International Perspectives on the Teaching and Learning of Mathematical Modelling

Gloria Ann Stillman
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Teaching Mathematical Modelling: Connecting to Research and Practice

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Part III Pedagogical Issues for Teaching and Learning

Chapter 14
Comparison of Mathematical Modelling Skills of Secondary and Tertiary Students

Juntao Fu and Jinxing Xie

Abstract The primary motivation of this study is to investigate whether there are any differences, and what they are, between the mathematical modelling skills of secondary and tertiary students entering a highly academic university in China. With this aim in mind, we used a multiple-choice question instrument, which has been extensively used in experimental studies in various countries, to assess students' modelling skills. Two tests, each consisting of six questions, were conducted with approximately 200 first-year students from Tsinghua University. The first test was carried out as the students entered the University, and the second test was carried out at the end of the first semester after an introductory unit in mathematical modelling. Statistical analyses were conducted to compare the differences in the results of these two tests.

1 Introduction

During the last two decades, research on mathematical modelling and applications has attracted an increasing interest in the mathematics education field (e.g., Biehler and Leiss 2010; Blum et al. 2002; Frejd and Ärlebäck 2011; Kaiser et al. 2006; Stillman et al. 2010). This increasing trend can be noted from the fact that in recent years there is much research focusing on the teaching and learning of mathematical modelling and applications, ranging across all education levels including primary, secondary, tertiary and teacher education. However, among all the research studies, it seems only a few (e.g., Kaiser 2007) focus explicitly on the comparison of mathematical modelling skills between the students at the secondary and the tertiary levels. This is a little puzzling and the reason might be due to that until now there

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are actually no roadmaps to sustained implementation of modelling education at all levels. Just as Blum et al. (2002) point out:

> In spite of a variety of existing materials, textbooks, etc., and of many arguments for the inclusion of modelling in mathematics education, why is it that the actual role of applications and mathematical modelling in everyday teaching practice is still rather marginal, for all levels of education? How can this trend be reversed to ensure that applications and mathematical modelling is integrated and preserved at all levels of mathematics education? (p. 165)

This is a big issue and it appears it remains to be resolved. The primary motivation of the current study is to investigate whether there are any differences, and what they are, between the mathematical modelling skills of secondary and tertiary students. To assess students' modelling skills, we adopted a multiple-choice question instrument which has been extensively used in experimental studies in various countries.

As an early application of the multiple-choice question instrument, Haines et al. (2000, 2001) designed two similar tests (Test 1 and Test 2, each consisting of six questions) to examine and measure students' mathematical modelling skills at different stages of a modelling process. As has been clarified in Houston and Neill (2003b), the six questions in each quiz try to identify six types of students' modelling sub-competencies, namely:

> Type 1: Making simplifying assumptions… . Type 2: Clarifying the goal… . Type 3: Formulating the problem… . Type 4: Assigning variables, parameters and constants… . Type 5: Formulating mathematical statements… . Type 6: Selecting a model. (pp. 156–158)

The work by Haines et al. (2000, 2001), including research methods and tests designed, gave an introductory reference for follow-up studies in the United Kingdom (Haines and Crouch 2001, 2005) and spreading to Australia, Ireland, Japan, Germany, Sweden and China (Dan and Xie 2011; Frejd and Ärlebäck 2011; Ikeda et al. 2007; Houston and Neill 2003a, b; Kaiser 2007; Lingefjärd and Holmquist 2005). In particular, Haines and Crouch (2001) paid more attention to the effectiveness of their research tool. They mixed these two tests into one with questions in a random order and asked 42 mathematics undergraduates to finish them within 30 min. According to their analyses, they found that scores with the same question number in these two tests are close, and the comparable manner of the analogue pairs indicates their tests are generally robust. Houston and Neill (2003b) added another question in each test to require students to sketch the graph of a function. They used Test 1 as a pre-test and Test 2 as a post-test in the first year. When it came to the second year, they created a new test comparable to the former ones and took it as a new pre-test. Test 2 was used again as a post-test at the end of the fourth semester. They found that over a 1 year course of study, students generally improved their test scores; however, there was a larger decline on the third test than the second one, indicating that most students did not have their modelling skills consolidated effectively over their summer holiday. The number of test questions was extended to 22 questions by Houston and Neill (2003b), and the issue of assessment validity was addressed in Lingefjärd and Holmquist (2005). Dan and Xie (2011) carried out a study with the students from Logistics Engineering College, an average level institution among Chinese engineering universities. Significant positive correlation between mathematical modelling skills and innovative thinking levels was found. However, correlation between mathematical modelling skills and scores achieved in basic mathematics courses was not so strong.

In the current study, we adopted the two tests (Test 1 and Test 2) originally designed by Haines et al. (2001). We used the data from the tests with students from Tsinghua University, a highly academic university in China, to check whether there are any differences between the mathematical modelling skills of secondary and tertiary students. We also examined whether the students' modelling skills are influenced by other factors such as the scores they achieved in basic mathematics courses.

2 Experiment

The tests were conducted in fall semester of 2010 with 193 first-year students of Tsinghua University. Among these, 157 were boys and the rest were girls. All the students were taking a course called Introduction to Mathematical Modelling, which is a one-semester course aiming to introduce some basic modelling concepts and examples to the first year students. The course consists of fifteen 90-min lectures, once every week. In each lecture, the teacher introduces a modelling instance with close relationship with the students' daily life, including some basic models arising from engineering, public administration, operations management, marketing and economics. Most of the instances are taken from a Chinese textbook by Jiang et al. (2011), which is the most popular one used in Chinese universities for mathematical modelling courses. Here are some examples for the modelling tasks: How do you allocate the number of seats in a committee (i.e., the apportionment problem)? How do you predict the growth of the population size in China and in the world? How much data can red and blue laser compact disks hold? What is the best design for the shape of a Cola can? What's the best time to sell a pig to the market? What is the best policy for inventory management (in particular, the classical economic ordering quantity (EOQ) model and the newsboy model)? Why and how does an airline company use an overbooking strategy? Why are prices usually subject to periodic fluctuations in certain types of markets (in particular, the cobweb model)? How do you calculate the repayment scheme for a fixed rate mortgage? How do you estimate the explosive power for the Trinity detonation with only a video of the detonation (in particular, using the dimensional analysis approach)? What are the advantages of using multi-stage rockets (and why usually are three-stage rockets used)?

During a lecture, the teacher first introduces to the students the modelling context for a specific modelling task, and asks the students to think about and try to solve the problem by themselves for about 15 min. Next the teacher asks some students to report their ideas in front of their classmates, followed by students' discussions guided by the teacher. Finally the teacher presents some well-established mathematical models for the task, pointing out the basic assumptions of the models and analysing the advantages and disadvantages of the models. Although out of the class no homework is assigned to students after each week's lecture, the students are asked to form groups (each group consists of three or four students) to independently pose, discuss and build up mathematical models to solve real world problems they are interested in. At the end of the course, each group is required to submit a course report to the teacher, presenting the out of class achievement of the group. Usually, a very large number of problems and topics from various contexts are covered in the students' reports.

Test 1 was carried out as the first year students entered the University and had just attended the first lecture of the course. Test 2 was carried out at the end of the first semester. All students finished both tests within 20 min. We used the results from the first test as a measure of secondary students' modelling skills, and the results from the second test as a measure of tertiary students' modelling skills. Questions with the same number in the two tests were quite similar and had comparable difficulty, which was proved in Haines and Crouch (2001). Take Question 6 for example.

Question 6 in Test 1. Which one of the following options most closely models the height of a sunflower while it is growing (in terms of time t)?

$$\text{A. } 1-e^{-t} \quad \text{B. } (1-t)^2 \quad \text{C. } t \quad \text{D. } t-t^2 \quad \text{E. } 1/(1+e^{-t})$$

Question 6 in Test 2. Which one of the following options most closely models the speed of a car starting from rest (in terms of time t)?

$$\text{A. } 1-e^{-t} \quad \text{B. } (1-t)^2 \quad \text{C. } t \quad \text{D. } t-t^2 \quad \text{E. } 1/(1+e^{-t})$$

Both of the questions ask the modeller to choose a reasonable model, by reflecting upon the mathematical model to be used in a particular situation and evaluating a solution or comparing the model with the real world problem. So it is reasonable to compare the scores in the two tests to check if students made any progress in modelling skills.

Together with the second test, we also asked the students to report their scores achieved at midterm examinations in Calculus and Linear Algebra, which are the two major basic mathematical courses for the first year students. This information was used to check whether modelling abilities are related to the knowledge of basic mathematics courses.

3 Results

3.1 Comparison of the Two Tests

Each question in the tests has one correct answer worth 2 points and one or several partial correct answers worth 1 point, so full marks for each test is 12. The average scores achieved by the students on the two tests are summarised in Table 14.1.

Table 14.1 Summary of Test 1 and Test 2

Question number	1	2	3	4	5	6	Overall average	Standard deviation
Test 1	1.17	1.35	1.68	1.42	1.91	1.78	9.32	1.86
Test 2	1.69	1.26	1.62	1.47	1.73	1.75	9.53	1.79
Sum	2.87	2.61	3.30	2.89	3.64	3.53	–	–

As can be seen from Table 14.1, the difference of the overall average scores of the two tests is within a gap of only 0.21 points, with almost the same standard deviations. Statistical analyses (in the statistical analyses in this chapter, we use *t*–tests with the significance level of 0.1) show that there is no significant difference between scores achieved by the students on the two tests. We also checked the number of students who performed better in Test 2 and found that 90 students achieved higher scores on Test 2 than Test 1, while 71 students' performance was worse. The difference was statistically insignificant. Therefore, we come to the conclusion that there is no statistically significant difference between the mathematical modelling skills of (upper) secondary and (first year) tertiary students. More precisely, first year students in Tsinghua University did not make any progress in mathematical modelling ability through one semester's study. It disappointed us more or less, as in an analogous study done by Kaiser (2007), the students scored 8.4 in pre-test and 9.6 in post-test (both tests consisted of eight testing questions other than six questions as in our study), which was a significant improvement. Nevertheless, a similar conclusion to our study was also made in Haines et al. (2000) with data from UK students. Some of the possible reasons we think are as follows:

- For quality teaching, "teachers ought to support students' individual modelling routes and encourage multiple solutions" (Blum 2011, p. 24); but we did not do much work along this line in our class or assign such kind of homework for our students. In fact, we put more attention on imposing the well-established models to the students in our class. This motivates us to improve our teaching methods in the future, making our teaching more effective.
- The students entering Tsinghua University are among the best students in China. They already achieved very high scores (with an overall average score of 9.32 out of 12) in the pre-test (Test 1), so it is difficult to improve further. In contrast, German students in Kaiser's study (2007) only scored 8.4 out of 16 on average in the pre-test so they had more of a gap to close.
- One semester is only a short time. It is difficult to improve the students' modelling skills in such a short time.

Since questions with the same number examine the same type of sub-competencies in the process of modelling, if we sum up the scores achieved for two questions with the same number in the two tests, we can identify advantages and disadvantages of our students in the corresponding sub-competency. It is reasonable to add the scores up directly since we have found that our students made little progress through one semester, although the two tests were finished at different times. A similar approach

Table 14.2 Relative scores of Tsinghua and UK students

	Test 1		Test 2		Tests 1 & 2	
Question	UK (%)	Tsinghua (%)	UK (%)	Tsinghua (%)	UK (%)	Tsinghua (%)
1	15.8	12.6	18.0	17.9	17.0	15.2
2	12.0	14.5	7.2	13.2	9.5	13.9
3	14.7	18.1	22.9	17.0	19.1	17.5
4	21.4	15.2	19.7	15.5	20.5	15.3
5	22.6	20.5	20.5	18.1	21.4	19.3
6	13.5	19.1	11.7	18.3	12.5	18.7
Sum	100.0	100.0	100.0	100.0	100.0	100.0

was used by Haines et al. (2000) to check students' understanding at stages within the modelling process. Haines and Crouch (2001) also followed this idea in order to develop a measure of attainment for stages of modelling. Examination of Table 14.1 shows that the students were most successful in solving Questions 3, 5 and 6 (82.6 %, 91.1 % and 88.2 % of the total marks were achieved), whilst Questions 1, 2 and 4 (only 71.6 %, 65.3 % and 72.3 % of the total marks were achieved) seemed to be more challenging for them. This observation suggests, in order to improve the students' modelling skills, we should put more attention to training students' ability in making reasonable assumptions for the real world problems, understanding the goals of the modelling tasks, and identifying required parameters, variables and constants. Among them, the problem of understanding the goals of the modelling tasks was also reported by Houston and Neill (2003a), and Kaiser (2007).

3.2 Comparison with UK Students

Now we compare the differences of the achievements of Tsinghua students (in our experiment) with those achieved by the students from two universities in UK (we refer to them as UK students hereafter) as reported by Haines and Crouch (2001). On average, Tsinghua students in our experiment performed better in each question than the UK students. Noticing that the students in Tsinghua are among the best in China, it is unreasonable and not meaningful to compare their scores directly with those of the UK students. Therefore, we decided to analyze the relative scores as shown in Table 14.2. Here the relative score achieved by students in a question means the average score achieved in this question divided by the overall average score for all test questions. For example, Tsinghua students scored 1.63 points in the third question of Test 1 on average and their overall average score is 9.32. Then the relative score in the third question of Test 1 equals 18.1 % (=1.63/9.32). If the relative score achieved by Tsinghua students in a question is much higher than that by UK students, it is reasonable to consider Tsinghua students performed better according to the sub-competency corresponding to the question in the process of modelling.

Table 14.2 shows that UK students performed relatively better in Question 4, and Tsinghua students performed relatively better in Question 6. This indicates that UK students have relatively better understanding than Tsinghua students in assigning parameters, variables and constants, whilst Tsinghua students have relatively better understanding than UK students in assessing and choosing a proper model. The differences in other questions are only marginal. In addition to this, Question 2 seems to be the most difficult one for all the students and Question 5 is the easiest one for them.

3.3 Relationship with Basic Mathematics Courses

Serious mathematical modelling needs to make use of much "pure" mathematical knowledge and skills. Questions in our tests also reflect some mathematics thinking, so it is natural to check the relationship between modelling skills and basic mathematics levels of the students. Similarly as in Dan and Xie (2011), firstly, we calculated the correlation coefficients of mathematical modelling skills and both the marks achieved by the students in their midterm examinations for two basic mathematics courses – Calculus and Linear Algebra. We found that the correlation coefficients were both less than 0.2, so no meaningful conclusion can be drawn from the data. However, we achieved a meaningful result after reanalyzing our data from another perspective as below.

We regard those who scored 12 points in Test 1 or Test 2 as skilled modellers, and those who scored no more than 6 points as having poor modelling skills. Similarly, when dealing with the total scores for the two tests, we regard 22–24 points as an indicator for strong modeling skills, and no more than 15 points as an indicator for poor modelling skills. The comparison results are listed in Table 14.3.

Examination of Table 14.3 indicates that the scores achieved by students in Calculus have no significant relationships with modelling skills in our experiment. In contrast, for the Linear Algebra course, all the p-values are less than 0.1, which leads to a positive conclusion: Students with strong modelling skills also possess strong skills in Linear Algebra and those with poor modelling skills possess relatively weak skills in Linear Algebra. The reason why this happens is not very clear and needs further investigation and discussion. One possible explanation might be that all the questions in our tests for modelling skills do not need to use knowledge in Calculus, but knowledge in Linear Algebra is relevant to the test questions we used.

4 Discussion and Conclusion

This chapter reports some initial results of comparison of the differences in modelling skills between upper secondary and first year tertiary students, from an experiment conducted at Tsinghua University in China. We recognize the difficulties that

Table 14.3 Relationship between modelling skills and mean scores on basic mathematics courses

		Number of students	Mean scores of calculus	Mean scores of linear algebra
Test 1	12 points	20	89.65	88.13
	No more than 6 points	15	87.60	82.00
	p-value	–	0.26	0.07
Test 2	12 points	18	87.50	85.61
	No more than 6 points	13	83.38	77.88
	p-value	–	0.23	0.08
Tests 1 & 2	22~24 points	22	88.23	87.20
	No more than 15 points	21	88.76	80.48
	p-value	–	0.43	0.07

students met in the process of modelling, which will help us improve our teaching approach. The comparison with UK students (Haines and Crouch 2001) clarifies the distinction of modelling competencies between Chinese and UK students, which to some extent reflects the difference between thinking patterns of the two countries' students. In addition, it needs to be noted that the tests used in our study did not examine some competencies which are necessary for a holistic way of conducting modelling processes (Kaiser 2007). So a research tool that addresses the full range of modelling skills is expected to be developed (Haines and Crouch 2001). For example, it might be reasonable to use an authentic modelling task to test students' modeling skills (e.g., Ludwig and Xu 2010).

Due to the limitations of our experiment (e.g., it was only a very short time between the pre-test and the post-test, and the students were all from a highly academic university in China), findings from the study are not very stimulating. However, the authors think it is a very interesting area to investigate whether there are any differences, and what they are, in modelling skills between students at different levels of education, since the answers to these questions can benefit the curriculum design and qualified teaching for mathematical modelling courses of students at different levels. More studies are expected to be done in the future along this line.

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