CUMCM-2001 Problems

(Note: University-level teams can choose any one from problems A and B; College-level teams can choose any one from problems C and D)

Problem A: 3-D Rebuilding for Blood Vessel

Section is used to find out the tissue and organ shape of the living things. For example, dying the sample and cut it into sections with about 1*um* thick, and then observe the shape of this section under the microscope. If the sample is continuously cut into lots of parallel sections, they can be observed in their given order. In the light of their digital images obtained by photographing and sampling, the accurate 3-D shapes of tissue and organ can be rebuilt by using computer.

Suppose that some blood vessels are regard as a kind of special pipeline, and its surface is formed by a sphere rolling when its center rolling along a curve (called center axis) such as cylinder, whose center axis is a straight line and sphere with fixed radius.

There are 100 pieces of images in succession for the parallel sections of a pipeline, which recorder the intersections between the pipeline and the sections. Image filenames in given order are 0.bmp, 1.bmp, …, 99.bmp, with same size (width 512 pixels, height 512 pixels) in BMP format. For simplification, suppose that there is just one intersection point between the center axis and every piece of section; the radius of sphere is fixed; the section distances and the pixel sizes are 1.

Let the Z-axis meet the section at right angle, and the first section be Z=0, the 100th be Z=99. The pixel coordinates in Z=z image in their given order in the file are

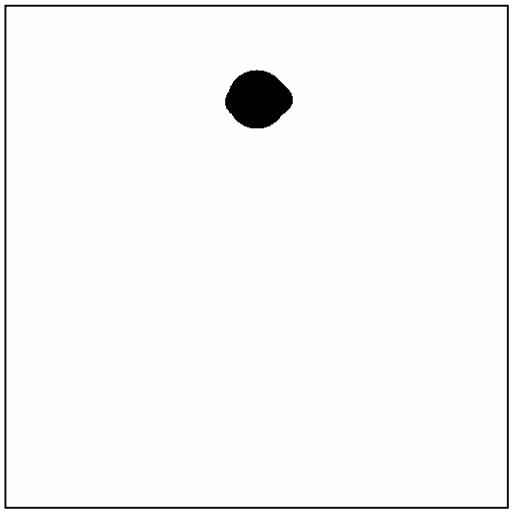
(-256, -256, z), (-256, -255, z), …(-256, 255, z),
(-255, -256, z), (-255, -255, z), …(-255, 255, z),
……
(255, -256, z), (255, -255, z), …(255, 255, z).

Try to compute the center axis and radius, give a concrete algorithm and draw the projection drawing of center axis in the XY-, YZ-, XZ-plane.

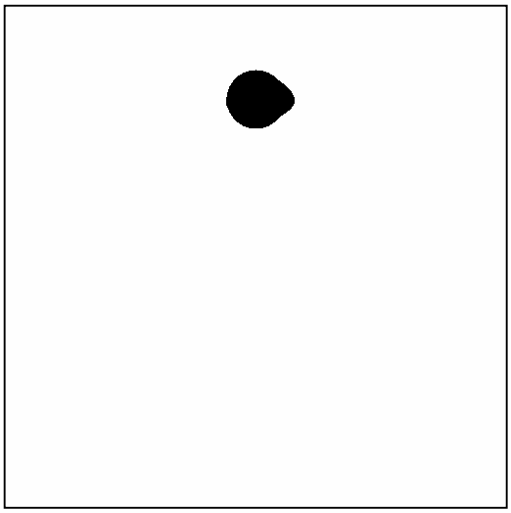
As an example, 6 pieces among 100 pieces are shown in next page. Please download all images from the web (http://mcm.edu.cn/mcm01/a01bmp.zip). About the explanations for BMP format files, please refer to any one of the following:

1. <<Visual C++ Digital Image Processing >>, Page 12, Section 2.3.1. Wrote by He Bin, etc. People's Post and Telecommunications Press, April 2001.
2. http://www.dcs.ed.ac.uk/home/mxr/gfx/2d/BMP.txt

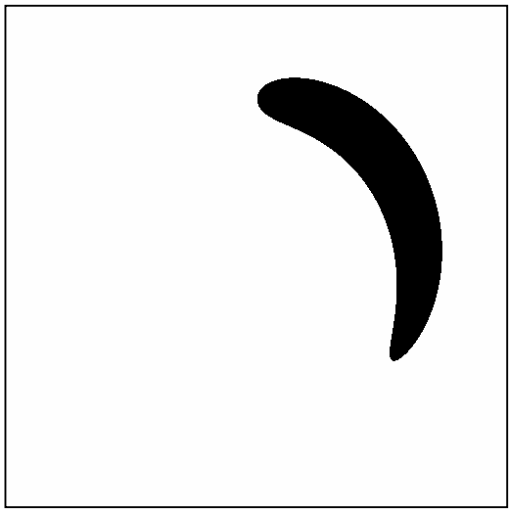
(Problem A is proposed by Prof. Wang Guo-zhao from Zhejiang University)



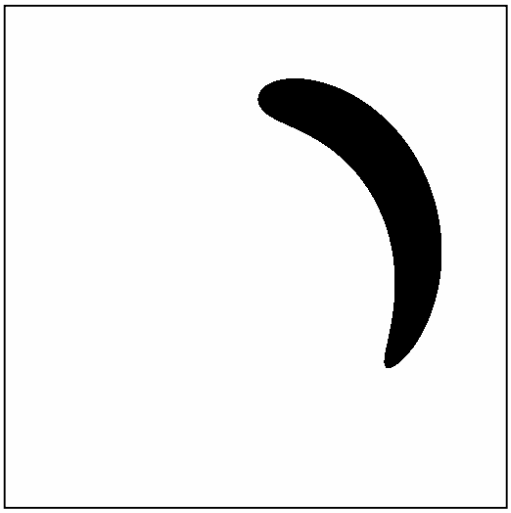
Z=0



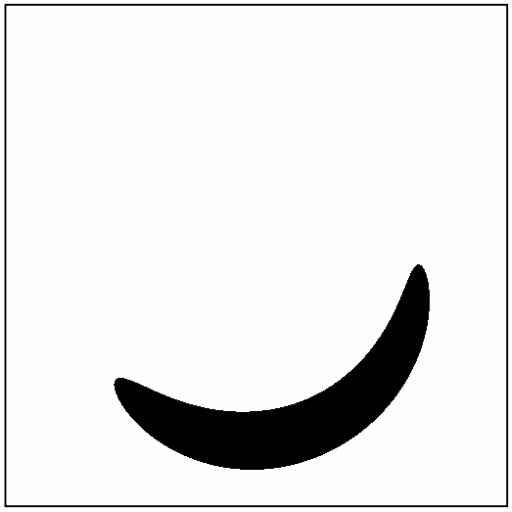
Z=1



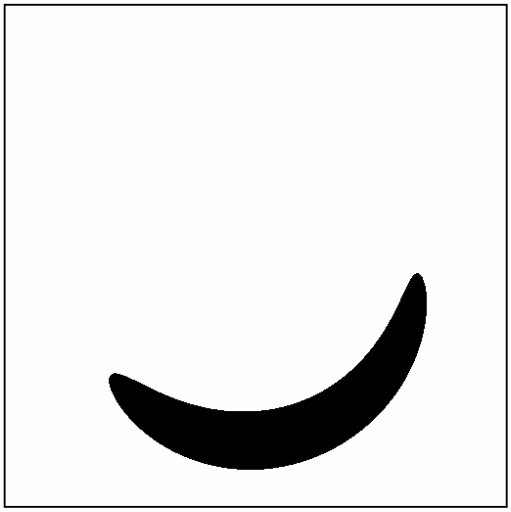
Z=49



Z=50



Z=98



Z=99

Problem B: Buses Scheduling

Public Traffic is an important component part of city traffic. To dispatch buses well is very significant for perfecting the environment of city traffic, improving people's journey, and enhancing economic returns and social effect of public traffic company. Now consider the problem of buses dispatch on a bus line, whose data come from investigation of the flow of passengers and running data on a bus line in a bid city.

In this line, there are 14 upward bus stops and 13 downward bus stops. The next two pages provide the statistics of passengers from the two running directions of bus stops in a workday. The company uses the same type bus on this line. Each bus is available for 100 passengers. According to statistics, bus's average speed is 20 km/h. Running dispatchers ask that on average the time waiting for a bus is not more than 10 minutes, 5 minutes at peak hours in morning, the rate of fully loaded is not more than 120% and not less than 50%.

According to these data, design an available bus dispatch schedule in a workday for this line and propose a method to solve this model. In light of demands for practical problem, how to collect data to better develop a schedule?

(Problem B is proposed by Prof. Tan Ze-guang from Tsinghua University)

Numbers of passengers (get-on and get-off)		Up direction: From A13 to A0													
Bus Stop Name		A13	A12	A11	A10	A9	A8	A7	A6	A5	A4	A3	A2	A1	A0
Distance Between Stops (km)			1.6	0.5	1	0.73	2.04	1.26	2.29	1	1.2	0.4	1	1.03	0.53
5:00-6:00	On	371	60	52	43	76	90	48	83	85	26	45	45	11	0
	Off	0	8	9	13	20	48	45	81	32	18	24	25	85	57
6:00-7:00	On	1990	376	333	256	589	594	315	622	510	176	308	307	68	0
	Off	0	99	105	164	239	588	542	800	407	208	300	288	921	615
7:00-8:00	On	3626	634	528	447	948	868	523	958	904	259	465	454	99	0
	Off	0	205	227	272	461	1058	1097	1793	801	469	560	636	1871	1459
8:00-9:00	On	2064	322	305	235	477	549	271	486	439	157	275	234	60	0
	Off	0	106	123	169	300	634	621	971	440	245	339	408	1132	759
9:00-10:00	On	1186	205	166	147	281	304	172	324	267	78	143	162	36	0
	Off	0	81	75	120	181	407	411	551	250	136	187	233	774	483
10:00-11:00	On	923	151	120	108	215	214	119	212	201	75	123	112	26	0
	Off	0	52	55	81	136	299	280	442	178	105	153	167	532	385
11:00-12:00	On	957	181	157	133	254	264	135	253	260	74	138	117	30	0
	Off	0	54	58	84	131	321	291	420	196	119	159	153	534	340
12:00-13:00	On	873	141	140	108	215	204	129	232	221	65	103	112	26	0
	Off	0	46	49	71	111	263	256	389	164	111	134	148	488	333
13:00-14:00	On	779	141	103	84	186	185	103	211	173	66	108	97	23	0
	Off	0	39	41	70	103	221	197	297	137	85	113	116	384	263
14:00-15:00	On	625	104	108	82	162	180	90	185	170	49	75	85	20	0
	Off	0	36	39	47	78	189	176	339	139	80	97	120	383	239
15:00-16:00	On	635	124	98	82	152	180	80	185	150	49	85	85	20	0
	Off	0	36	39	57	88	209	196	339	129	80	107	110	353	229
16:00-17:00	On	1493	299	240	199	396	404	210	428	390	120	208	197	49	0
	Off	0	80	85	135	194	450	441	731	335	157	255	251	800	557
17:00-18:00	On	2011	379	311	230	497	479	296	586	508	140	250	259	61	0
	Off	0	110	118	171	257	694	573	957	390	253	293	378	1228	793
18:00-19:00	On	691	124	107	89	167	165	108	201	194	53	93	82	22	0
	Off	0	45	48	80	108	237	231	390	150	89	131	125	428	336
19:00-20:00	On	350	64	55	46	91	85	50	88	89	27	48	47	11	0
	Off	0	22	23	34	63	116	108	196	83	48	64	66	204	139
20:00-21:00	On	304	50	43	36	72	75	40	77	60	22	38	37	9	0
	Off	0	16	17	24	38	80	84	143	59	34	46	47	160	117
21:00-22:00	On	209	37	32	26	53	55	29	47	52	16	28	27	6	0
	Off	0	14	14	21	33	78	63	125	62	30	40	41	128	92
22:00-23:00	On	19	3	3	2	5	5	3	5	5	1	3	2	1	0
	Off	0	3	3	5	8	18	17	27	12	7	9	9	32	21

Numbers of passengers (get-on and get-off) Down direction: From A0 to A13														
Bus Stop Name		A0	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13
Distance Between Stops (km)			1.56	1	0.44	1.2	0.97	2.29	1.3	2	0.73	1	0.5	1.62
5:00-6:00	On	22	3	4	2	4	4	3	3	3	1	1	0	0
	Off	0	2	1	1	6	7	7	5	3	4	2	3	9
6:00-7:00	On	795	143	167	84	151	188	109	137	130	45	53	16	0
	Off	0	70	40	40	184	205	195	147	93	109	75	108	271
7:00-8:00	On	2328	380	427	224	420	455	272	343	331	126	138	45	0
	Off	0	294	156	157	710	780	849	545	374	444	265	373	958
8:00-9:00	On	2706	374	492	224	404	532	333	345	354	120	153	46	0
	Off	0	266	158	149	756	827	856	529	367	428	237	376	1167
9:00-10:00	On	1556	204	274	125	235	308	162	203	198	76	99	27	0
	Off	0	157	100	80	410	511	498	336	199	276	136	219	556
10:00-11:00	On	902	147	183	82	155	206	120	150	143	50	59	18	0
	Off	0	103	59	59	246	346	320	191	147	185	96	154	438
11:00-12:00	On	847	130	132	67	127	150	108	104	107	41	48	15	0
	Off	0	94	48	48	199	238	256	175	122	143	68	128	346
12:00-13:00	On	706	90	118	66	105	144	92	95	88	34	40	12	0
	Off	0	70	40	40	174	215	205	127	103	119	65	98	261
13:00-14:00	On	770	97	126	59	102	133	97	102	104	36	43	13	0
	Off	0	75	43	43	166	210	209	136	90	127	60	115	309
14:00-15:00	On	839	133	156	69	130	165	101	118	120	42	49	15	0
	Off	0	84	48	48	219	238	246	155	112	153	78	118	346
15:00-16:00	On	1110	170	189	79	169	194	141	152	166	54	64	19	0
	Off	0	110	73	63	253	307	341	215	136	167	102	144	425
16:00-17:00	On	1837	260	330	146	305	404	229	277	253	95	122	34	0
	Off	0	175	96	106	459	617	549	401	266	304	162	269	784
17:00-18:00	On	3020	474	587	248	468	649	388	432	452	157	205	56	0
	Off	0	330	193	194	737	934	1016	606	416	494	278	448	1249
18:00-19:00	On	1966	350	399	204	328	471	289	335	342	122	132	40	0
	Off	0	223	129	150	635	787	690	505	304	423	246	320	1010
19:00-20:00	On	939	130	165	88	138	187	124	143	147	48	56	17	0
	Off	0	113	59	59	266	306	290	201	147	155	86	154	398
20:00-21:00	On	640	107	126	69	112	153	87	102	94	36	43	13	0
	Off	0	75	43	43	186	230	219	146	90	127	70	95	319
21:00-22:00	On	636	110	128	56	105	144	82	95	98	34	40	12	0
	Off	0	73	41	42	190	243	192	132	107	123	67	101	290
22:00-23:00	On	294	43	51	24	46	58	35	41	42	15	17	5	0
	Off	0	35	20	20	87	108	92	69	47	60	33	49	136

Problem C: Fund Plan

A school foundation has a fund of *M* yuan and wants to deposit it or buy state treasury bond. Present interest rates of bank savings and state treasury bond refer to the following table. Supposed that treasury bill issues at least one time each year and the issue time is not fixed. The policies of drawing money refer to bank present policies.

The foundation plans that each year they give awards to excellent teachers and students using the interests during n years. The bonus amounts are asked to equal each year, and at the end of nth year the original funds will be still reserved. The foundation wishes to get an optimal fund using plan to raise the bonus amount. Please you help him design fund using plan under the following conditions and give concrete results to the case *M*=50000000 (RMB Yuan); *n*=10:

1. Just deposit the funds.
2. Deposit and buy treasury bill.
3. The school will celebrate it has been found for 100 years after the funds reach. The foundation hope this year the bonus is more 20% than other years.

	Saving Interest Rate (Tax Excluded, %)	Interest Rate for Treasury Bond (%)
Saving	0.792	
Half Year	1.664	
1-year	1.800	
2-year	1.944	2.55
3-year	2.160	2.89
5-year	2.304	3.14

(Problem C is proposed by Prof. Chen En-shu and Sun Zhi-zhong from Southeast University)

Problem D:

(Same as Problem B)