Multilevel Lot-Sizing Heuristics and Freezing the Master Production Schedule in Material Requirements Planning Systems

(Subject Areas: Material Requirements Planning, Master Production Scheduling, Lot-Sizing, Simulation)

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Abstract

This paper compares the performance of a new improved heuristic (STIL) proposed by Coleman and McKnew (1991) against those of the Wagner-Whitin (WW), cost-modified Wagner-Whitin (MWW) and some simple lot-sizing rules found to perform well by Zhao et al. (1995). It also investigates the impact of the lot-sizing rules on the selection of the parameters for freezing the master production schedule (MPS). The experimental settings include forecasting, master production scheduling and material requirements planning under a rolling time horizon. The result of the study shows that the STIL heuristic, the WW rule and the MWW rule do not really outperform the simple cost-modified Silver-Meal (MSM) and the Silver-Meal/Lot-for-Lot (SM/LFL) even though they require substantially higher computation time. The study also shows that the selection of the parameters for freezing the MPS is not significantly influenced by the selection of the lot-sizing rules.

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1. Introduction

Multilevel lot sizing is an important technical issue in Material Requirements Planning (MRP) systems. Because of the interdependencies of the lot-sizes at different levels of the bill of material, optimal solutions to the multi-level lot-sizing problems are very difficult to find. As a result, many heuristic procedures have been developed to solve the multilevel lot-sizing problems (Blackburn and Millen 1982a, 1982b). Many of these heuristic procedures involve modifications of the single level lot-sizing rules such as the Wagner-Whitin (WW) rule (Wagner and Whitin 1958), and the Silver-Meal (SM) rule (Silver and Meal 1973). Although considerable efforts have been devoted to the development and evaluation of multilevel lot-sizing procedures, so far no satisfactory procedures have been identified for practical applications.

Coleman and McKnew (1991) proposed a new improved heuristic procedure (STIL) for determining the lot-sizes in multi-level MRP systems. They evaluated the performance of this improved heuristic rule against the Wager-Within (WW) rule, the cost-modified Wager-Whitin (MWW) rule, the Incremental Part-Period Algorithm (IPPA) rule (Boe and Yilmaz 1983), the Cumulative Incremental Part-Period Algorithm (CIPPA) rule under deterministic demand and fixed time horizon. The STIL heuristic is a sequential application of TOPS (Techniques for Order Placement and Sizing) proposed earlier by the same authors (Coleman and McKnew 1990) with modifications allowing for analysis of ramifications on component item costs before an action is taken.

Through simulations, they showed that the new heuristic procedure was superior in both cost performance and computational efficiency than other heuristic procedures tested in their study and suggested in the literature. They also suggested that the STIL heuristic procedure could be used as "an aid to MRP researchers and a foundation for future efforts to confront more complex cases". However, the environment in which the heuristic procedure was tested excluded demand uncertainty and the fact that production schedule needs to be developed on a rolling time horizon. To prove that the STIL heuristic is really as useful as the authors claimed it, we need to evaluate its performance against other lot-sizing rules in more realistic operational settings.

Zhao et al. (1995) investigated the performance of 14 heuristic bt-sizing rules in multiple stage MRP systems under demand uncertainty and rolling time horizon. The study shows that the forecasting models, the cost structure and the product structure significantly influences the performance of the lot-sizing rules. The cost modified lot-sizing rules were shown to reduce both total cost and schedule instability. Although the ranking of the lot-sizing rules is influenced by the other operating parameters, the Cost-modified Silver-Meal/Lot-For-Lot (MSM/LFL), the cost-modified Silver-Meal (MSM), the Silver-Meal/Lot-For-Lot (SM/LFL), the cost-modified Part-period Balancing/Lot-For-Lot (MPPB/LFL) and the Periodic Ordering Quantity/Lot-For-Lot (POQ/LFL) rules were shown to outperform other rules under most conditions. The study also indicated that the selection of the lot-sizing rules significantly influenced the selection of parameters for freezing the master production schedule. However, because of computation time considerations, they did not include the Wagner-Whitin rule (Wagner and Whitin 1958) and the STIL heuristic rule suggested by Coleman and McKnew (1991).

The purpose of this study is two folds: 1). To compare the performance of the STIL heuristic rule with those of Wagner-Whitin (WW), cost-modified Wagner-Within (MWW), and the seven best rules (MSM/LFL, MSM, SM/LFL, MPPB/LFL and POQ/LFL) found by Zhao et al. (1995) using total cost, schedule instability, and service level as the performance criteria; 2). To examine the impact of lot-sizing rule selection on the selection of the parameters for freezing the master production schedule using the best lot-sizing rules selected in this paper. In the following sections, we will first present the research design and research hypotheses, then show the result. Finally we will summarize and discuss the research findings.

2. Research Design

The methodology used in this study is computer simulation. This section describes the design and implementation of the MRP simulation model and summarizes the independent and dependent variables of the experimental design.

2.1 The Simulation Model

This simulation model was adapted from the MRP model developed by Zhao and Lee (1993, 1996), Zhao and Lam (1996) and Zhao et al. (1995). The simulation model consists of three phases: Demand Generation and Parameters Estimation; Development of the Master Production Schedule; and Material Requirements Planning. The three phases of the simulation model are discussed below.

2.1.1. Demand Generation and Parameter Estimation

The first phase of simulation generates demands and makes parameter estimation for the forecasting models. The following demand generation function is used to generate demands for 400 periods.

$$A_t = A_0 + T_{mag} * t + S_{mag} * SIN(PI/P*t) + N_{mag} * R$$

Where,

A_t = demand for period t; t = time period; A_0 = initial demand; T_{mag} = slope of the trend component; S_{mag} = magnitude of the seasonal component; N_{mag}= magnitude of the noise component; PI = a constant equal to 3.14; P = period of the seasonal variations; R = a normal random variate.

The initial demand, slope of the trend component, the magnitudes of the seasonal and noise components and the period of the seasonal variation can be changed to generate different demand patterns. In this study, P is fixed at 7 periods, A_o, T_{mag}, S_{mag}, $S_{mag,}$ and N_{mag} are assumed to have high and low amplitudes of variations. The values of the parameters, the average and standard deviations of demands for the two demand patterns generated during the 400 periods are shown in Table 1.

(***** insert table 1 in about here *********)

Once the demands are generated for four hundred periods, a specific forecasting model is used to make forecasts for the first 49 periods. The mean absolute deviation (MAD) is calculated to evaluate the performance of forecasting models. To optimize the performance of forecasting models, parameters for each of the two forecasting models are changed in small increments within a specified range of values. The best set of parameters for each of the forecasting models are selected based on their abilities to minimize the forecasting error measure (MAD) in the first 49 periods. Table 2 shows the best set of parameters selected for each forecasting model based on their abilities to minimize the one-period-ahead forecasting error measure.

(**** insert table 2 in about here *********)

2.1.2. MPS Development Phase

The second phase of the simulation model forecasts the demand for future periods using the best set of parameters identified in phase I, then develops the master production schedule (MPS). The MPS is developed in a rolling time horizon environment. The procedure for developing the MPS is well described by Zhao and Lee (1993, 1996), Zhao and Lam (1996) and Zhao et al. (1995). A brief description is provided in this section.

After the demand forecast is made for a number of periods into the future (planning horizon), the MPS is developed for these periods. As time goes by, more demand information will become available, the demand forecast is revised and the MPS is revised accordingly. To avoid excess changes, management often chooses to implement a portion of MPS according to the original plan. The portion of the MPS that is not changed is referred as "frozen." Figure 1 illustrates the major parameters for freezing the MPS under a rolling time horizon.

(--------- insert figure 1 in about here ---------------)

The lead-time (LT) is the cumulative production lead-time for finishing an end item starting from ordering raw materials. Schedules within the production lead-time are always frozen. The planning horizon (PH) is defined as the number of periods beyond the total production lead-time for which the production schedules are developed in each replanning cycle. We intentionally excluded the lead-time in the planning horizon in order to make a fair comparison between the two products with different lead times. In a make-to-stock environment, forecasts provide the demand information. As the planning horizon increases, the accuracy of the forecast near the end of the horizon decreases. In a make-to-order environment, the planning horizon is heavily influenced by how early in advance customers are willing to place their orders. The frozen interval is the number of scheduled periods within the planning horizon for which the schedules are implemented according to the original plan. The free interval is the number of scheduled periods beyond the frozen interval. This portion is subject to change based on new demand information. The frozen proportion (FP) refers to the ratio of the frozen interval relative to the planning horizon. The higher the frozen proportion, the more stable the production schedules, and the lower the nervousness of the MRP systems. However, a higher frozen proportion may increase stock-outs for finished products due to low responsiveness to demand changes.

The replanning periodicity (RP) is the number of periods between successive replannings. When the replanning periodicity is equal to 4 periods, demands for future periods are forecasted in every four periods and the MPS and the MRP plans are revised. The greater the RP, the less frequently the replanning will occur, the less responsive the system will be to the demand changes, and the higher the stock-outs. However, a greater RP will result in fewer changes in production schedules and lower nervousness.

In addition to the parameters shown in Figure 1, Sridharan et al. (1987, 1988) suggested two methods of freezing MPS (ZM): a period-based method and an order-based method. Using the period-based method, orders within a certain number of periods in the planning horizon are implemented according to the original plan and the freezing proportion is equal to the number of periods in the frozen interval divided by the total number of periods in the planning horizon. Using the order-based method, a certain number of orders placed in the planning horizon are implemented as originally planned and the freezing proportion is equal to the number of orders frozen divided by the number of orders in the planning horizon.

2.1.3. Material Requirements Planning

The MRP simulation module takes the MPS as the input and develops production and ordering schedule for the dependent demand items under a rolling time horizon. The master production schedules are developed based on demand or demand forecasts within the planning horizon (PH). Within the frozen interval, the MPS is implemented and cannot be changed. In contrast, beyond the frozen interval the MPS is subject to revision. As the production schedule is rolled RP periods ahead, more demand information is available, previously forecasted demands are revised, and forecasted demands for more distant future periods (not previously forecasted) are appended to the schedule. Net requirements for the non-frozen periods within the new planning horizon are calculated using the following equation:

$$\text{Nrequire}(t)=\text{Grequire}(t)-\text{Endinv}(t-1)$$

where

Nrequire(t) is the net requirement for period t

Grequire(t) is the demand or forecast of demand for period t

Endinv(t-1) is the ending inventory for period t-1

When Nrequire(t) is less than 0, Endinv(t) is set to minus Nrequire(t) and Nrequire(t) is set to zero and based on the net requirement, lot-sizing rules are used to determine lot sizes within the planning horizon. This process is repeated until master production schedules are developed for all 346 periods (400 periods minus 49 periods for parameter estimation and 5 period for the cumulative lead-time). The number of periods for which MPS schedules are frozen depends on the frozen proportion (FP), the planning horizon (PH), and the freezing method (ZM).

The production schedules for the dependent items are developed using the standard MRP logic and lot-sizing rule. After production schedules for all items have been developed and implemented, performance measures are calculated to evaluate the performance of the MRP systems. The simulation procedure is summarized in Figure 2.

(--------Insert Figure 2 about here------)

2.2. Independent and Dependent Variables

2.2.1 *Independent Variables*

The computer model allows many parameters to vary in the simulation experiment. There are three major groups of independent variables in this simulation experiment. The first group is the forecasting error variables, which include demand pattern, forecasting models, and forecasting error measures used to make parameter estimates for the forecasting models. The second group of variables are the parameters for freezing the master production schedule (MPS), which include planning horizon (PH), the replanning periodicity (RP), the freezing proportion (FP) and the method of freezing (ZM). The third group of independent variables is the MRP parameters, which include MRP structure, cost structure and lot-sizing rules. The number of levels of these parameters and their values are shown in Table 3 and are discussed below:

(---------- insert table 3 in about here----------)

2.2.1.1. *Forecasting Error Variables*

Demand pattern (DP): Two demand patterns (shown in table 1) are used in this study. The two demand patterns represent the low and high amplitudes of variations in the trend, seasonality and noise components of the demand patterns. When forecasting models are used to forecast demands with different amplitudes of variations, different levels of forecasting errors will appear.

Forecasting Models (FM): Although many forecasting models can be used to forecast demand, only two forecasting models are used in this study. The two forecasting models used are the simple moving average (MA) model and Winters' mode (WM). The first model is simple and easy to use, but does not perform very well when there are trends and seasonal variations in the demand patterns, the later is also relatively easy to use and have been shown to perform well with seasonal and trend components of variations (Groff 1973, Hibon 1984).

2.2.1.2. *Parameters for Freezing the Master Production Schedule*

Planning Horizon (PH): Previous research found that the performance of the MRP system was improved when the planning horizon is a multiple of the natural ordering cycle (Carlson et al. 1982, Blackburn & Millen 1980). Zhao and Lee (1993) and Zhao et al. (1995) found that the length of the planning horizon had a significant impact on total cost and instability within multilevel MRP systems. The selection of the planning horizon is contingent upon MRP structure, lot sizing rule, and the natural ordering cycle for the end item. To reduce the number of combinations of the independent variable, the planning horizon was set at 4 and 8 natural ordering cycles respectively.

Freezing Proportion (FP): Freezing proportion has been found to significantly influence the performance of multilevel MRP systems. The freezing proportion is set at 0.50 and 1.00 respectively.

Freezing Method (ZM): Previous research by Sridharan et al. (1987, 1988), Zhao and Lee (1993) and Zhao et al. (1995) used two alternative methods for freezing the MPS. They are also used in this study.

Replanning Periodicity (RP): Replanning Periodicity is the time periods between replanning cycles. Zhao and Lee (1993) and Zhao et al. (1995) found that a replanning periodicity equal to the frozen interval resulted in the best system performance. Therefore the replanning periodicity was set at 1.00 (replanning after the entire frozen interval is passed).

2.2.1.3. MRP Operating Parameters

MRP Structure (MS): The structure of an MRP system has been shown to influence the performance of not only the system but also the lot sizing rules (Lee & Adam 1986, Zhao et al. 1995). Two distinctive MRP structures (shown in Figure 3) were used in this study. MRP 1 reflects a pure fabrication process in which the purchased part (item 5) is fabricated through five stages to get the final product. In contrast, MRP2 reflects the pure assembly type of process in which four parts are assembled together to get the final product. Lead times between successive stages are assumed to be 1 period for both structures. However, the cumulative production lead times are 5 and 2 periods for MRP1 and MRP2 respectively.

Lot Sizing Rules (LSR): Lot-sizing rules selected in this study includes the seven best lot-sizing rules that were found to perform well by Zhao et al (1995), the STIL heuristic proposed by Coleman and McKnew (1991), the Wagner-Whitin rule (WW) (Wagner and Whitin 1958) and the cost-modified Wagner-Whitin (MWW) as proposed by Blackburn and Millen (1980). These rules are listed in Table 3. Detailed descriptions of the lot-sizing rules can be obtained from the author.

Cost Parameters (T): Inventory carrying cost, productions setup cost/ordering cost and shortage costs are three cost parameters in MRP settings. Blackburn et al. (1985, 1986, 1987) used the echelon holding cost concept in evaluating the performance of different strategies for dampening MRP nervousness. A similar approach is used in this study.

The echelon holding cost (EHC) for the end item is fixed at \$1 per unit per period and production setup costs are varied so that the natural ordering cycle (T) is 4 and 8 periods, respectively. Echelon holding costs for the four dependent items are randomly generated from the set [0.1, 0.5, 1.0, 2.0]. Setup costs for these items are designed so that the natural ordering cycles (T) for the four items are randomly selected from the set [2, 4, 6, 8]. Neither the echelon holding cost nor the production setup costs for the dependent items are changed. The shortage cost for the end item is set at 5 times the end item value, and the end item value is assumed to be 10 times the inventory holding cost. Thus the shortage cost is (1.0+0.5+1.0+2.0+0.1)*10*5, or \$230/unit. In this simulation experiment, no shortages are allowed for the dependent items. Table 4 shows the cost parameters generated using this procedure.

(--------Insert Table 4 about here----)

2.2.2. Dependent Variables

The following three criteria will be used as the dependent variables of the experimental design:

1) Total Cost (TC), which is the sum of the production setup cost, inventory carrying cost and stock-out cost for all items within the length of the simulation run.

2) Schedule Instability or Nervousness (SI), measured by the following equation:

$$SI = \frac{\sum_{i=1}^{n}\sum_{k>1}\sum_{t=M_k}^{M_k+N-1} |Q_{ti}^{k} - Q_{ti}^{k-1}|}{S}$$

where, i= item index

n= total number of items in the MRP structure

t= time period

k= planning cycle

Q_{ti}^{k} = scheduled order quantity for item i in period t during planning cycle k

Q_{ti}^{k-1} = scheduled order quantity for item i in period t during planning cycle k-1

M_k= beginning period of planning cycle k

N= planning-horizon length

S= total number of orders in all planning cycles

A similar formula was used by Sridharan et al. (1988) to measure MPS instability in single level systems.

3). Service Level (SL), which is the percentage of end item demands that are satisfied.

The values of the dependent variables are computed for each combination of the independent variables. For each combination of the independent variables, five runs are made to reduce the random effects. The data will be analyzed using Analysis of Variance (ANOVA) procedure to test the hypotheses presented in the next section.

3. Research Hypotheses

Two general hypotheses are tested in this study. The first hypothesis is on the impact of the lot-sizing rules on the performance of the MRP systems. The second hypothesis is concerned with the impact of the lot-sizing rules on the selection of the MPS freezing parameters.

Hypothesis 1:

Lot sizing rule selection will significantly influence the performance of MRP system. The sophisticated lot-sizing rules such as Wagner-Whitin rule(WW), cost-modified Wagner-Whitin rule (MWW) and improved STIL heuristic rule proposed by Coleman and McKnew may not significantly outperform the simple rules under most conditions.

Hypothesis 2:

The interaction effects between the lot-sizing rules and the parameters for freezing the MPS significantly influence the performance of MRP systems. The selection of lot sizing rules significantly influences the selection of parameters for freezing the MPS and vise versa.

4. Results

In order to test the hypotheses above, the output from the simulation program was analyzed using the analysis of variance (ANOVA) procedure and Duncan's multiple range tests. The results are summarized as follows.

4.1. The impact of the lot-sizing rule selection on the performance of MRP systems.

The ANOVA results show that the lot-sizing rule selection has a significant impact on the total cost (TC), schedule instability (SI) and service level (SL). The relative performance of the lot-sizing rules is ranked according to the three performance measures using Duncan's Multiple Range test under different operating conditions. These conditions are represented by 32 settings with different combinations of the independent variables (MS=MRP1, MRP2; DP=HV, LV; T=4, 8;

PH=4, 8; FP=0.5, 1.0). Because of the space limitations, the ANOVA results and the performance rankings of the lot-sizing rules under each of these are not presented here. They are available upon request from the corresponding author. The result in these tables shows that the selection of lot-sizing rules significantly influence the total cost and the schedule instability, but does not result in significantly different service level. The service level is, however, significantly influenced by the demand pattern, length of the planning horizon, the freezing proportion, and the natural ordering cycle of the end item. The performance rankings of the ten lot-sizing rules according to total cost and schedule instability are significantly influenced by the demand patterns (DP), the natural ordering cycles of the end item (T), the MRP structure (MS) and the freezing parameters such as freezing proportion (FP) and planning horizon (PH).

To examine the overall performance of the lot-sizing rules, we summarized the performance results according to total cost and schedule instability in Table 5. Since lot-sizing rule selection does not make a significant difference in service level, we have not included service level in Table 5. To compare the computation efficiency of the lot-sizing rules, mean computation times (in milliseconds) required to finish a simulation run using the ten lot-sizing rules are also reported in Table 5.

The performance measures in Table 5 include the relative average total cost (RATC), relative average schedule instability (RASI) across all levels of other independent variables and the corresponding performance ranking using the Duncan's multiple range test (RANK). The relative average total cost and relative average schedule instability are calculated using the average total costs or schedule instabilities during the length of the simulation run (346 periods) across all levels of other independent variables. The relative cost or schedule instability figures are calculated by dividing the lowest average cost or schedule instability (the cost or instability using the best performing lot-sizing rule) into the average costs or schedule instability using a specific lot-sizing rule. Table 5 also reports the number of times that the lot-sizing rule produced the lowest total cost (#BEST) and the maximum relative total cost (MRTC) among the 32 settings. Similar performance measures are also shown for schedule instability.

To help us compare the overall performance of the lot-sizing rules, we plotted the relative average total cost (RATC) and relative average schedule instability (RASI) for different lot-sizing rules in Figure 4. From Figure 4, we can see that the MWW, STIL, MSM/LFL, MSM and MPPB/LFL rules are ranked as the top five according to both average total cost and average schedule instability. The MWW rule is the best according to average total cost and the STIL rule is best according to average schedule instability. The non-modified WW rule is the second worst according to both average cost and schedule instability.

Figure 5 displays the number of times that each lot-sizing rule achieved the lowest relative total cost and the number of times that it achieved the lowest schedule instability among all 32 settings. The MWW results in the highest number of best performance (28 out of 32) among all lot-sizing rules when total cost is the performance criterion, followed by MSM/LFL (26 out of 32), MSM and MPPB/LFL (24 out of 32), and the STIL (16 out of 32). The WW rule results in the lowest number of best performances (3 out of 32) according to total cost. When the schedule instability is used as the performance criterion, the STIL results in the highest number of best performance (25 out of 32), followed by MSM (5 out of 32), MSM/LFL (4 out of 32), POQ/LFL (3 out of 32) and MWW (2 out of 32). Again, the MWW, STIL, MSM/LFL and MSM are ranked among the top five according to both total cost and schedule instability, but MPPB/LFL did not result in lowest schedule instability in any case. The WW rule had the lowest number of best performance among all lot-sizing rules.

The highest relative total cost and relative schedule instability among all 32 cases are plotted against the lot-sizing rules in Figure 6. The relative total costs and schedule instability in Figure 6 represent the worst case performance for each of the lot-sizing rules. In the worst case, MSM/LFL, MSM and MPPB/LFL only produced 1% higher total cost than the best lot-sizing rule and thus have the best worst case performance among all lot-sizing rules. Both the MWW and STIL rules result in 6% higher total cost than the lowest total cost in the worst case. WW is the worst performer with 9% higher total cost than the best. For schedule instability, the STIL heuristic rule has the best worst case performance with 56% higher schedule instability than lowest instability in the worst case, followed by the MSM/LFL (66% higher), MSM (67% higher) and MPPB/LFL (80% higher). In the worst case, MWW and WW rules result in 90% and 135% higher schedule instability than the best performing rule respectively. According to the worst case performance, STIL, MSM/LFL, MSM, MWW and MPPB/LFL are all ranked among the top five

Overall the results in figures 4 through 6 indicate that the MWW, MSM/LFL, MSM and STIL rules are all ranked among the top five according to all three performance criteria: average performance, the number of times of best performance and the best worst case performance. It is very interesting to note that the simple MSM and MSM/LFL rules perform very well in comparison with the more sophisticated MWW and the STIL rules. Although the STIL heuristic is significantly better than MSM and MSM/LFL in terms of its ability to minimize schedule instability, its ability to reduce cost is often worse than that of the MSM and MSM/LFL. The MWW rule is consistently worse than the MSM and MSM/LFL rules in terms of reducing schedule instability even though it often results in lower total cost. Considering both total cost and schedule instability, the performance of the MSM and the MSM/LFL rules is not worse than the STIL and MWW rules. This result supports the first research hypothesis.

Comparison of the CPU times in Table 5 shows that MSM/LFL requires the shortest time to complete the simulation run (45.59 ms) among the four best rules, followed by the MSM rule (50.55 ms). The MWW rule requires 136.02 milliseconds, which is almost three times of the time required for the MSM/LFL rule. The STIL heuristic rule requires substantially lower CPU time than the MWW rule, but still significantly higher computation time (43% higher) than the MSM/LFL. Considering the computation time, apparently the MSM/LFL rule is superior to the MWW and STIL heuristic rules.

4.2. Interaction effect between the lot-sizing rules and the MPS freezing parameters.

The ANOVA result shows that the interactions between lot-sizing rules and the MPS freezing parameters significantly influence the total cost and the schedule instability. Therefore, the performance of different MPS freezing parameters is examined for some of the best performing lot-sizing rules under different operating conditions. Among the four rules (STIL, MWW, MSM/LFL, and MSM) that are ranked among the top five according to both total cost and schedule instability, the MSM rule is very similar to the MSM/LFL rule except that the MSM/LFL rule requires significantly lower computation time. Therefore we only used the STIL, MWW, and MSM/LFL rules in examining the impact of lot-sizing rules on the selection of MPS freezing parameters.

4.2.1. Performance of planning horizons

Table 6 presents the relative performance of two different planning horizons using the three best lot-sizing rules (MWW, STIL and MSM/LFL) under different conditions. When the freezing proportion is equal to 0.50, a planning horizon of 8 natural ordering cycles results in both higher total cost and schedule instability than a planning horizon of 4 natural ordering cycles under all conditions using all three lot-sizing rules. When the freezing proportion is equal to 1.00, a planning horizon of 8 natural ordering cycles results in a higher total cost, but a lower schedule instability than that of 4 natural ordering cycles under all conditions. Therefore, the selection of the planning horizon has to be based on the trade-off between total cost and schedule instability. The result in Table 6 indicates that the selection of the planning horizon is not significantly influenced by the lot-sizing rules used, but is significantly influenced by the freezing proportion. When 50% of the planning horizon is frozen, a planning horizon of 4 natural ordering cycles is better than that of 8 natural ordering cycles under all conditions. However, the selection of the planning horizon is more difficult if the freezing proportion is 100%. The trade-off between total costs and schedule instability has to be made in selecting the best freezing planning horizon.

4.2.2. Performance of freezing proportions

The relative performance of the two freezing proportions using the three lot-sizing rules (MWW, STIL and MSM/LFL) under different conditions is presented in Table 7. Results in the table indicate that a freezing proportion of 1.0 always results in a higher total cost and a lower schedule instability than a freezing proportion of 0.50 using all three lot-sizing rules under all conditions. Therefore the selection of the freezing proportion has to be based on the trade-off between total cost and schedule instability regardless of the lot-sizing rules used. Therefore the selection of the lot-sizing rule does not influence the selection of the freezing proportion.

In term of the magnitudes of differences in total costs and schedule instability between the two freezing proportions, Table 7 indicates that the demand patterns (DP), natural ordering cycle

(T), and the length of the planning horizon (PH) all significantly influence the difference in total cost and schedule instability between the two freezing proportions. In general, we can say the increases in the amplitudes of the demand variation, the natural ordering cycle, and the planning horizon length all increase the difference in both total cost and schedule instability between the two freezing proportions, thus making the selection of the freezing proportion more important for improving the performance of the system.

Although the results of the ANOVA indicate that the interaction between lot-sizing rules and the MPS freezing parameters significantly influence the system performance, the results in Tables 6 and 7 indicate the selection of lot-sizing rules will not influence the selection of the freezing proportion when the three best lot-sizing rules are used.

5. Discussions and conclusions

This study evaluated the performance of the multi-level heuristic rule proposed by Coleman and McKnew (1991) against the Wagner-Whitin (WW) rule, the cost-modified Wagner-Whitin (MWW) rule, and some simple lot-sizing rules found to perform well by Zhao et al (1995). The experimental settings of this simulation study include forecasting, master production scheduling and material requirements planning under a rolling time horizon. The master production schedule is frozen under a rolling horizon to reduce schedule instability. The impact of the lot-sizing rules selection on the selection of the MPS freezing parameters is also examined. The study has the following significant findings:

1. Lot-sizing rule selection significantly influences total cost and schedule instability in multilevel MRP systems, but does not significantly influence the service level. The STIL heuristic rule significantly outperforms the other rules in terms of reducing schedule instability under most conditions. However, it does not outperform the simple cost-modified Silver-Meal rule and the Silver-Meal/lot-for-lot rule in terms of total cost. Actually the cost modified Silver-Meal rule and the Silver-Meal/lot-for-lot rule have better and more stable performance according to total cost than the STIL heuristic rule. The cost-modified Silver-Meal rule also requires a significantly lower computation time than the STIL heuristic rule. The Wagner-Whitin rule, which is popularly used by researchers, performs poorly according to both total cost and schedule

instability in multi-level MRP system under rolling horizon. However, the cost modification procedure proposed by Blackburn and Millen (1982a) significantly improves its performance. However, the computation time needed using the MWW rule is much higher than that using the STIL, MSM, and MSM/LFL. Considering the computation time, the ease of use, the total cost and schedule instability, the MSM/LFL and MSM seem to be the best lot-sizing rules.

2. The length of the planning horizon and the freezing proportion both significantly influence the total cost and schedule instability. Although the interaction effects between the lot-sizing rules and MPS freezing parameters are significant, the selection of the planning horizon and the freezing proportion does not seem to be significantly influenced by the selection of the lot-sizing rules when MWW, STIL and MSM/LFL are used as the lot-sizing rules. In the experimental settings of this study, a freezing proportion of 1.00 always results in a higher total cost, but lower schedule instability than a freezing proportion of 0.50. Therefore the selection of the freezing proportion has to be made based on the needs of the company to reduce schedule instability or total cost. If the firm selects a freezing proportion of 0.50, then a planning horizon of 4 natural ordering cycles performs better than a planning horizon of 8 natural ordering cycles according to both total cost and schedule instability. However, if a freezing proportion of 1.00 is selected, the choice of the planning horizon has to be made based on the trade-off between total cost and the schedule instability because a planning horizon of four natural ordering cycles results in a lower total cost, but a higher schedule instability.

The findings from this study clearly indicate that many sophisticated lot-sizing rules may not improve system performance when some practical factors such as forecasting inaccuracy, the need to plan under a rolling time horizon, and to reduce schedule instability are considered. The study also shows that the parameters for freezing the master production schedule need to be properly selected in order to improve the performance of the system. The findings from this study will enhance the understanding of the performance impact of lot-sizing rules and the parameters for freezing the master production schedule in multilevel MRP systems under demand uncertainty. The guidelines provided in this study will help practitioners select the lot-sizing rules and freezing parameters to improve the performance of MRP systems in a make-to-stock environment.

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Table 1
Characteristics of the demand data

DP\PARAMETERS	A_0	T_{mag}	S_{mag}	P	N_{mag}	Average	Std. Dev.
Low Variation (LV)	763.3	1	100	7	50	1000.06	133.84
High Variation (HV)	537.3	2	200	7	100	999.91	267.23

Table 2
Parameters Used in the Forecasting Models

DP\FM	Moving Average	Winters'		
	N	α	β	γ
LV	1	0.05	0.10	0.10
HV	6	0.05	0.05	0.10

Table 3 Independent Variables of the Experimental Design

Variable Number	Variable Name	Label	Number of Levels	Values
Forecasting Error Variables				
1	Demand Pattern	DP	2	Low Variation (LV) High Variation (HV)
2	Forecasting Models	FM	2	Moving Average (MA) Winters' Model (WM)
Parameters for Freezing the Master Production Schedule				
3	Planning Horizon	PH	2	4 and 8 natural ordering cycles
4	Freezing Proportion	FP	2	0.50 and 1.0
5	Freezing Method	ZM	2	order-based and period-based methods
6	Replanning Periodicity	RP	1	the entire frozen interval 1.00*FP*PH*T
MRP Operating Parameters				
7	MRP Structure	MS	2	MRP1, MRP2
8	Cost Parameters	T	2	4 and 8 natural ordering cycles respectively
9	Lot Sizing Rules	LSR[1]	10	MSM; MSM/LFL; SM/LFL; SM; WW; MWW; PPB/LFL; POQ/LFL; MPPB/LFL; STIL

Note:

1. The abbreviations of the lot-sizing rules are:

MSM: Cost-Modified Silver-Meal; MSM/LFL: Cost-Modified Silver-Meal/Lot-For-Lot;
SM: Silver-Meal; SM/LFL: Silver-Meal/Lot-For-Lot ;
WW: Wagner-Within; MWW: Cost-modified Wagner-Whitin
PPB/LFL: Part-Period Balancing/Lot-For-Lot; POQ/LFL: Periodic Ordering Quantity/Lot-for-lot;
MPPB/LFL: Cost Modified Part-Period Balancing/Lot-For-Lot;
STIL: The multi-level heuristic rule proposed by Coleman and McKnew (1991)

Table 4
Cost Parameters

Item	Item 1	Item 2	Item 3	Item 4	Item 5
Echelon Holding Cost ($/unit/period)					
Set #1, T=2	1.00	0.50	1.00	2.00	0.10
Set #2, T=4	1.00	0.50	1.00	2.00	0.10
Production Setup Cost($/setup)					
Set #1, T=4	8000	4000	18000	4000	1800
Set #2, T=8	32000	4000	18000	4000	1800
Shortage Cost for the End Item: (1.0+0.5+1.0+2.0+0.1)*10*5=$230/unit					

Table 5 Summary of Lot-sizing Rules Performance

LSR	Total Cost				Schedule Instability				Computation Time (ms)[5]	
	RATC[1]	rank[1]	#best[3]	MRTC[4]	RASI[2]	Rank[2]	#best[3]	MRSI[4]	mean	rank
MPPB/LFL	102	3	24	101	118	4	0	180	44.88	2
MSM	101	2	24	101	106	3	5	167	50.55	7
MSM/LFL	103	4	26	101	105	2	4	166	45.59	5
MWW	100	1	28	106	118	4	2	190	136.02	9
POQ/LFL	105	6	16	106	133	7	3	234	44.10	1
PPB/LFL	105	6	12	107	155	9	0	263	45.08	3
SM	103	4	14	105	131	6	0	231	50.04	6
SM/LFL	105	6	13	105	130	5	0	231	45.47	4
STIL	102	3	16	106	100	1	25	156	65.08	8
WW	104	5	3	109	142	8	0	235	181.45	10

Notes:

1. RATC refers to the relative average total cost. The average total cost is the arithmetic mean of the total cost for all simulation runs using the lot-sizing rule. The lot-sizing rule that results in the lowest average total cost is used as the benchmark lot-sizing rule. The relative average total cost using a specific lot-sizing rule is calculated by dividing the average total cost for this lot-sizing rule by the average total cost using the benchmark lot-sizing rule. The rank indicates the relative ranking of the lot-sizing rule according to the average total cost.
2. RASI refers to the relative average schedule instability. The average schedule instability is the arithmetic mean of the schedule instability for all simulation runs using the lot-sizing rule. The lot-sizing rule that results in the lowest average schedule instability is used as the benchmark lot-sizing rule. The relative schedule instability using a specific lot-sizing rule is calculated by dividing the average schedule instability for this lot-sizing rule by the average schedule instability using the benchmark lot-sizing rule. The rank indicates the relative ranking of the lot-sizing rule according to the average schedule instability.
3. # best is the number of times that a specific lot-sizing rule achieves the lowest total cost or schedule instability out of the 32 settings.
4. RMTC and RMSI are the maximum relative total cost and maximum relative schedule instability respectively. It indicates the worst case performance of the lot-sizing rule relative to the best performing rule.
5. The computation time is the mean of the CPU times (in milliseconds) required to complete a simulation run using a specific lot-sizing rule. The rank indicates the relative ranking of the lot-sizing rule among the ten rules used according to the mean computation time.

Table 6. Relative Performance of Planning Horizon Using Different Lot-sizing Rules

MS	LSR (PH[4])	DP=HV[2]								DP=LV[2]							
		T=4[3]				T=8[3]				T=4[3]				T=8[3]			
		FP=0.5[5]		FP=1[5]		FP=0.5[5]		FP=1[5]		FP=0.5[5]		FP=1[5]		FP=0.5[5]		FP=1[5]	
		RTC[1]	RSI[1]	RTC[1]	RSI[1]	RTC[1]	RSI[1]	RTC[1]	RSI[1]	RTC[1]	RSI[1]	RTC[1]	RSI[1]	RTC[1]	RSI[1]	RTC[1]	RSI[1]
MRP1	MWW(PH=4)	100	100	100	106	100	100	100	108	100	100	100	107	100	100	100	111
	MWW(PH=8)	125	108	141	100	130	102	100	100	120	108	136	100	132	102	114	100
	STIL(PH=4)	100	100	100	106	100	100	100	109	100	100	100	105	100	100	100	110
	STIL(PH=8)	126	120	140	100	129	103	102	100	121	112	136	100	129	103	118	100
	MSM/LFL(PH=4)	100	100	100	100	100	100	100	103	100	100	100	105	100	100	100	106
	MSM/LFL(PH=4)	125	103	139	99	130	106	102	100	122	108	136	100	132	106	102	100
MRP2	MWW(PH=4)	100	100	100	106	100	100	100	110	100	100	100	106	100	100	100	110
	MWW(PH=8)	125	106	141	100	130	104	102	100	121	106	137	100	131	105	102	100
	STIL(PH=4)	100	100	100	102	100	100	100	106	100	100	100	101	100	100	100	103
	STIL(PH=8)	126	111	141	100	129	101	102	100	121	105	137	100	131	102	102	100
	MSM/LFL(PH=4)	100	100	100	100	100	100	100	105	100	100	100	100	100	100	100	105
	MSM/LFL(PH=4)	125	109	139	100	130	107	102	100	121	100	134	100	131	104	101	100

Notes:

1. RTC and RSI represent relative total cost and relative schedule instability respectively. They are calculated by dividing the lowest total cost (cost for the benchmark freezing proportion) or the lowest instability (instability for the benchmark freezing proportion) under that specific condition into the total cost or schedule instability using a specific freezing proportion.
2. DP=HV, LV represents the high variation and low variation demand respectively.
3. T=4 and 8 represents the natural ordering cycle for the end item to be 4 and 8 periods respectively.
4. PH=4 and 8 represents the planning horizons to be 4 and 8 natural ordering cycles respectively.
5. FP=0.5 and 1 represents the freezing proportion to be 50% and 100% of the planning horizons respectively.

Table 7. Relative Performance of Freezing Proportions Using Different Lot-sizing Rules

		DP=HV[2]								DP=LV[2]							
		T=4[3]				T=8[3]				T=4[3]				T=8[3]			
		PH=4[4]		PH=8[4]		PH=4[4]		PH=8[4]		PH=4[4]		PH=8[4]		PH=4[4]		PH=8[4]	
MS	LSR (FP[5])	RTC[1]	RSI[1]	RTC[1]	RSI[1]	RTC[1]	RSI[1]	RTC[1]	RSI[1]	RTC[1]	RSI[1]	RTC[1]	RSI[1]	RTC[1]	RSI[1]	RTC[1]	RSI[1]
MRP1	MWW(FP=0.5)	100	144	100	164	100	188	100	162	100	108	100	125	100	188	100	212
	MWW(FP=1)	126	100	141	100	139	100	107	100	124	100	138	100	118	100	102	100
	STIL(FP=0.5)	100	157	100	201	100	190	100	213	100	164	100	193	100	165	100	185
	STIL(FP=1)	126	100	141	100	134	100	109	100	122	100	138	100	116	100	102	100
	MSM/LFL(FP=0.5)	100	147	100	152	100	192	100	210	100	124	100	141	100	193	100	218
	MSM/LFL(FP=1)	127	00	141	100	136	100	107	100	123	100	138	100	132	100	102	100
MRP2	MWW(FP=0.5)	100	152	100	172	100	193	100	220	100	159	100	174	100	185	100	218
	MWW(FP=1)	126	100	142	100	136	100	107	100	122	100	138	100	131	100	102	100
	STIL(FP=0.5)	100	152	100	172	100	195	100	209	100	158	100	176	100	187	100	197
	STIL(FP=1)	126	100	142	100	135	100	107	100	122	100	138	100	131	100	102	100
	MSM/LFL(FP=0.5)	100	159	100	174	100	192	100	215	100	166	100	165	100	194	100	212
	MSM/LFL(FP=1)	127	100	141	100	136	100	107	100	124	100	137	100	132	100	102	100

Notes:

1. RTC and RSI represent relative total cost and relative schedule instability respectively. They are calculated by dividing the lowest total cost (cost for the benchmark freezing proportion) or the lowest instability (instability for the benchmark freezing proportion) under that specific condition into the total cost or schedule instability using a specific freezing proportion.
2. DP=HV, LV represents the high variation and low variation demands respectively.
3. T=4 and 8 represents the natural ordering cycle for the end item to be 4 and 8 periods respectively.
4. PH=4 and 8 represents the planning horizons to be 4 and 8 natural ordering cycles respectively.
5. FP=0.5 and 1 represents the freezing proportion to be 50% and 100% of the planning horizons respectively.

Figure 1

Demonstration of MPS freezing parameters

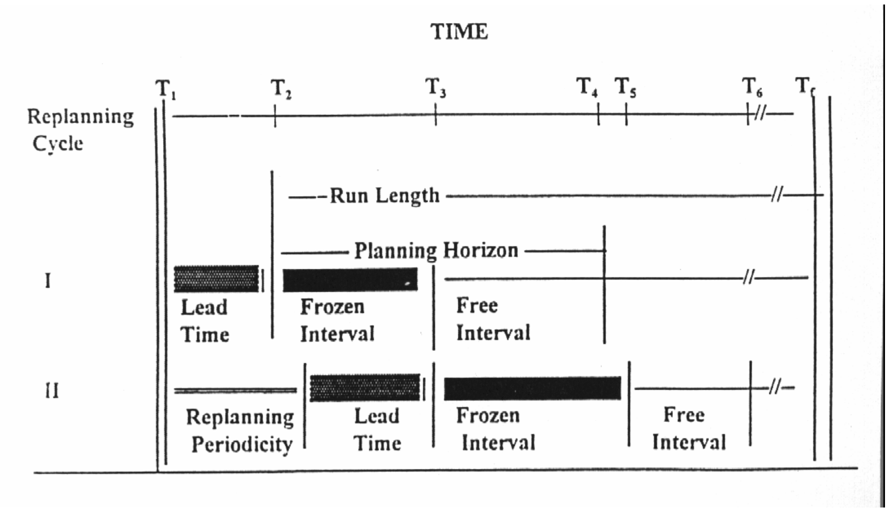


Figure 2
Simulation Procedure

Step A:	Select a forecasting model(FM), a demand pattern(DP), a cost parameter set (T), a planning horizon(PH), a freezing proportion(FP), a method to freeze MRP schedule (ZM), a replanning periodicity(RP), and a lot-sizing rule (LSR) to decide the lot sizes, then go to step B.
Step B:	Calculate the starting period (LS) and the finishing period (LF) for lot-sizing decisions; Develop MPS using lot-sizing rule for periods between LS and LF; Use the standard MRP logic and the lot-sizing rule to develop MRP schedules for all other items; Implement the MRP schedules within the frozen interval, calculate the ending inventories, and update performance measures, then go to step C.
Step C:	If the end of the simulation has not been reached, roll the schedule RP periods ahead and go to Step B. Otherwise, record the performance measures and go to Step D.
Step D:	If all the combinations of different FM, DP, T, PH, ZM, FP, RP, and LSR have been exhausted, stop; Otherwise, go to step A and select at least one different value of FM, DP, T, PH, ZM, FP, RP, or LSR.

Figure 3
MRP Structure

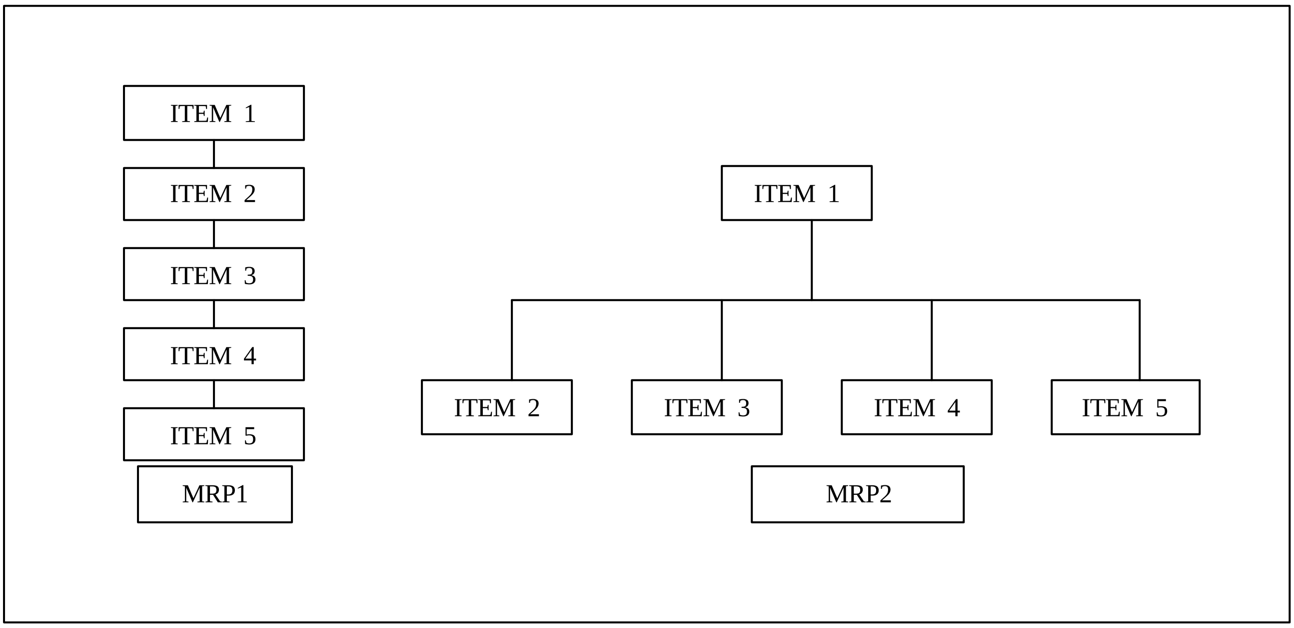
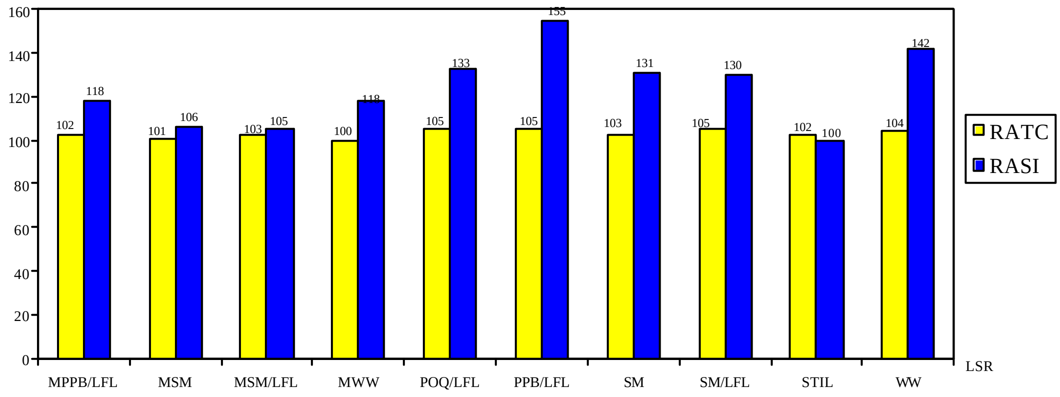
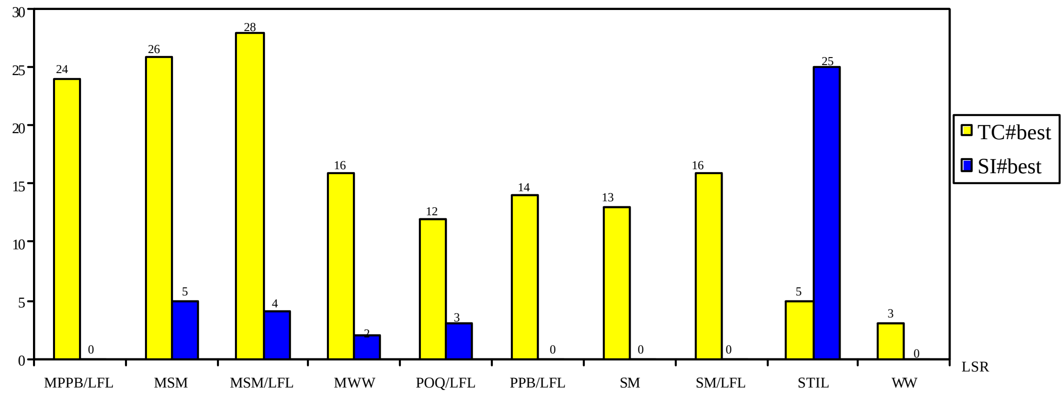


Figure 4 : Relative average total cost and schedule instability



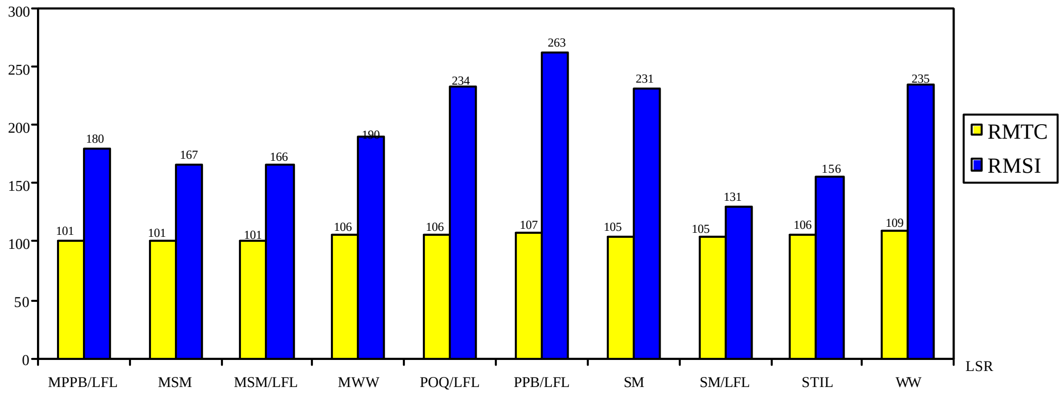
RATC ranking	3	2	4	1	6	6	4	6	3	5
RASI ranking	4	3	2	4	7	9	6	5	1	8

Figure 5 : Number of best



TC ranking	3	3	2	1	4	7	5	6	4	8
SI ranking	6	2	3	5	4	6	6	6	1	6

Figure 6 : Relative total cost or schedule instability of the worst performing case



TC ranking	1	1	1	3	3	4	2	2	3	5
SI ranking	4	3	2	5	7	9	6	6	1	8