

## CUMCM-2024 Problem A

### Bench Loong

Bench Loong is a traditional local cultural activity in Zhejiang and Fujian provinces. Dozens or even hundreds of benches are connected end to end, forming a meandering Bench Loong. When the Bench Loong is coiled, the head leads the way, the bodies and the tail rotate together, forming a disciform. Generally, on the premise that the Bench Loong can enter in and out freely, smaller the required area and faster the travel speed are, more aesthetic the performance is.

A certain Bench Loong consists of 223 sections, with the first section being the head, the following 221 sections being the bodies, and the last section being the tail. The length of the head is 341 cm, the length of the bodies and the tail is 220 cm. The width of each bench is 30 cm. There are two holes on each bench, with a diameter of 5.5 cm each hole. The distance between the center of each hole and the center of the nearest end of the bench is 27.5 cm (see Figure 1 and Figure 2). Two adjacent benches are connected by handles (see Figure 3).

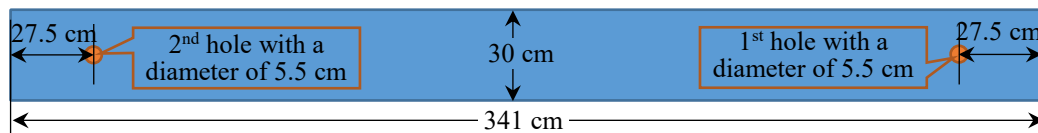


Figure 1 Top view of the head

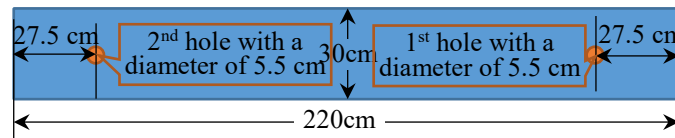


Figure 2 Top view of the body and the tail

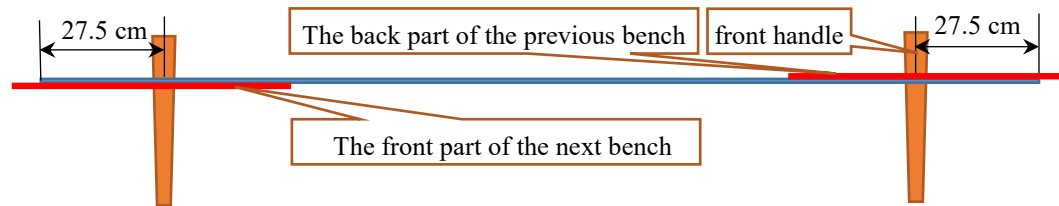


Figure 3 Front view of bench

Please establish mathematical models to solve the following problems:

**Problem 1** The Bench Loong rotates in clockwise along an equidistant spiral with a pitch of 55 cm. The center of each handle is located on the spiral. The speed of the front handle of the head keeps at 1 m/s. Initially, the head is located at point A on the 16<sup>th</sup> turn of the spiral (see Figure 4). Please give the position and speed of the Bench Loong per second from the initial moment to 300 seconds (i.e., the positions and speeds of the front handles of the head, bodies, and tail, as well as the center of the rear handle of the tail, the same below). Please save the results to the file result1.xlsx

(the templet file is in Annex. “tail (back)” refers to the back handle of the tail, and the rest are all front handles. Keep the results rounded to 6 decimal places, the same below). Meanwhile, in the paper, please give the positions and speeds of the front handles of the head and the 1<sup>st</sup>, 51<sup>st</sup>, 101<sup>st</sup>, 151<sup>st</sup>, 201<sup>st</sup> bodies as well as the rear handle of the tail at 0 s, 60 s, 120 s, 180 s, 240 s, and 300 s, respectively.

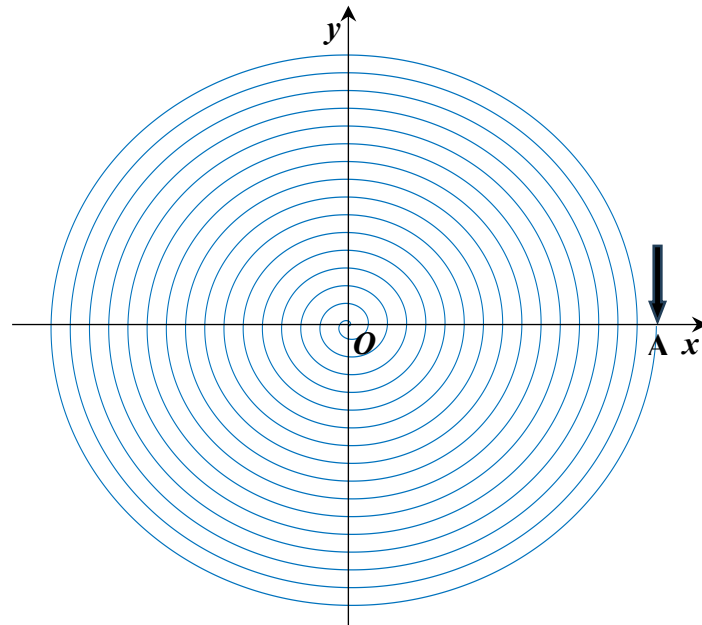


Figure 4 The spiral curve with rotating in

**Table 1 Format of the positions in the paper**

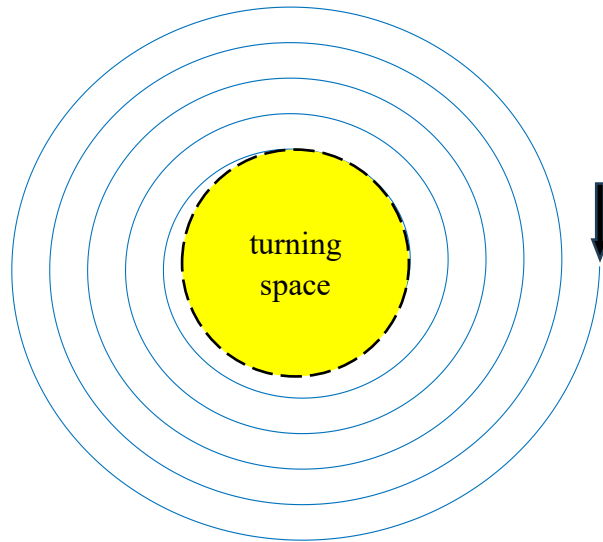
	0 s	60 s	120 s	180 s	240 s	300 s
head x (m)						
head y (m)						
1 <sup>st</sup> body x (m)						
1 <sup>st</sup> body y (m)						
51 <sup>st</sup> body x (m)						
51 <sup>st</sup> body y (m)						
101 <sup>st</sup> body x (m)						
101 <sup>st</sup> body y (m)						
151 <sup>st</sup> body x (m)						
151 <sup>st</sup> body y (m)						
201 <sup>st</sup> body x (m)						
201 <sup>st</sup> body y (m)						
tail (back) x (m)						
tail (back) y (m)						

**Table 2 Format of the speeds in the paper**

	0 s	60 s	120 s	180 s	240 s	300 s
head (m/s)						
1 <sup>st</sup> body (m/s)						
51 <sup>st</sup> body (m/s)						
101 <sup>st</sup> body (m/s)						
151 <sup>st</sup> body (m/s)						
201 <sup>st</sup> body (m/s)						
tail (back) (m/s)						

**Problem 2** The Bench Loong rotates in along the spiral curve given in Problem 1. Please determine the termination time, so that there would be no collision between the benches. Please give the position and speed of the Bench Loong at this termination time, and save the results to the file result2.xlsx (the templet file is in Annex). Meanwhile, in the paper, please give the positions and speeds of the front handles of the head and the 1<sup>st</sup>, 51<sup>st</sup>, 101<sup>st</sup>, 151<sup>st</sup>, 201<sup>st</sup> bodies as well as the rear handle of the tail.

**Problem 3** The Bench Loong will switch from clockwise turning while rotating in to counterclockwise turning while rotating out, which requires a certain turning space. If the turning space is a circular area with a diameter of 9 m centered on the center of the spiral curve (see Figure 5), please determine the minimum pitch so that the front handle of the head can reach the boundary of the turning space along the corresponding spiral curve.

**Figure 5 Turning space**

**Problem 4** The pitch of the rotating-in spiral curve is 1.7 m, the rotating-out spiral curve and rotating-in spiral curve are symmetrical about the center of the spiral curve. The Bench Loong completes a turn within the turning space in Problem 3. The turning path is an S-shaped curve formed by two tangential connection arcs. The radius of the former arc is twice that of the latter arc, and it is tangent to both the rotating-in and rotating-out spiral curves. Keeping the tangency of all parts, can the arcs be adjusted to shorten the turning curve?

The speed of the front handle of the head remains at 1 m/s. Let the time of turning around be zero moment, please give the position and speed of the Bench Loong per second from  $-100$  s to  $100$  s. Save the results to the file result4.xlsx (the templet file is in Annex). Meanwhile, in the paper, please give the positions and speeds of the front handles of the head and the 1<sup>st</sup>, 51<sup>st</sup>, 101<sup>st</sup>, 151<sup>st</sup>, 201<sup>st</sup> bodies as well as the rear handle of the tail at  $-100$  s,  $-50$  s,  $0$  s,  $50$  s, and  $100$  s, respectively.

**Problem 5** The Bench Loong travels along the path given in Problem 4, and the speed of the head remains constant. Please determine the maximum speed of the head so that the speed of each handle of the bench does not exceed 2 m/s.