE 8

Performance of LSRs Under Different Demand Patterns

Dependent Variables / Independent Variables		Total Cost		Schedule Instability		Service Level	
DV	LSR	RTC*	RANK§	RSI†	RANK§	SL‡	RANK§
Low	DPA	100.0	1	1,214	3	99.821	2
	DS	100.3	2~3	100	1~2	99.803	3~4
	GU	100.4	2~3	108	1~2	99.803	3~4
	KR	102.8	4	4,652	4	99.873	1
Medium	DPA	112.7	1	1,129	3	99.472	2
	DS	115.2	2~3	131	1~2	99.434	3~4
	GU	114.9	2~3	141	1~2	99.433	3~4
	KR	119.3	4	4,411	4	99.564	1
High	DPA	164.7	1	1,267	3	98.549	2
	DS	170.2	3	210	1~2	98.445	3~4
	GU	168.7	2	219	1~2	98.449	3~4
	KR	172.3	4	4,248	4	98.674	1

Notes: * RTC represents relative total cost. The lowest total cost is set at 100. The RTC of the demand variations (or product mix variations) and LSRs are obtained by dividing the lowest TC into the TC of the specific conditions. † RSI represents relative schedule instability. The lowest SI among all its values is set at 100. The RSIs of the demand variations (or product mix variations) and LSRs are obtained by dividing the lowest SI into the SIs of the specific conditions. ‡ SL represents service level. It is the percentage of demand satisfied. § RANK represents relative rank of the four LSRs obtained using Duncan's multiple range test with a significance level of 5%.

2. As T increases, TC, SI, and SL all increase. The reason that SL increases is that the PH (in periods) is longer for T = 8 than that for T = 4. Under a capacity constraint, a longer PH will allow the manager to better utilize capacity to meet demand and reduce unfeasibility caused by limited capacity. This leads to a higher SL, lower TC, and higher SI.

TABLE 9

Performance of LSRs Under Different Cost Parameters

Dependent Variables / Independent Variables		Total Cost		Schedule Instability		Service Level	
T	LSR	RTC*	RANK§	RSI†	RANK§	SL‡	RANK§
4	DPA	102.6	2~4	366	3	99.103	2
	DS	103.7	2~4	100	1~2	99.062	3~4
	GU	103.7	2~4	103	1~2	99.062	3~4
	KR	100.0	1	1,582	4	99.212	1
8	DPA	185.9	1	1,798	3	99.459	2
	DS	191.2	2~3	165	1~2	99.392	3~4
	GU	189.8	2~3	177	1~2	99.394	3~4
	KR	201.5	4	6,399	4	99.528	1

Notes: * RTC represents relative total cost. The lowest total cost is set at 100. The RTC of the maximum natural ordering cycles (or shortage cost parameters) and LSRs are obtained by dividing the lowest TC into the TC of the specific conditions. † RSI represents relative schedule instability. The lowest SI among all its values is set at 100. The RSIs of the maximum natural ordering cycles (or SC parameters) and LSRs are obtained by dividing the lowest SI into the SIs of the specific conditions. ‡ SL represents service level. It is the percentage of demand satisfied. § RANK represents relative rank of the four LSRs obtained by using Duncan's multiple range test with a significance level of 5%.

TABLE 10
Performance of MPS Freezing Parameters (MPSFP) Under Different LSRs

Dependent Variables / Independent Variables		Total Cost		Schedule Instability		Service Level	
LSR	MPSFP[e]	RTC[a]	RANK[d]	RSI[b]	RANK[d]	SL[c]	RANK[d]
DPA	PH = 4	112.6	2	836	1	99.135	2
	PH = 8	100.0	1	882	2	99.426	1
DS	PH = 4	116.3	2	100	1	99.069	2
	PH = 8	101.0	1	110	2	99.385	1
DPA	FP = 0.00	100.0	1	11,262	5	99.591	1~2
	FP = 0.25	106.2	2	2,834	4	99.559	1~2
	FP = 0.50	111.2	3	1,192	3	99.498	3
	FP = 0.75	118.2	4	459	2	99.372	4
	FP = 1.00	173.8	5	0	1	98.383	5
DS	FP = 0.00	106.6	1~2	1,167	5	99.552	1
	FP = 0.25	109.4	1~2	443	4	99.502	2
	FP = 0.50	113.7	3	215	3	99.422	3
	FP = 0.75	121.5	4	100	2	99.278	4
	FP = 1.00	171.8	5	0	1	98.383	5
DPA	RP = 0.25	111.8	4	1,185	4	99.107	4
	RP = 0.50	105.3	3	961	3	99.263	3
	RP = 0.75	101.4	1~2	875	1~2	99.357	1~2
	RP = 1.00	1~2	831	1~2	99.395	1~2	
DS	RP = 0.25	115.2	4	146	4	99.055	4
	RP = 0.50	107.8	3	118	3	99.209	3
	RP = 0.75	103.3	1~2	107	2	99.303	1~2
	RP = 1.00	101.4	1~2	100	1	99.342	1~2

Notes: [a] RTC represents relative total cost. The lowest total cost is set at 100. The RTC of the MPSFP and LSRs are obtained by dividing the lowest TC into the TC of the specific conditions. [b] RSI represents relative schedule instability. The lowest SI among all its values is set at 100. The RSIs of the other MPSFP and LSRs are obtained by dividing the lowest SI into the SIs of the specific conditions. [c] SL represents service level. It is the percentage of demand satisfied. [d] RANK represents relative rank of the MPSFM obtained using Duncan's multiple range test with a significance level of 5%. [e] MPSFM represents the master production scheduling freezing parameters.

Overall, the results in Table 9 support research Hypothesis 3 as it relates to T. When T = 4, the KR rule is the best choice if the company wants to reduce TC and improve SL. However, the KR rule also produces the highest level of SI. If the company wants to consider all three criteria in selecting LSRS, DS and GU might be the best choice. When T = 8, DPA is the best rule if the company wants to reduce TC. If the company wants to balance the interest between TC, SI, and SL, again the best choice of LSR is DS or GU.

5.4. *Impact of LSR Selection on Selection of MPS Freezing Parameters*

Table 4 shows that almost all the two-way interactions between the LSR and the MPS freezing parameters are significant in terms of all three performance measures, except for the two-way interaction between the LSR and RP, where there is no significant influence on SL. To examine the impact of LSR selection on the selection of MPS freezing parameters, we ranked the performance of the MPS freezing parameters using the Duncan's test when different LSRS are used. Because of the space limitations, we present in Table 10 only the results obtained from using the DPA and the DS rules. The impact of the LSR on the performance of the freezing parameters is discussed below.

LOT-SIZING RULES (LSR) AND PLANNING HORIZON (PH). Examination of the results in Table 10 shows that a PH of eight times of T always produces lower TC, higher SL, but also higher SI than a PH of four times T regardless of the LSRS. This observation reveals that the selection of LSR has no impact on the selection of PH, thus in terms of PH, Hypothesis 4 is not supported. If a company wants to reduce TC, a planning horizon of eight natural ordering cycles should be used. If it wants to reduce SI, a planning horizon of four natural ordering cycles is preferred.

LOT-SIZING RULES (LSR) AND FREEZING PROPORTION (FP). The performance ranking of FP according to all three criteria using different LSRS is also shown in Table 10. The results indicate that a larger FP produces a higher TC, lower SL, and lower SI under all LSRS. TC increases slowly and SL deteriorates slowly when the freezing proportion increases from 0.00 to 0.25, 0.50, and 0.75. However, TC increases quickly and SL decreases more dramatically when the freezing proportion increases from 0.75 to 1.00. This result agrees with the overall TC performance of FP found in Section 5.2. This observation reveals that the selection of LSR has no impact on the selection of FP; thus Hypothesis 4 is not supported in terms of FP.

LOT-SIZING RULES (LSR) AND REPLANNING PERIODICITY (RP). The results in Table 10 indicate that a higher RP produces a lower TC, higher SL, and also lower SI using all four LSRS. This result agrees with the overall TC performance of RP found in Section 5.2. This observation reveals that the selection of LSR has no impact on the selection of RP. Replanning more frequently will worsen the system performance according to all three performance criteria. To achieve the best system performance, a company should replan after the entire frozen interval is passed.

Overall, the result in Table 10 shows that the selection of LSR does not significantly influence the selection of the MPS freezing parameters. These results do not support Hypothesis 4. Just as in the case of unlimited capacity, the selection of PH and FP has to be based on trade-offs between TC and SI. However, RP selection is much easier. When RP is equal to the frozen interval, the system can achieve the best performance.

6. Discussion and Conclusions

This paper investigates the impact of capacitated lot sizing rules and the parameters for freezing the MPS on the total cost, schedule instability, and service level of multi-item single-level systems with a single resource constraint. It also examines the impact of demand pattern, cost structure, and capacity tightness on the selection of the lot-sizing rules and MPS freezing parameters under rolling time horizons and deterministic demand. This paper makes the following contributions.

1. This study extends earlier studies of lot-sizing rules and freezing the MPS from a single end-item system without any capacity constraints to a multiple end-item system with a single resource capacity constraint. Through simulation and subsequent analyses of the simulation output, we found that most of the findings concerning the performance of MPS freezing parameters under the assumption of no capacity constraint (Sridharan, Berry, and Udayabhanu 1987, 1988; Sridharan and Berry 1990a, 1990b) are also true under a single resource constraint. Therefore this study fills a gap in the literature on the impact of MPS freezing under a capacity constraint.

2. This study also evaluates the performance of four capacitated lot-sizing rules (the DPA, DS, GU, and KR rules) under a rolling time horizon with MPS freezing. These lot-sizing rules have never been evaluated in such an environment before. Among the four lot-sizing rules considered, the DPA rule is the best according to TC under most conditions and thus should be the best lot-sizing rule if a company wants to minimize TC. If a company wants to achieve a proper balance of TC, SI, and SL, the DS and the GU rules are better choices. Although capacity tightness, demand variation, and cost structure do significantly influence the performance ranking of the lot-sizing rules, the DS and GU rules seem to perform well under most

conditions considering all three performance criteria. These findings can help a company improve system performance by selecting the proper lot-sizing rule.

3. This study also shows that the selection of the four lot-sizing rules tested in this study does not significantly influence the selection of MPS freezing parameters. This result enhances our knowledge and understanding of the impact of freezing the MPS under capacity constraints.

This study has several limitations. Future research is needed to better understand the impact of lot-sizing rules and MPS freezing parameters on system performance under capacity constraints. The following future research avenues may be pursued.

1. The experimental settings in this study include master production scheduling in multi-item single-level systems under a single resource constraint. While a single-level study will provide insights into the problem, still more insights can be gained by conducting the study on a multi-level system.

2. In this study, we assumed that demands are known with certainty. Actually very few companies know their demand with certainty and thus have to rely on demand forecasts in developing their MPS. Studies on the impact of MPS freezing without considering capacity constraints have shown that forecasting errors often significantly influence the selection of lot-sizing rules and MPS freezing parameters (Zhao and Lee 1993; Zhao, Lee, and Goodale 1995). Under capacity constraints, the impact of forecasting errors might be even more significant. It will be interesting to examine the impact of forecasting errors on the selection of MPS freezing parameters and lot-sizing rules under capacity constraints.

3. In this study, we assumed that all products have the same capacity consumption rate (1 per unit). In reality, different products may have different capacity consumption rates. When different consumption rates are used, the product mix factor (MV) may have a greater impact on system performance. The impact of the capacity consumption rates of different items should be investigated in the future.

4. To reduce the number of factors to be considered in this study, we did not include demand lumpiness in the design of the demand patterns. Demand lumpiness may influence the performance of lot-sizing rules. Future studies should examine the impact of demand lumpiness on the performance of the system and the selection of lot-sizing rules under capacity constraints.[1]

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